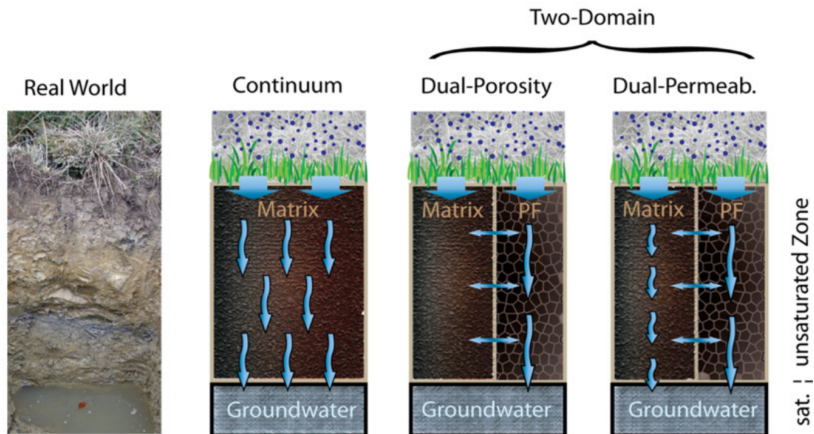


Hydrus 1D - introduction

143ESP - Soil Physics for Engineers

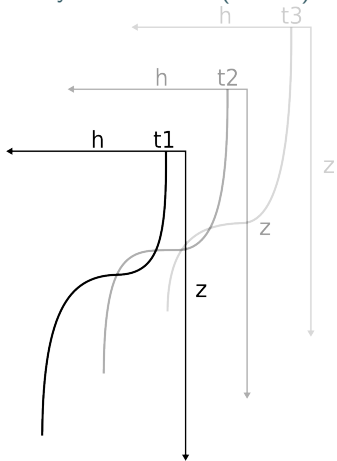
Jakub Jeřábek

1. Numerical modeling

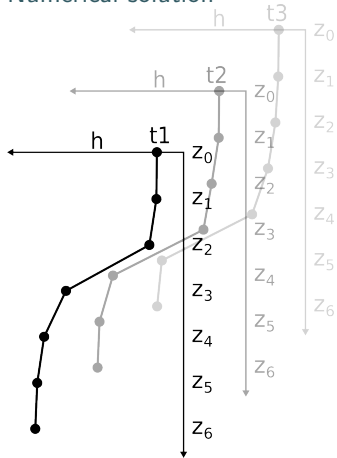


1. Numerical modeling

Analytical solution (nature)



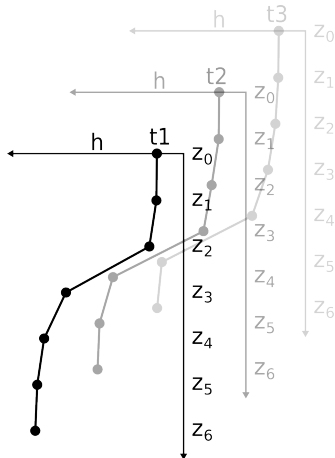
Numerical solution



1. Numerical modeling

Numerical solution

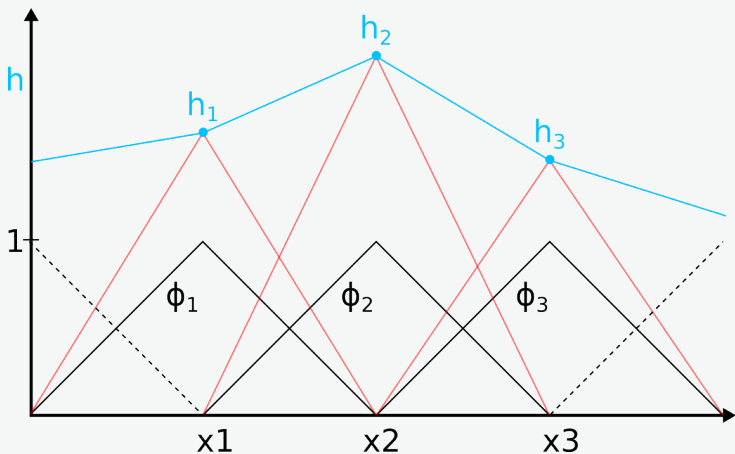
1. Partial differential equation
2. Spatial discretization leads to system of ordinary differential equations
3. Time discretization leads system of linear equations



1. Numerical modeling

Spatial discretization

Galerkin finite element method

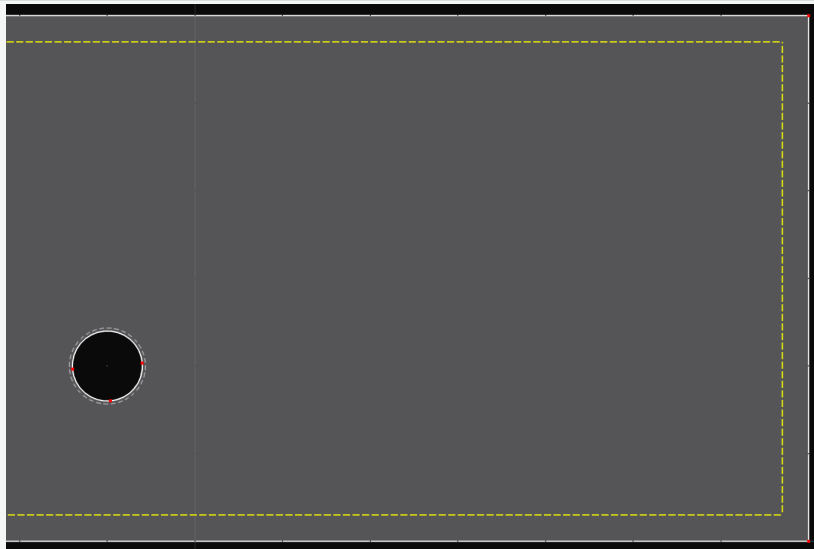


$$h(x) \approx h_1\phi_1 + h_2\phi_2 + h_3\phi_3 + h_4\phi_4$$

1. Numerical modeling

Spatial discretization

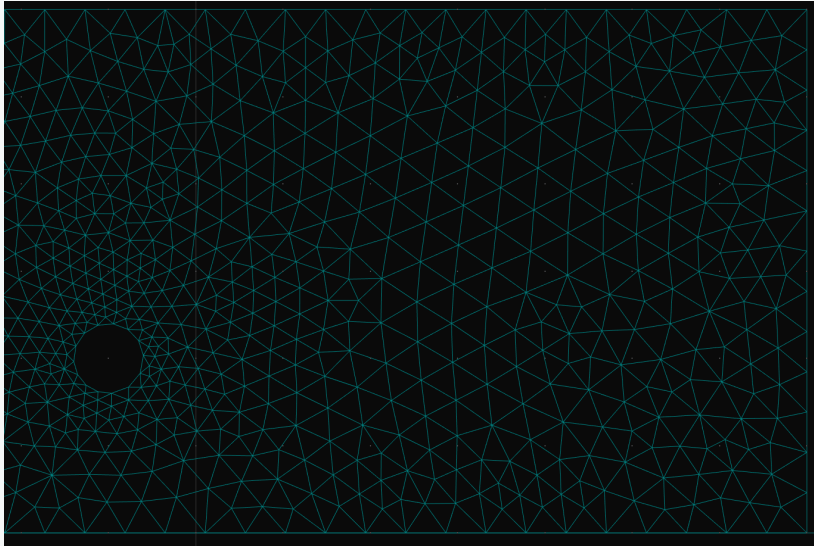
Galerkin finite element method



1. Numerical modeling

Spatial discretization

Galerkin finite element method



1. Numerical modeling

Spatial discretization

Galerkin finite element method



1. Numerical modeling

Time discretization

$$\frac{\partial h}{\partial t} = F(h)$$

Explicit method

$$\frac{h_{t+1} - h_t}{\Delta t} = F(h_t)$$

$$h_{t+1} = h_t + \Delta t F(h_t)$$

- + simple implementation
- sensitive to time step size

Implicit method

$$\frac{h_{t+1} - h_t}{\Delta t} = F(h_{t+1})$$

$$h_{t+1} - \Delta t F(h_{t+1}) = h_t$$

- + complicated implementation
- less sensitive to time step size

$$\boxed{C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left(K(h) \left(\frac{\partial h}{\partial x} + \cos(\alpha) \right) \right) - S}$$

1. Numerical modeling

1. Richards equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left(K(h) \left(\frac{\partial h}{\partial x} + \cos(\alpha) \right) \right) - S$$

2. Spatial discretization – > System of ordinary differential equations

$$- \int_{\Omega} \frac{d \sum_a \theta_a \phi_a}{dt} V_i d\Omega = - \int_{\Omega} \left(K(h) \sum_a h_a \frac{d\phi_a}{dx} \right) \frac{dV_i}{dx} d\Omega + \int_{\Omega} K(h) \cos(\alpha) \frac{dV_i}{dx} d\Omega$$

3. Time discretization – > System of linear equations

$$\begin{aligned} - \int_{\Omega} \frac{\sum_a \theta_a \phi_a - \theta_{i,pre}}{\Delta t} V_i d\Omega = & - \int_{\Omega} \left(K(h) \sum_a h_a \frac{d\phi_a}{dx} \right) \frac{dV_i}{dx} d\Omega + \\ & + \int_{\Omega} K(h) \cos(\alpha) \frac{dV_i}{dx} d\Omega \end{aligned}$$

2. Boundary conditions

Boundary conditions are equation at the edge points of the computation domain.

Dirichlet BC

$$h = h_{bc}$$

Neumann BC

$$-K \frac{\partial H}{\partial z} = q_{bc}$$

Free drainage (unit gradient)

$$\frac{\partial H}{\partial z} = \frac{\partial h}{\partial z} + \frac{\partial z}{\partial z} = 0 + 1 = 1$$

$$-K(h) = q_{bc}$$

Seepage face

$$-K \frac{\partial H}{\partial z} = 0 \quad \text{pro} \quad h < 0$$

$$h = h_{hb} \quad \text{pro} \quad h \geq 0$$

2. Boundary conditions

Boundary conditions are equation at the edge points of the computation domain.

Dirichlet BC

- Constant head infiltration
- Groundwater level GWL

Neumann BC

- Irrigation, Rainfall, ET
- Impermeable layer

Free drainage (unit gradient)

- Distant GWL

Seepage face

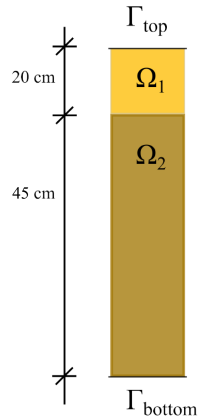
- Lower end of a soil sample
- Law of a dam

3. Task 1

Steady state

Soil profile of depth 65 *cm*. In 65 *cm* below soil surface is the ground water level. Soil surface is not covered with vegetation and is very dry. Plastic sheet is put to the soil surface, so no more evaporation takes place. The task is to estimate how long it takes until the soil water reaches the steady state conditions. Soil profile consists of two soil materials (see attachment).

- initial condition at the top of the soil profile $h = -120$ *cm*; bottom initial condition $h = ?$ *cm*. Initial conditions is linearly interpolated through the soil profile.
- top boundary (BC) condition flow = 0 *cm/den*
- lower BC pressure = ? *cm*



Steady state

Questions

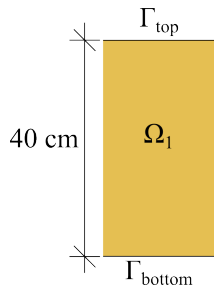
- How long it takes until the soil reaches steady state?
- How do you know?
- What is the total potential in steady state?
- What is the flow in the profile in the end of the computation?
- What will be the pressure at the lower BC if the GWL drops by 120 *cm*

4. Task 2

Constant head infiltration (in field)

Observe dynamics of infiltration experiment into soil profile. 2 cm of water are kept during the infiltration. GWL does not affect the shallow soil profile (use soil 1 from previous task). Observe dynamics of infiltration for 2 hours.

- initial condition $h = -8 \text{ cm}$ in the profile
- top BC is?
- bottom BC is?

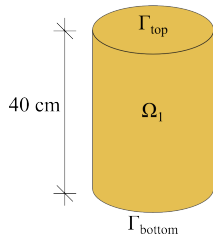


Plot the pressure (profile information) and the flow over boundaries (water flow - boundary fluxes and heads: all fluxes). how long takes the infiltration front to reach the bottom of the modeled soil material?

Constant head infiltration (in lab)

Observe dynamics of infiltration experiment into soil sample. 2 cm of water are kept during the infiltration. The water freely flow from the bottom part of the soil sample. Observe dynamics of infiltration for 2 hours.

- initial condition $h = -8$ cm in the profile
- top BC is?
- bottom BC is?



Plot the pressure (profile information) and the flow over boundaries (water flow - boundary fluxes and heads: all fluxes). how long takes the infiltration front to reach the bottom of the modeled soil material?

Constant head infiltration

Question (tasks 2 a 3)

- How differ the pressure in tasks 2 and 3?
- How differ the flow over the lower edge of the soil material in tasks 2 and 3?
- Why it differers?
- What is the difference in the total potential at the and of the infiltration experiment between tasks 2 and 3?

Soil 1

- h (cm)	Water cont.
1	0.365
10	0.232
30	0.177
58	0.149
89	0.137
500	0.119
6000	0.107

$K_s = 280 \text{ cm/day}$

Soil 2

- h (cm)	Water cont.
1	0.310
10	0.268
30	0.241
58	0.199
89	0.177
500	0.152
6000	0.137

$K_s = 65 \text{ cm/day}$