Groundwater hydraulics – lecture 3

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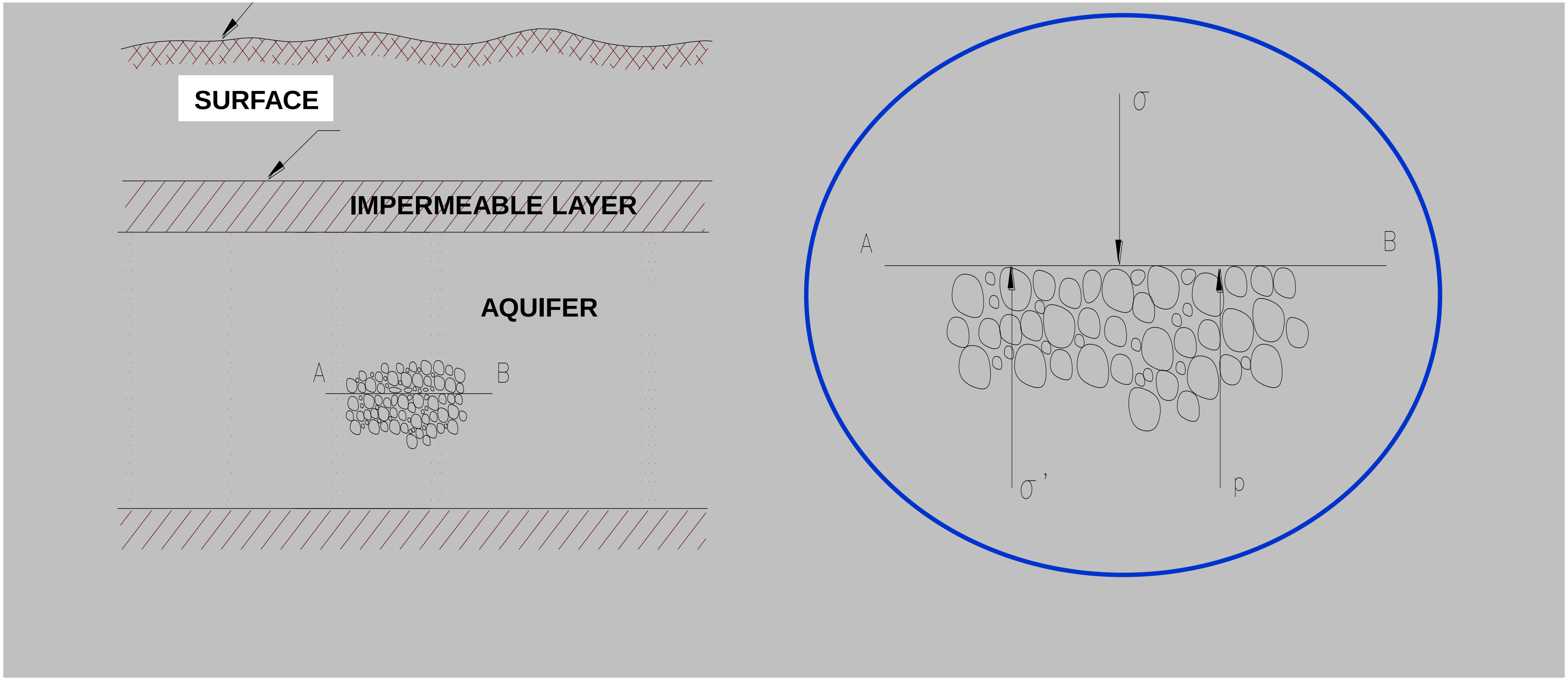
SPECIFIC STORATIVITY

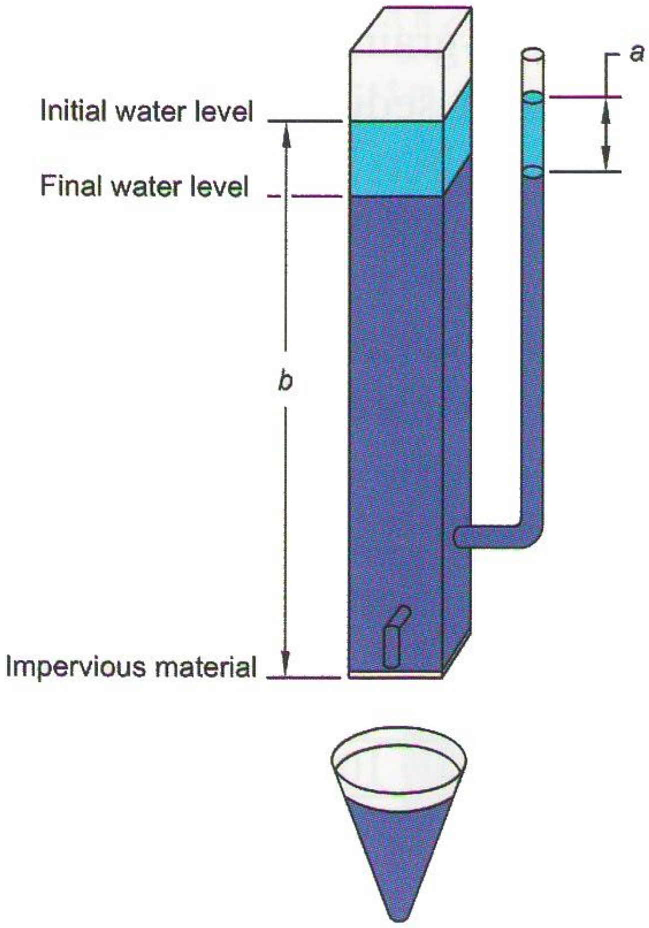
Ability of pores to hold the specific amount of water.

- *unconfined aquifer* – changes in water table – filling other pores with water
- *confined aquifer* – changes of water volume due to compressibility of porous media and water flow

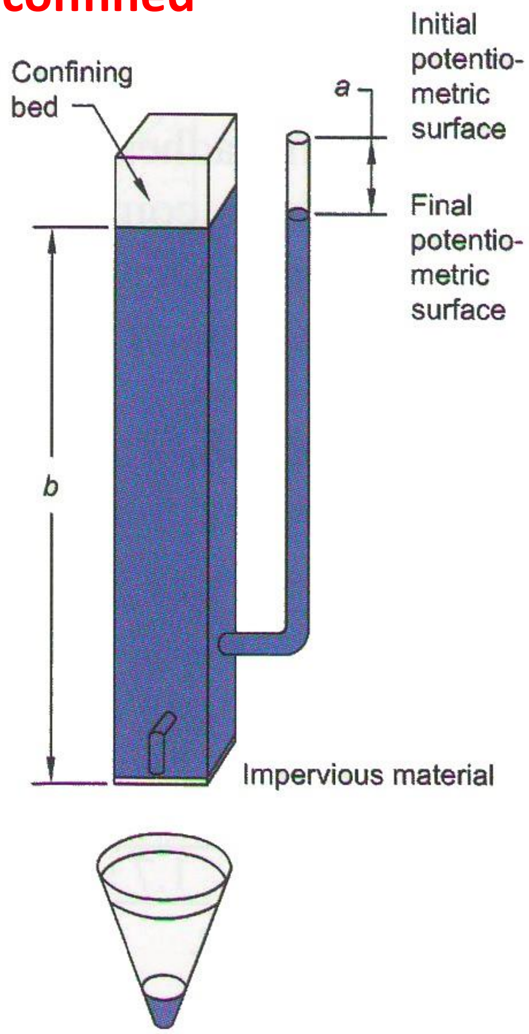


STORATIVITy – Magnitude?

unconfined



confined



Unconfined storativity >>>> confined storativity

Total tension **σ** is in equilibrium with tension among solids (intergranular tension) **σ‘** and with water pressure **p**:

intergranular tension

$$\sigma = \sigma' + p$$ water pressure

If the total tension **σ** of the surface AB is constant and the water pressure is changed e.g. due to water pumping – the change of intergranular tension **σ‘** occurs:

$$d\sigma = 0 = d\sigma' + dp\,, \quad d\sigma' = -dp$$

The change is the same with size, but with minus „**-**“.
The water pressure drop causes an increase in intergranular tension **σ** '.

COEFFICIENT OF VOLUMETRIC COMPRESSIBILITY OF WATER

$$\beta = -\frac{1}{V_v}\frac{\partial V_v}{\partial p} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

[Pa $^{-1}$]

$\beta = 4.79.10^{-10}$ Pa $^{-1}$

V_v ... water volume

ρ ... water density

p ... water pressure

COEFFICIENT OF VOLUMENTRIC COMPRESSIBILITY OF POROUS MEDIA

$$\alpha = -\frac{1}{V}\frac{\partial V}{\partial \sigma'}$$

[Pa $^{-1}$]

V ... volume of porous media

Increasing integranular tension cause structural changes of solids, replacement – changes of porosity.

Rock	*α 10^{-12} Pa $^{-1}$*
Consolidated sediments and mountain rocks	*30 - 300*
Less consolidated sediments	*200 - 1 000*
Loose sediments	*1 000 - 8 000*

Expression of coefficient **α** with porosity:

Total volume of porous media: $V = V_s + V_p$

Porosity: $n = V_p/V$

When volumetric volume of solids is constant: $V_s = constant$ then:

$$V_S = (1-n)V = \textit{constant} \quad \frac{\partial V_S}{\partial \sigma'} = 0 \,, \;\Rightarrow\; \frac{1}{V}\frac{\partial V}{\partial \sigma'} = \frac{1}{1-n}\frac{\partial n}{\partial \sigma'}$$

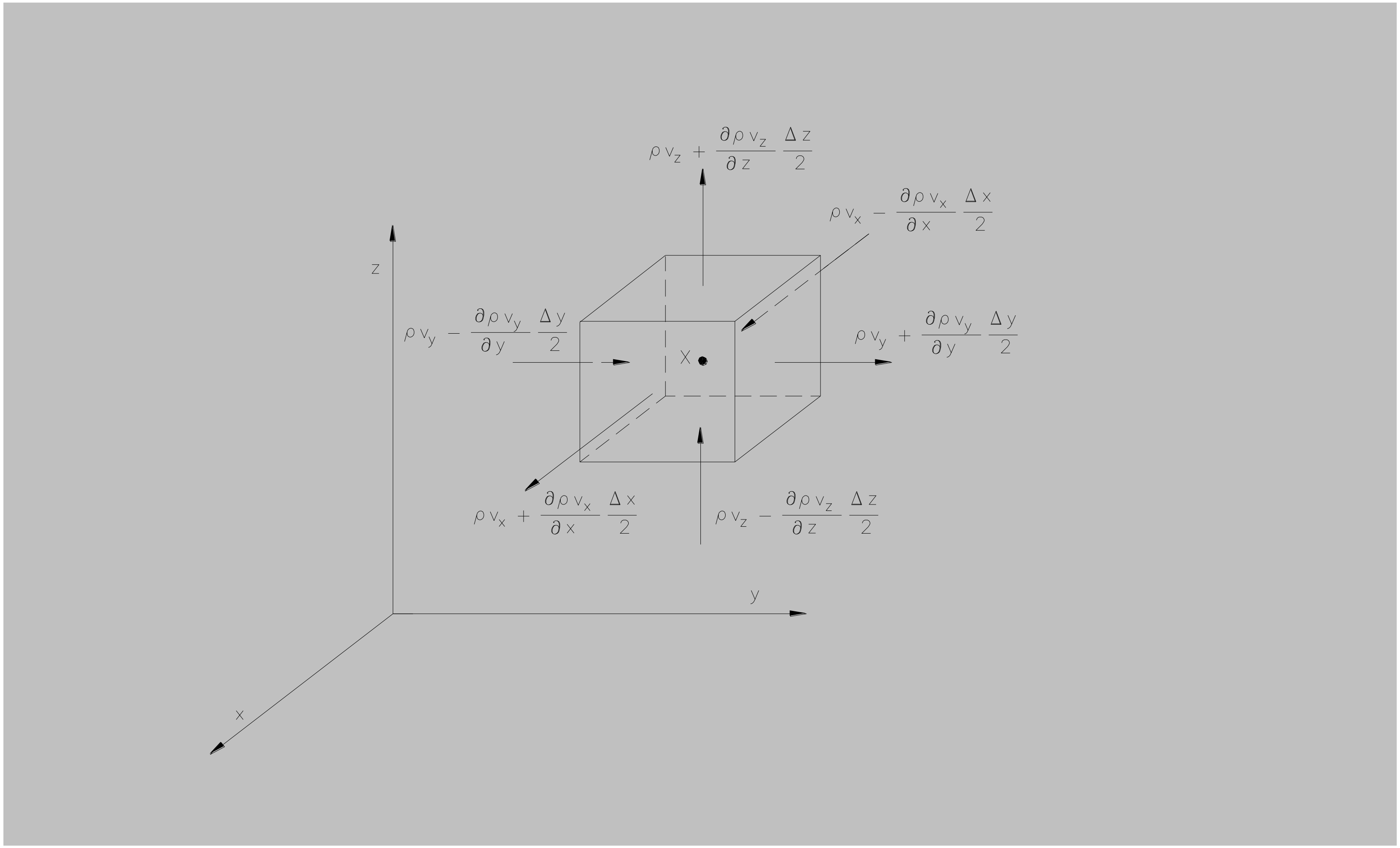
$$\alpha = -\frac{1}{1-n}\frac{\partial n}{\partial \sigma'} = \frac{1}{1-n}\frac{\partial n}{\partial p}$$

SPECIFIC STORATIVITY

is the volume of water released from storage per unit decline in hydraulic head in the aquifer, per unit area of the aquifer:

$$S_0 = \frac{\Delta V_v}{V \;\Delta H} \qquad [m^{-1}]$$

MATHEMATICAL DESCRIPTION OF 3D GROUND WATER FLOW



EQUATION OF CONTINUITY: $\frac{\partial \rho n}{\partial t} = -\left(\frac{\partial}{\partial x} \rho v_x + \frac{\partial}{\partial y} \rho v_y + \frac{\partial}{\partial z} \rho v_z \right)$

Amount of water flowing into the system minus amount of water coming out of the system for given time is equal to changes of retained amount of water. When the steady state water flow occurs than: **inflow = ouflow**

Left side of equation is adjusted by specific storativity:

$$\frac{\partial(\rho n)}{\partial t} = \rho \frac{\partial n}{\partial t} + n \frac{\partial \rho}{\partial t} = \left(\rho \frac{\partial n}{\partial p} + n \frac{\partial \rho}{\partial p} \right) \frac{\partial p}{\partial t}$$

PER PARTES

$$\alpha = \frac{1}{1-n} \frac{\partial n}{\partial p} \qquad \beta = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$$

$$\rho \left[\alpha (1-n) + \beta n \right] \frac{\partial p}{\partial t} = g \rho^2 \left[\alpha (1-n) + \beta n \right] \frac{\partial H}{\partial t}, \quad \frac{\partial H}{\partial t} = \frac{1}{\rho g} \frac{\partial p}{\partial t}$$

S_0^*

$$\boxed{S_0 * = \rho \; S_0}$$

EQUATION OF CONTINUITY

$$\boxed{-\left(\frac{\partial}{\partial x}\rho\, v_x+\frac{\partial}{\partial y}\rho\, v_y+\frac{\partial}{\partial z}\rho\, v_z\right)=\rho\; S_0\;\frac{\partial H}{\partial t}}$$

when *ρ = constant* then the continuity equation is:

$$S_0\;\frac{\partial H}{\partial t}=-\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)$$

unsteady flow

$$\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)=0$$

steady flow

EQUATION OF 3D GROUND WATER FLOW

Combination of continuum approach and Darcy´s law

- *homogeneous isotropic porous media*

  K = constant

$$K\left(\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}+\frac{\partial^2 H}{\partial z^2}\right)=S_0\frac{\partial H}{\partial t}$$

- *non-homogeneous isotropic porous media*

  K = fce(x,y,z)

$$\frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial H}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial H}{\partial z}\right)=S_0\frac{\partial H}{\partial t}$$

- *non-homogeneous anisotropic porous media*

  $K_{x,y,z}$ = fce(x,y,z)

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial H}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial H}{\partial z}\right)=S_0\frac{\partial H}{\partial t}$$

- *steady state water flow, homogeneous isotropic porous media*

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}+\frac{\partial^2 H}{\partial z^2}=0$$

SOLUTION OF EXAMPLES OF 3D GROUND WATER FLOW

What we need to know for solving the flow through a specified porous medium domain:

- *geometry of area*
- *value of physical parameters (K, S_0) and spatial layout*
- *initial and boundary conditions*

INITIAL CONDITIONS (IC)

It must be defined for solution of the unstable flow when the initial time **t = 0**.

Mathematically the initial condition is defined as:

$$H = f\ (x,y,z,0)$$

BOUNDARY CONDITION (BC)

If the problem si one in which the dependent variables are also time dependent , the boundary conditions must be specified for all times t ≥ 0.

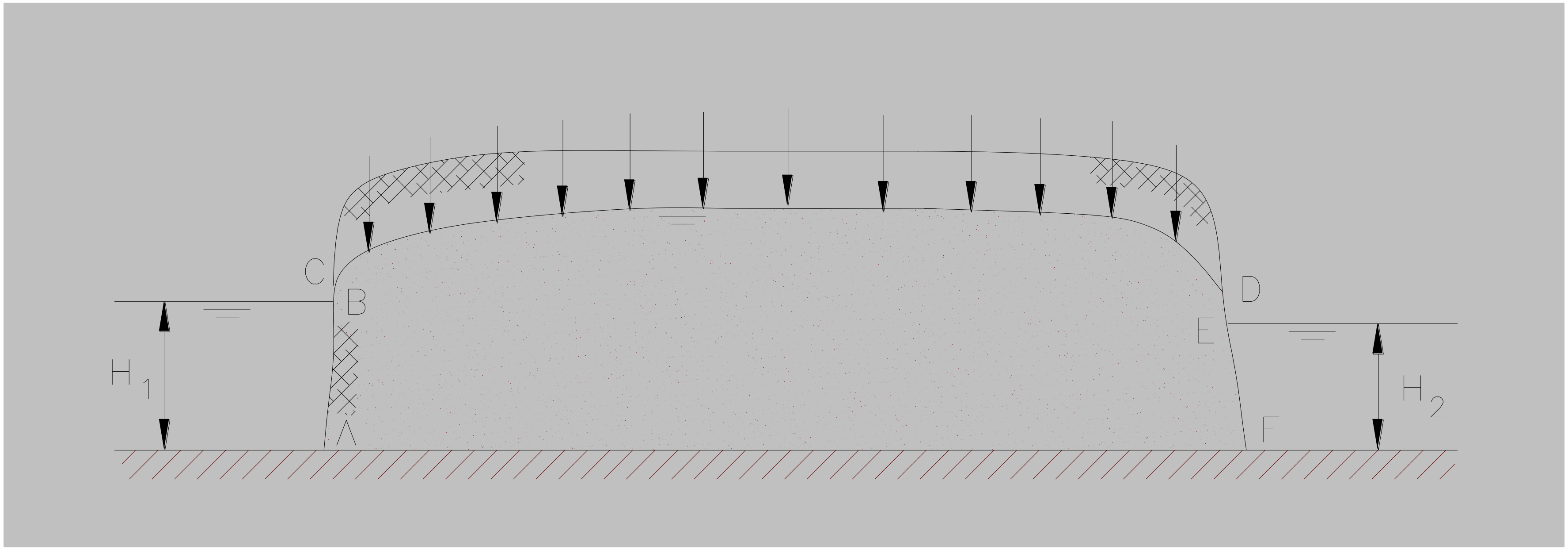
It have to be defined for solution of the steady and unsteady flow.
BO characterizes how the water flow is affected by surrounding area.
BO defines interactions on boundaries.

- BC of prescribed value of hydraulic head (Dirichlet´s boundary conditions):

$$H = f_1(x, y, z) \quad or \quad H = f_2(x, y, z, t) \quad on \quad S,$$

- BC of prescribed flux (Neumann´s boundary conditions): $v_n = f(x,y,z,t) \quad on \quad S$

    - water inflow – perpendicularly – value of flux is known



- Border with the prescribed value of hydraulic head: **A B** ($H = H_1$), **E F** ($H = H_2$)
  e.g. **river, lake**

- Border with the prescribed flow: **A F** = impermeable wall $v_z = 0$
  e.g. **impermeable layer**

- Phreatic surface:

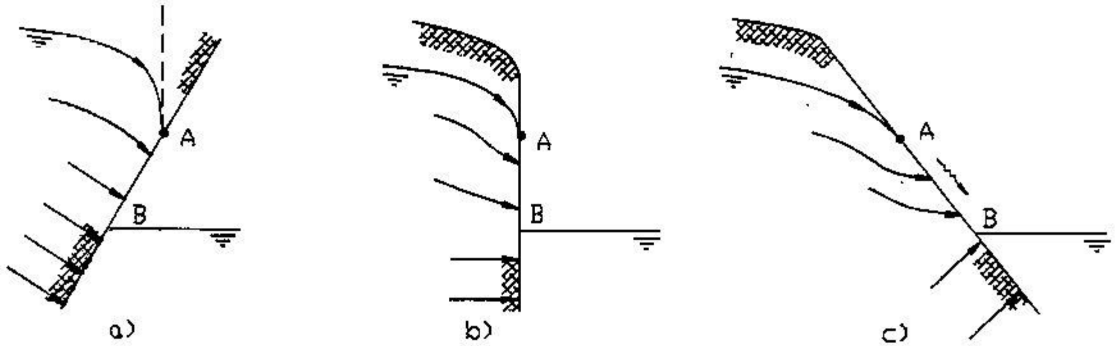
$$H(x,y,z,t)=z \quad \text{or} \quad H(x,y,z,t)-z=0 \quad \text{on} \quad S$$

Where free ground water table is formed.
On water table the water pressure is zero and hydraulic head is equal to geodetic head.
The position of free water table is not known – solution is done by iteration.

- Seepage face:

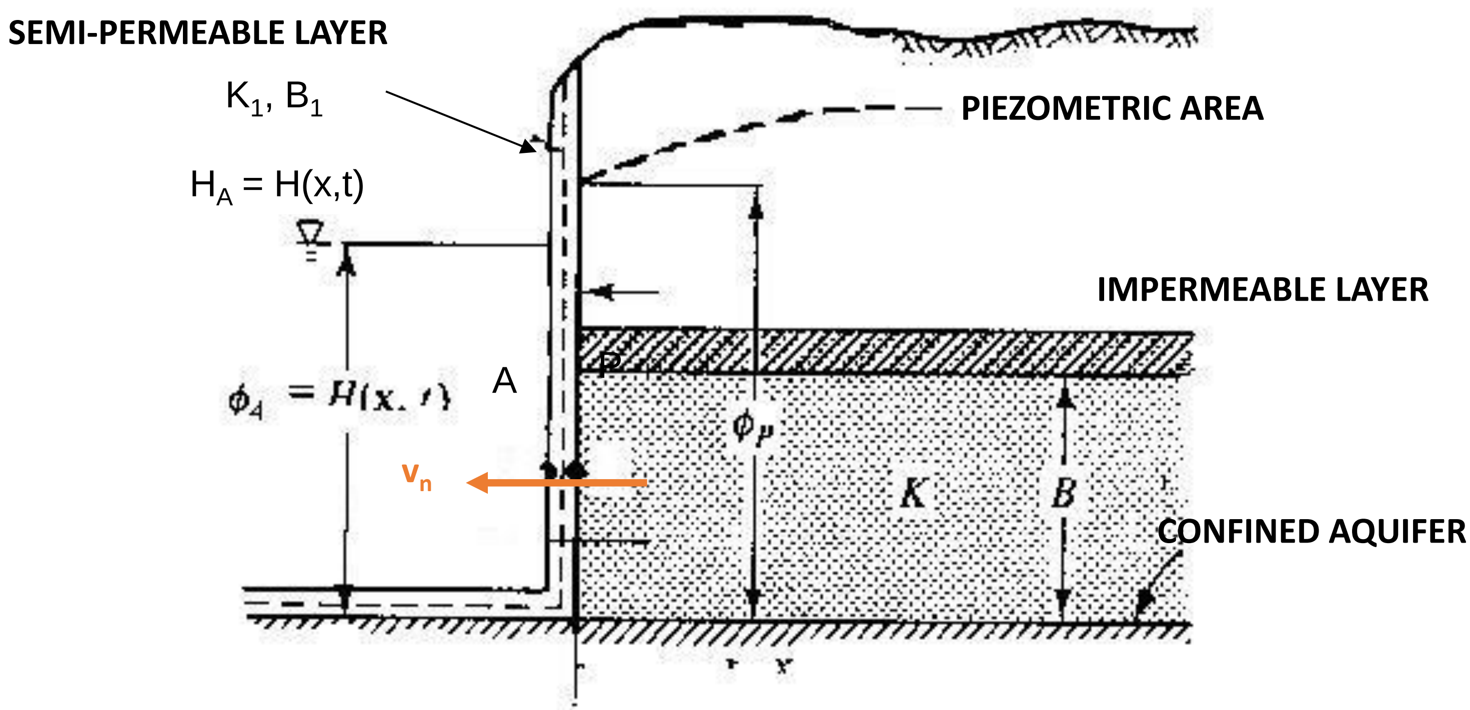
$$H(x,y,z,t)=z$$

It is part of phreatic surface.
Where the ground water flows to the surface.



- Semi-permeable boundary (Newton´s boundary condition):

Occurs where the water in the aquifer is indirectly in contact with surface water - there is a semi-permeable layer between the aquifer and river which causes a different position of the water level in the aquifer and the river.



Flow rate through boundary is done by:

$$v_n = (\phi - H(x,t))/c \;\; , kde \quad c = B_1 / K_1$$

H(x,t) ... known function of location of the water level in river

Φ ... demanded value of water level in aquifers

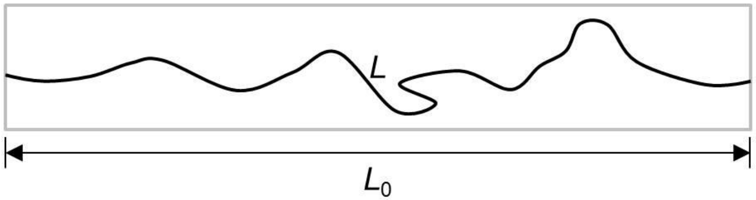
CHARACTERISTICS OF THE CURRENT FIELD

FLOWLINES – the instantaneous curves that are at every point tangent to the direction of the velocity.

PATHLINES - individual fluid particles follow. These can be thought of as "recording" the path of a fluid element in the flow over a certain period. The direction the path takes will be determined by the streamlines of the fluid at each moment in time.

TORTUOSITY - is the ratio of the length of a streamline—a flow line or path—between two points to the straight-line distance between those points.

$$\tau = \frac{\lambda^*}{\lambda}$$

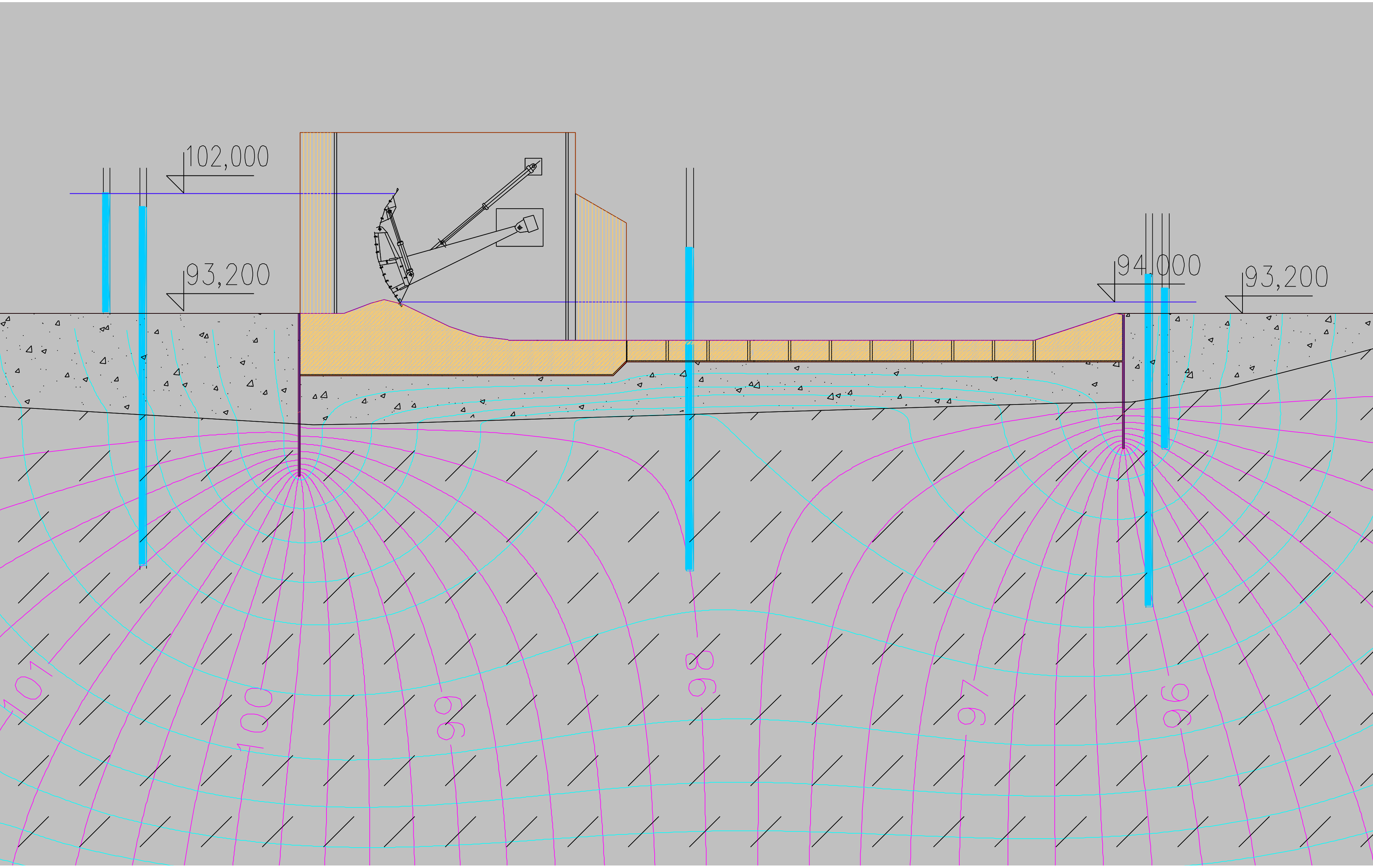


Tortuosity = L/L_0

Values: from 1,3 to 2,0.

Relationship with porosity: **$\tau = n^{-1/4}$**.

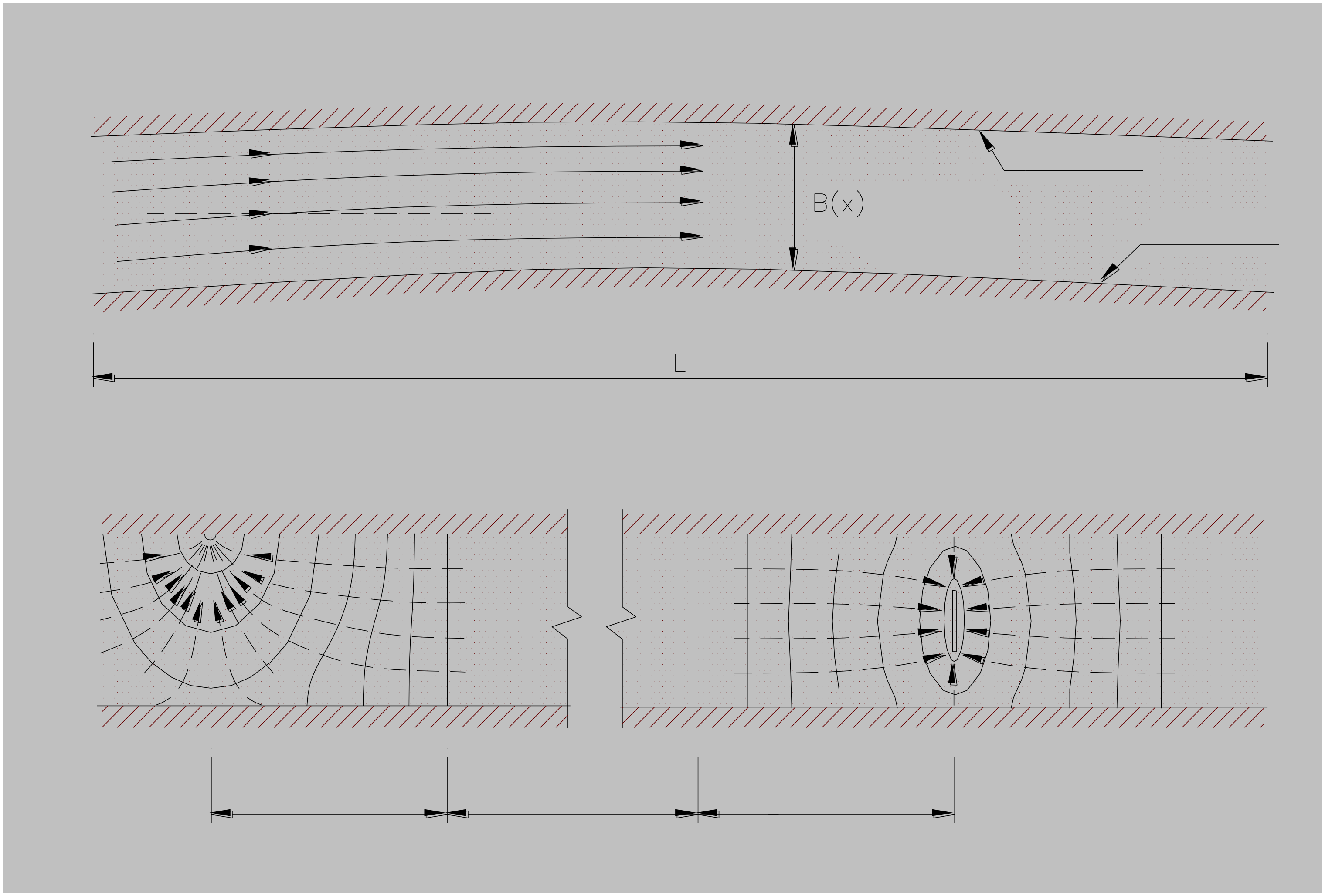
TIMELINES - lines formed by a set of fluid particles that were marked at a previous instant in time, creating a line or a curve that is displaced in time as the particles move.

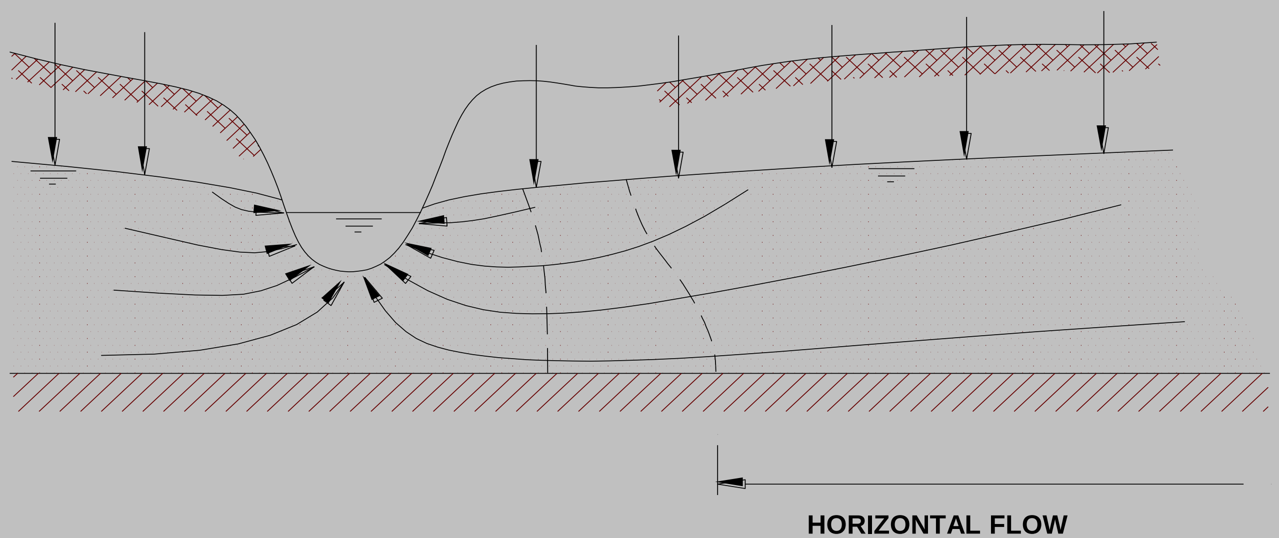

102,000
93,200
94,000
93,200
101
100
99
98
97
96

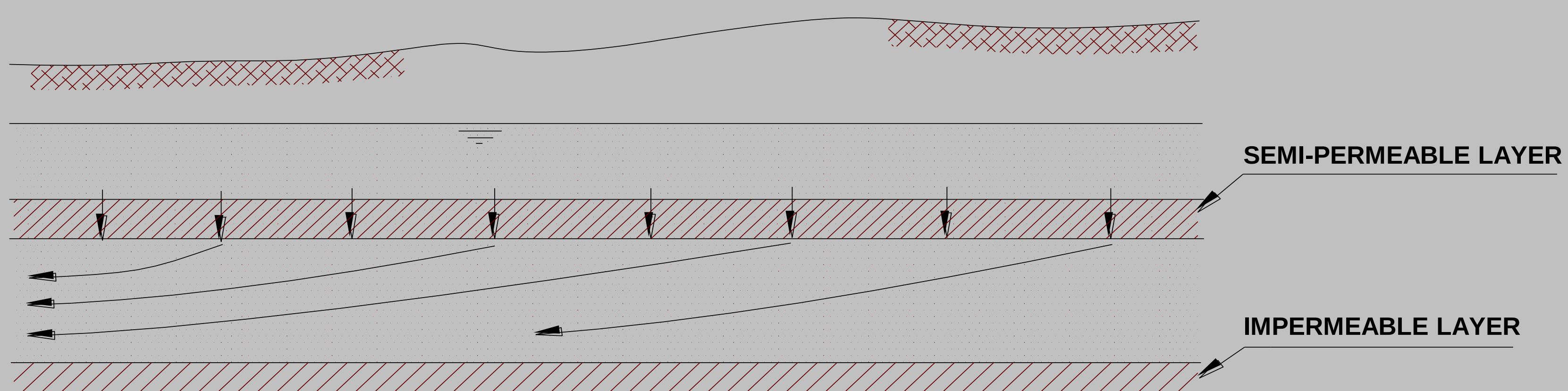
MATHEMATICAL DESCRIPTION OF 2D SPATIAL GROUND WATER FLOW

HYDRAULIC APPROACH

- approach to solve groundwater flow
- vertical components of velocity are negligible
- assumption of horizontal flow




HORIZONTAL FLOW

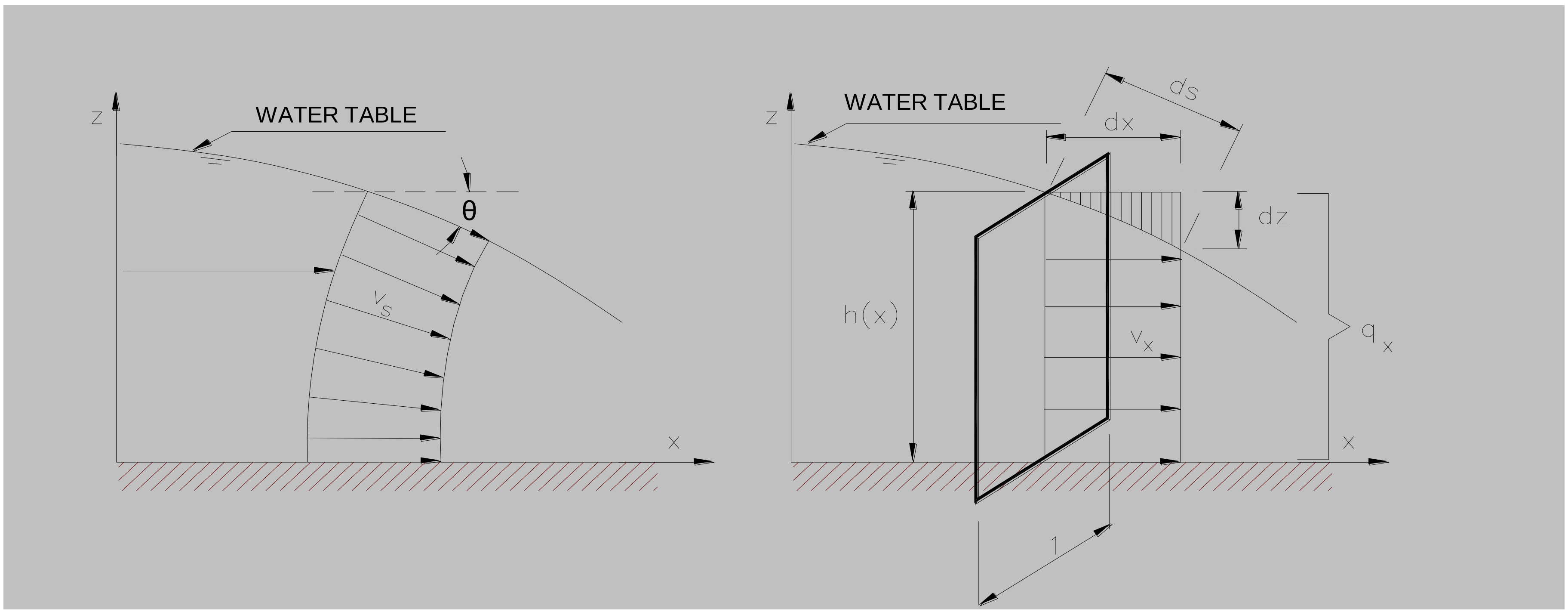

SEMI-PERMEABLE LAYER
IMPERMEABLE LAYER

DUPUIT ASSUMPTIONS

The hydraulic height H (x, y, z) is equal to the groundwater level h (x, y), the flow lines are horizontal lines, the vertical are equipotentials. The gradient of the potential is given by the slope of the plane and is constant after the vertical.

$$\frac{dH}{dx}(x,y,z) = \frac{dh}{dx}(x,y)$$

REALITY **HYDRAULIC APPROACH**



$$v_s = -K\frac{dH}{ds} = -K\frac{dz}{ds} = -K\sin\theta \qquad v_x = -K\frac{dh}{dx}, \quad h = h(x)$$

THANK YOU FOR YOU ATTENTION