Groundwater hydraulics 5
Dupuit assumptions
Flow in the dam/soil block

Martin Šanda, B673

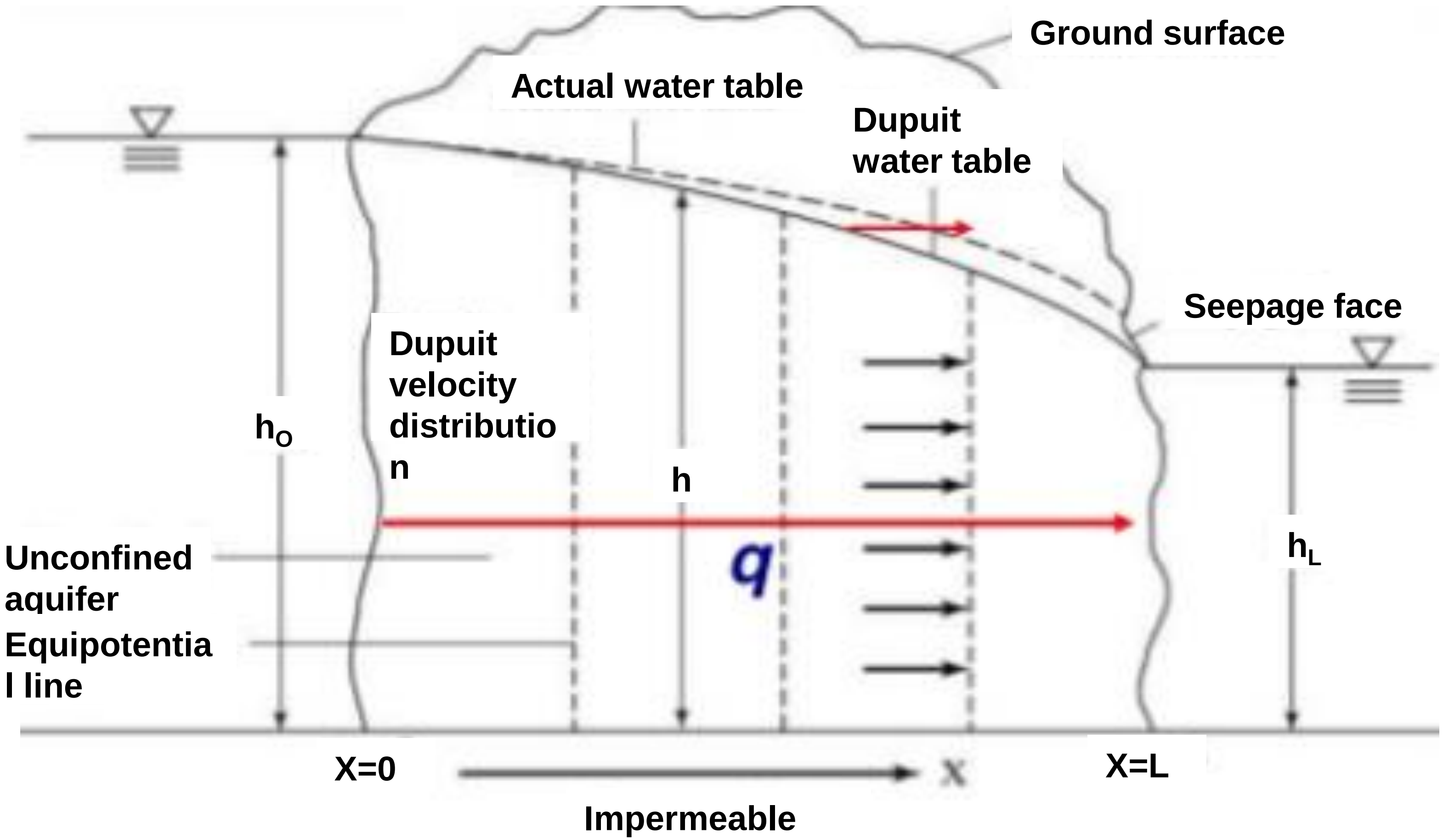
martin.sanda@fsv.cvut.cz

Martina Sobotková, B609

martina.sobotkova@fsv.cvut.cz

Dept. Landscape Water Conservation

CROSS SECTION OF FLOW

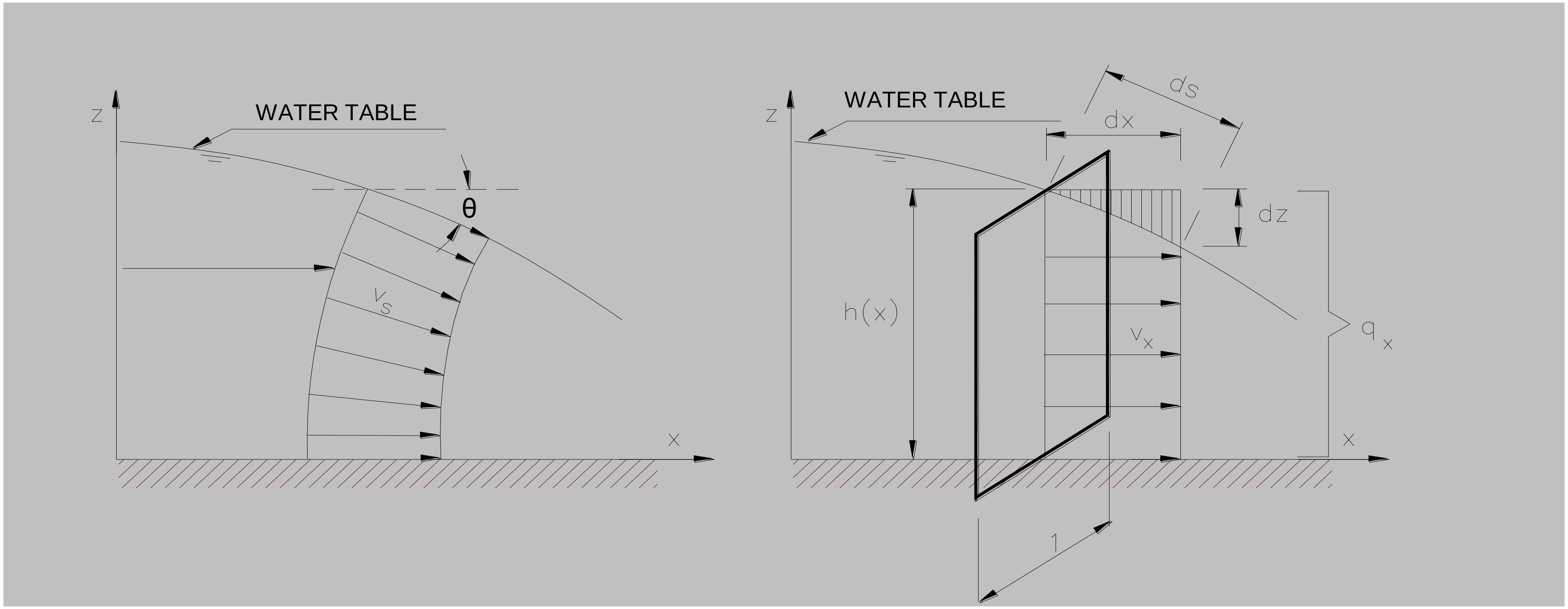


DUPUIT´S ASSUMPTION - EQUATIONS

The hydraulic height H (x, y, z) is equal to the groundwater level h (x, y), the flow lines are horizontal lines, the vertical are equipotentials. The gradient of the potential is given by the slope of the plane and is constant after the vertical.

$$\frac{dH}{dx}(x,y,z) = \frac{dh}{dx}(x,y)$$

REALITY | **HYDRAULIC APPROACH**



$$v_s = -K\frac{dH}{ds} = -K\frac{dz}{ds} = -K\sin\theta$$

$$v_x = -K\frac{dh}{dx}, \quad h = h(x)$$

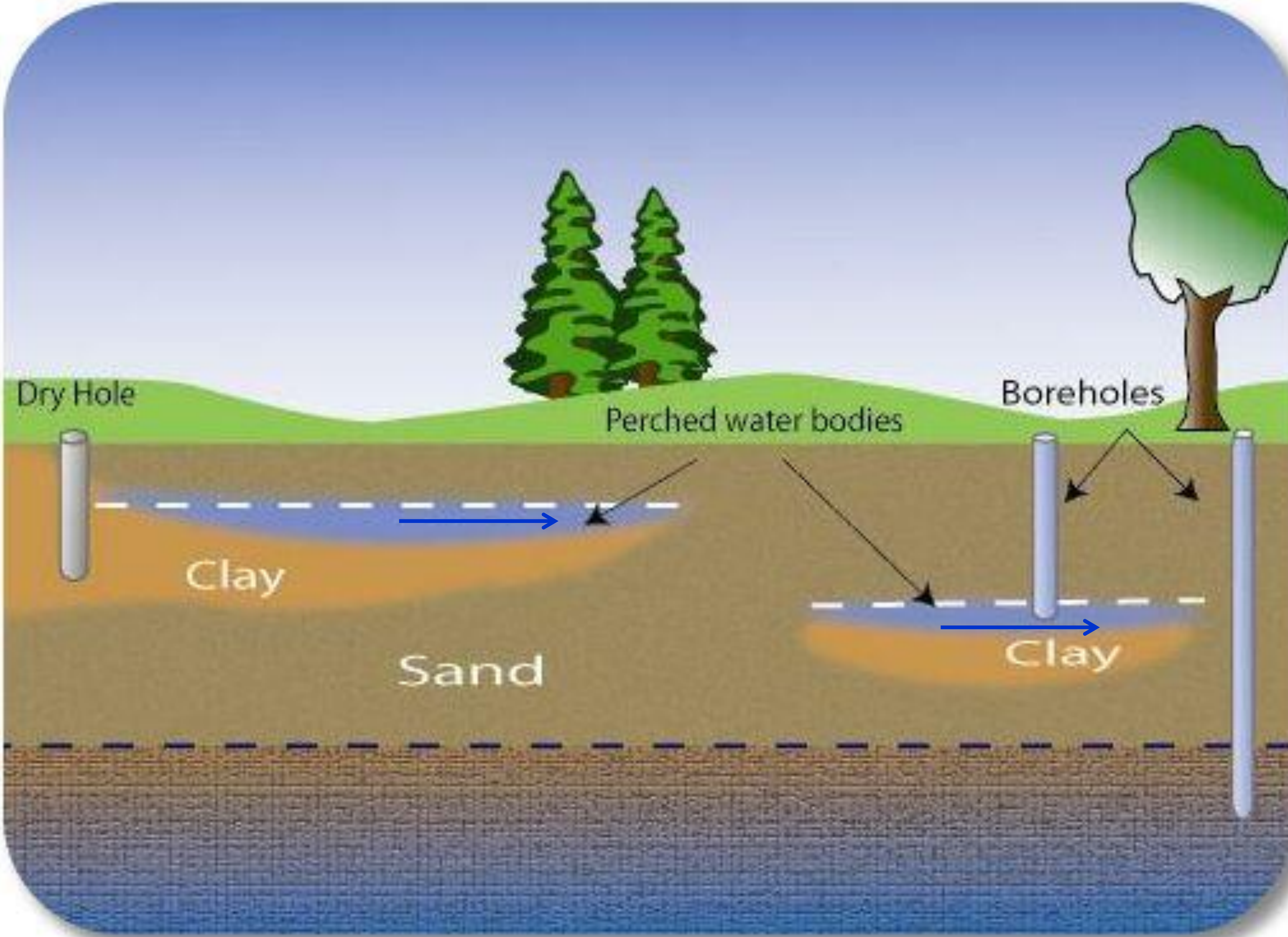
SIMPLIFIED SOLUTION OF GROUND WATER FLOW

STEADY STATE FLOW INSTEAD OF UNSTEADY STATE FLOW – if a particular problem is solved and we do not care about the water level and fluctuations depend on time

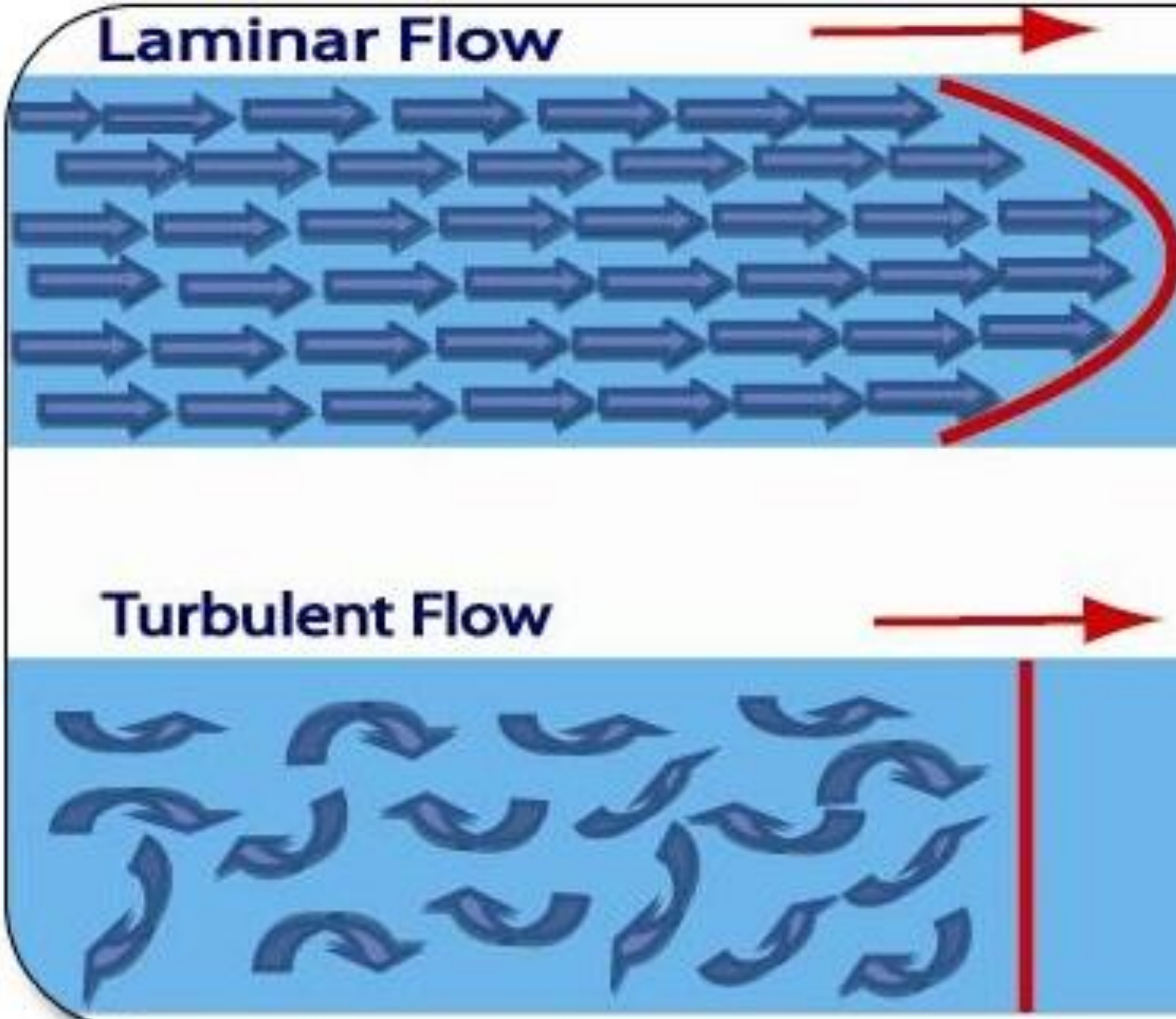
Spatial problem converted to 2D – due to symmetry – e.g. seepage of levee - the water flow occurs in the vertical planes perpendicular to the longitudinal axis of the levee - problem is solved as the water flow in 2D

Hydraulic approach – Dupuit concept

- conversion the **spatial flow into the horizontal system**
- used for the water flow in aquifers – **horizontal system**
- **slope** of the ground water level **is extremely small – 1/1000**
- **vertical components of the velocity are negligible**

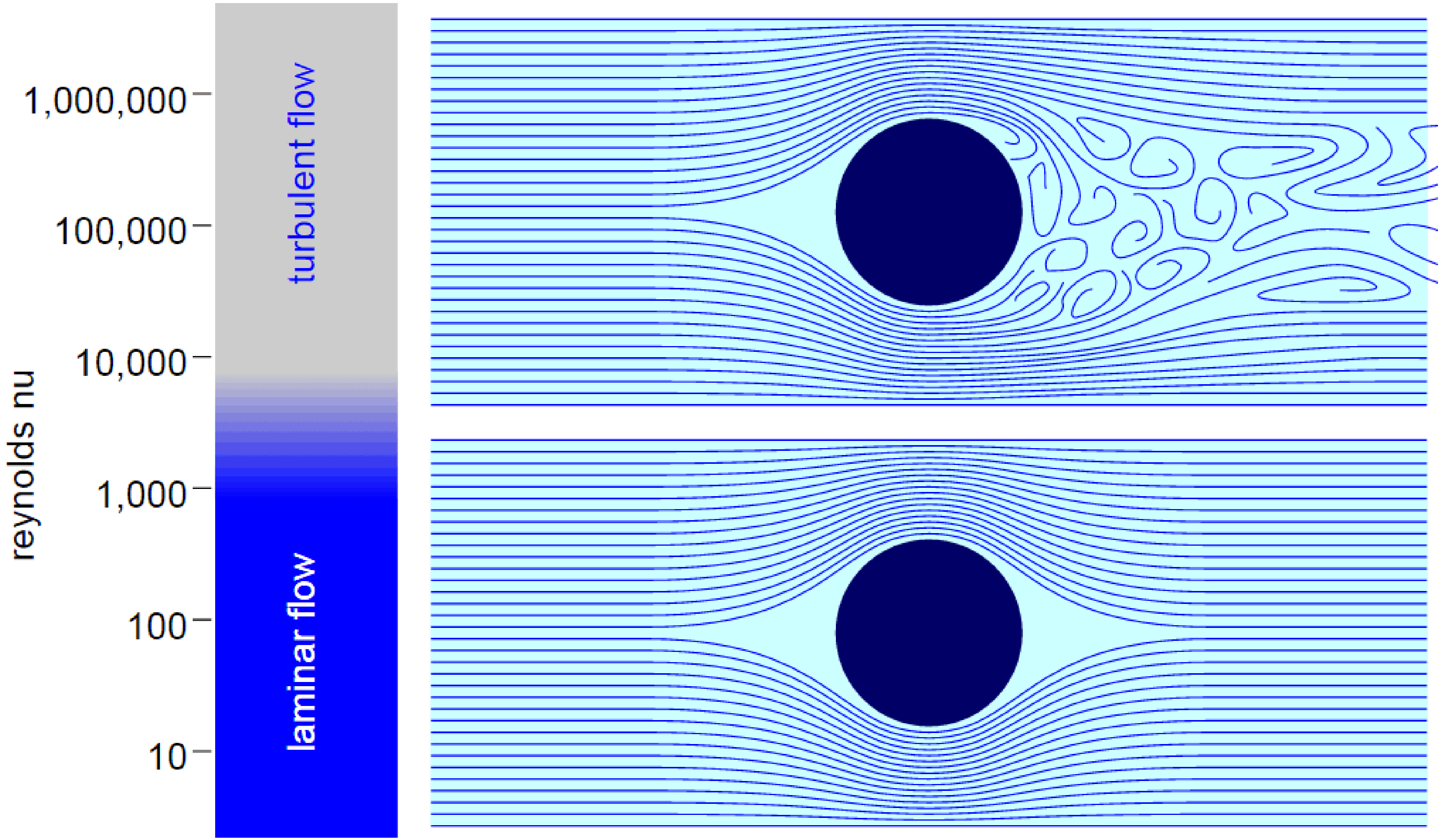


Dry Hole
Perched water bodies
Boreholes
Clay
Sand
Clay

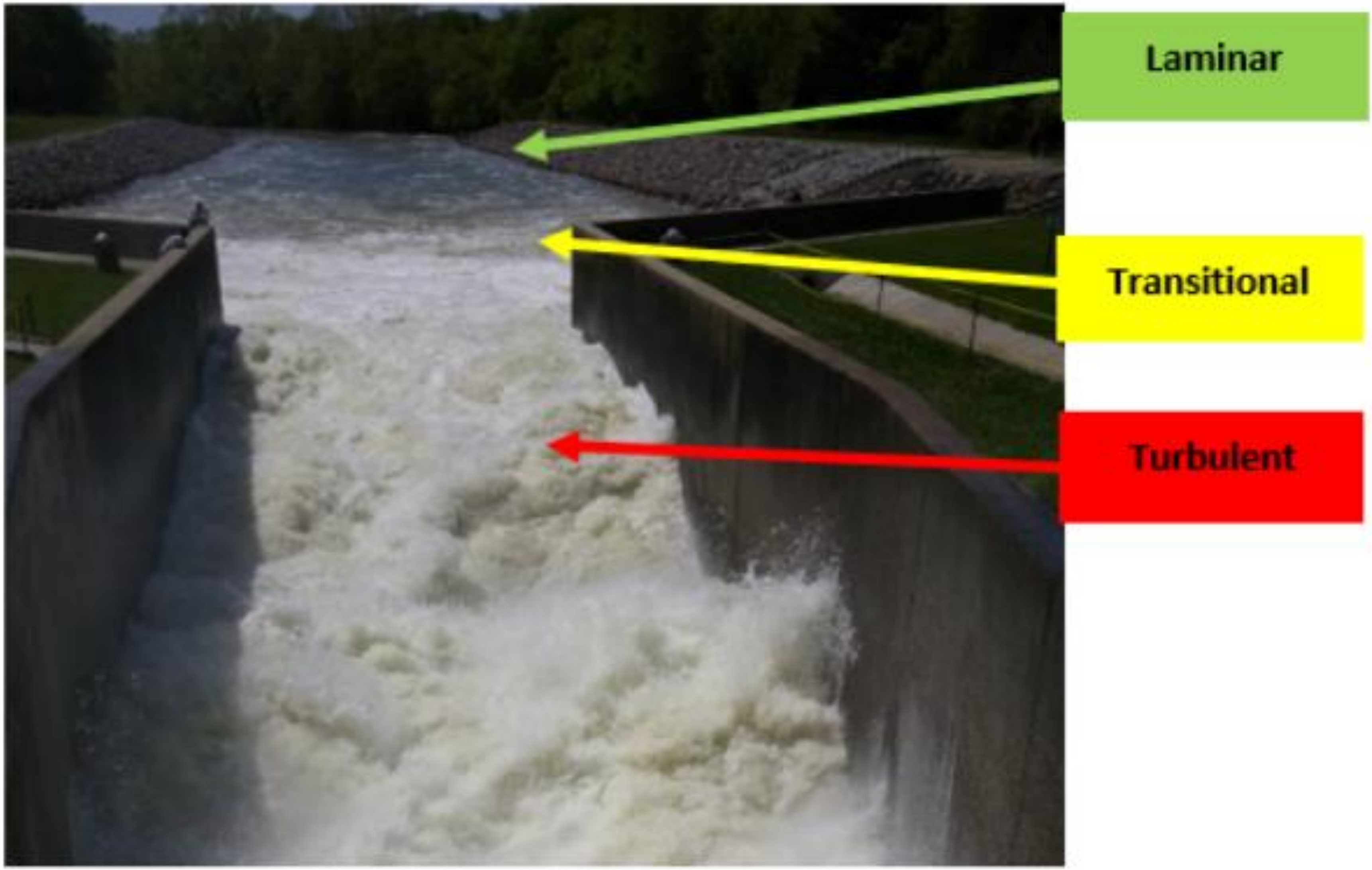


- LOW VELOCITIES
- FLOW WITHOUT LATERAL MIXING
- MOVING IN SMOOTH PATHS

- FLOW UNDERGOES IRREGULAR FLUCTUATIONS



1,000,000
100,000
10,000
1,000
100
10
reynolds nu
turbulent flow
laminar flow

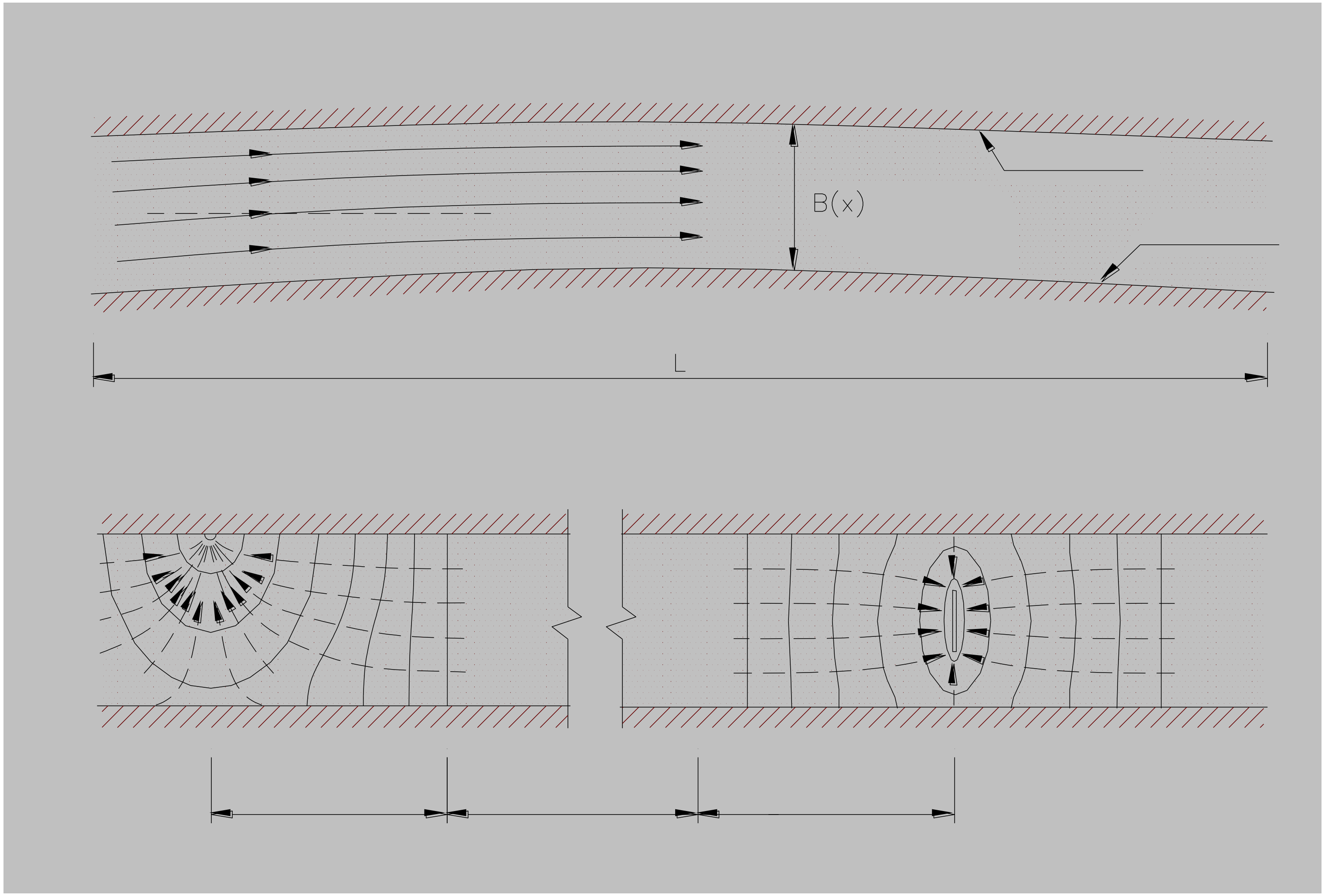


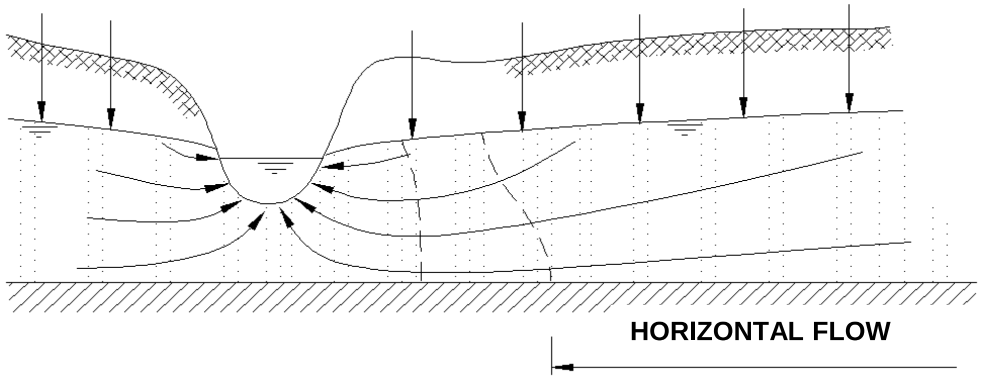
Turbulent to Laminar Flows

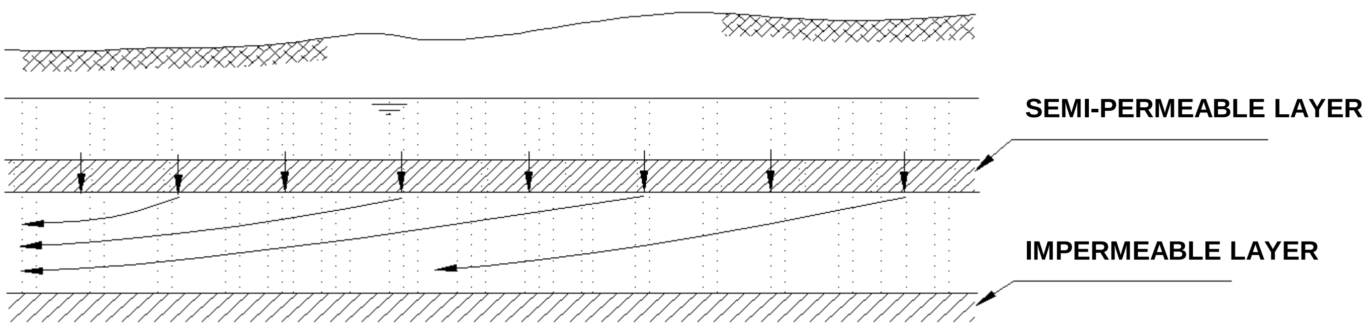
MATHEMATICAL DESCRIPTION OF 2D SPATIAL GROUND WATER FLOW

HYDRAULIC APPROACH

- approach to solve groundwater flow
- vertical components of velocity are negligible
- assumption of horizontal flow




HORIZONTAL FLOW


SEMI-PERMEABLE LAYER
IMPERMEABLE LAYER

DUPUIT´S ASSUMPTIONS

For unconfined ground water flow Dupuit developed a theory that allows for for a simple solution based based off the folowing assumptions:

1. The water table or free surface is only slightly inclined

2. The streamlines are considered horizontal and equipotential lines are considered vertical

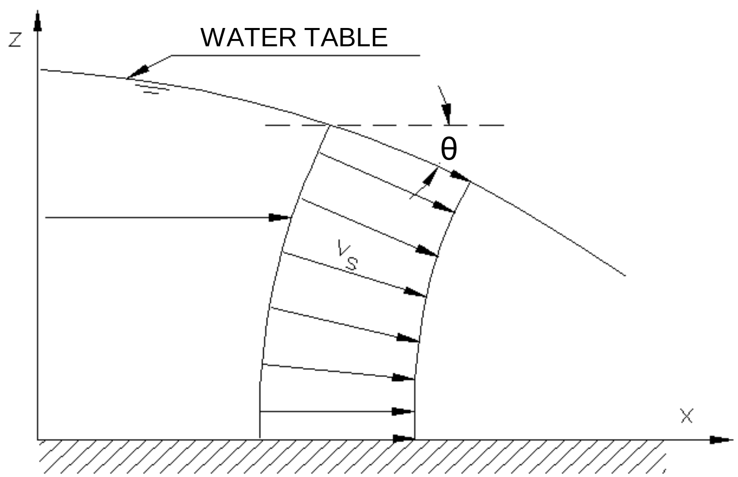
3. Slopes of the free surfaces and hydraulic gradient is equal

DUPUIT´S ASSUMPTION - EQUATIONS

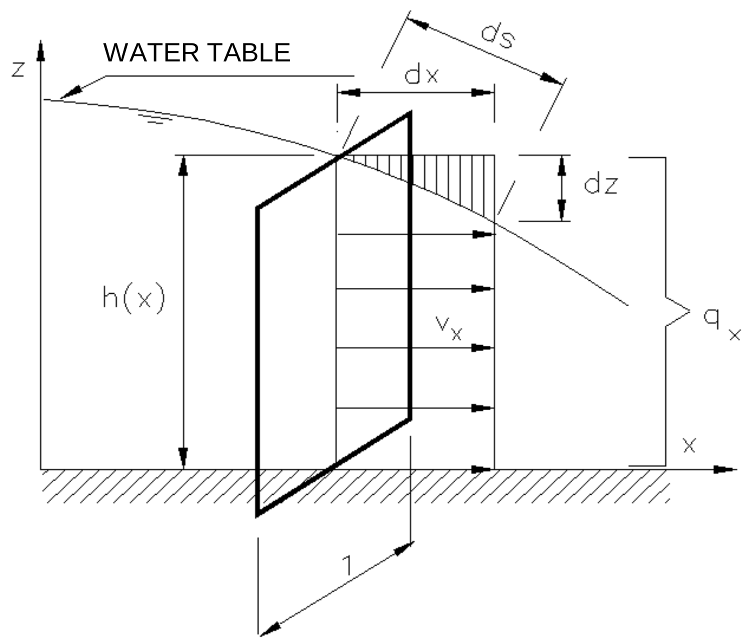
The hydraulic height H (x, y, z) is equal to the groundwater level h (x, y), the flow lines are horizontal lines, the vertical are equipotentials. The gradient of the potential is given by the slope of the plane and is constant after the vertical.

$$\frac{dH}{dx}(x,y,z) = \frac{dh}{dx}(x,y)$$

REALITY



HYDRAULIC APPROACH



$$v_s = -K\frac{dH}{ds} = -K\frac{dz}{ds} = -K\sin\theta$$

$$v_x = -K\frac{dh}{dx}, \quad h = h(x)$$

Specific flow q can be determined by integration of v_x by aquifer height:

$$q_x = \int_0^{h(x)} v_x(x)\, dz \qquad q_x = -K \frac{dh}{dx} h(x)$$

Equation of continuity describe equality of flow rate:

$$\frac{dq_x}{dx} = 0 \qquad \frac{d}{dx}\left(h(x) \frac{dh}{dx}(x) \right) = 0$$

$$\boxed{\frac{d^2 h^2(x)}{dx^2} = 0}$$

The equation describes steady state flow in the vertical plane and homogeneous isotropic unconfined aquifer (Dupuit concept).

When we solve problem in coordinates x, z the equation is:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \qquad \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} = 0$$

H is function of x and z.

Dupuit concept reduces the number of variables (vertical coordinate z).

$$\frac{\partial^2}{\partial x^2}\left(h^2(x,y)\right)+\frac{\partial^2}{\partial y^2}\left(h^2(x,y)\right)=0$$

h is function of x a y

For homogeneous and isotropic aquifer:

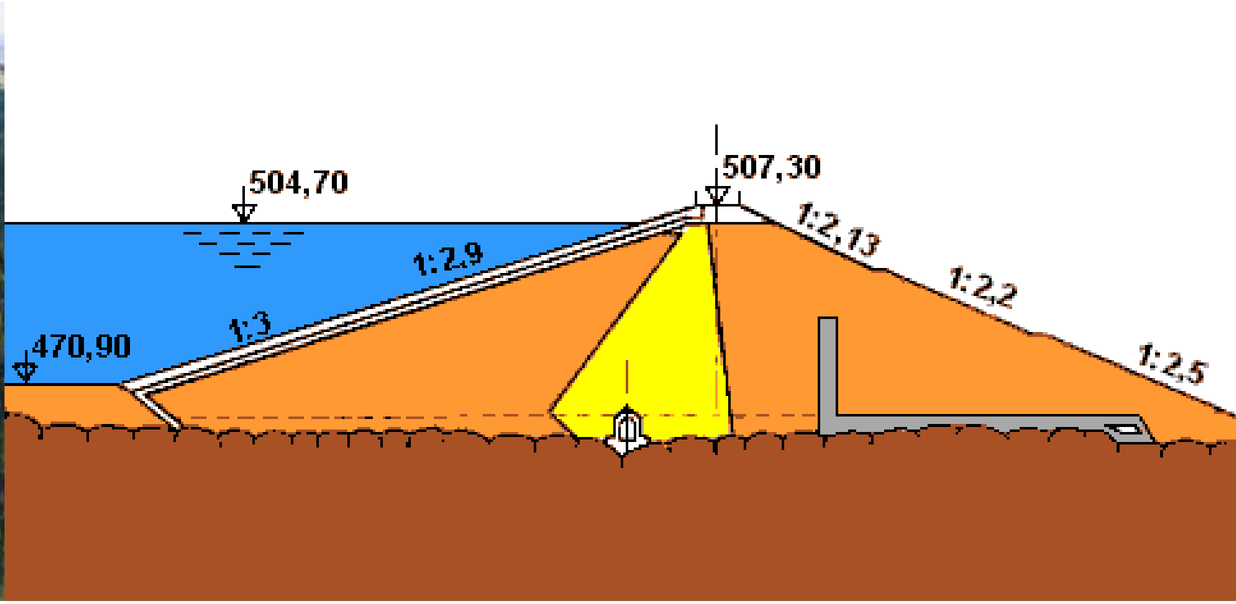
$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}+\frac{\partial^2 H}{\partial z^2}=0$$

H is function of x, y and z

The vertical component of velocity is neglected (Dupuit concept). It is in opposite direction of z axis. The equation is:

$$v_z(x,z)=-\frac{zq_x^2}{Kh^3(x)}$$

LEEVES DAM – WATER RESERVOIR HORKA on Libocký stream (built 1966 – 1970)



water reservoir – 1. source of drinking water for Sokolov (North Bohemia)
2. hydro electricity, flood protection

levees dam – stabilization part – gravel sand

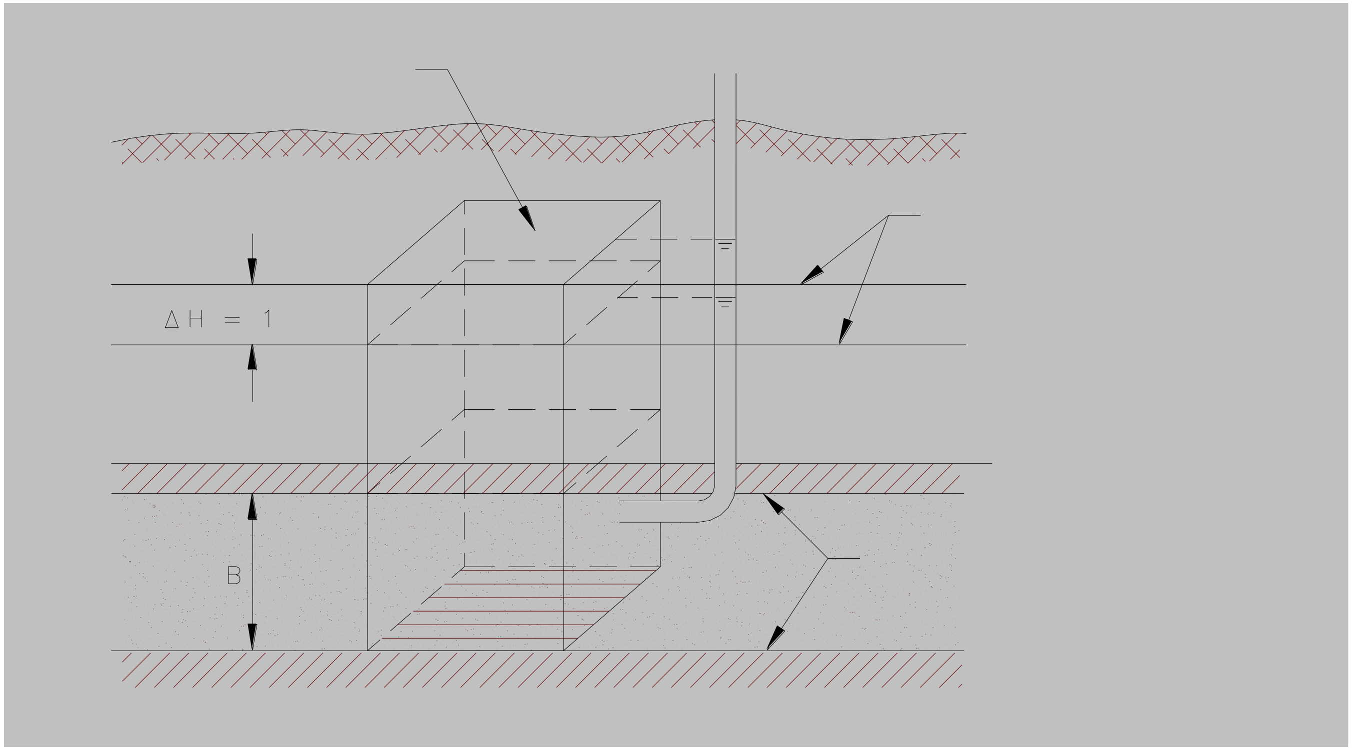
lenght: 278 m, wide of ...: 5.95 m, height: 40 m

STORATIVITY OF AQUIFERS

$$S = \frac{\Delta V_v}{A \;\; \Delta H} \qquad [-]$$

Storativity (storage coefficient) of confined aquifer is defined as: **volume of water released from storage per unit decline in hydraulic head in the aquifer, per unit area of the aquifer.**

Storativity is a dimensionless quantity, and ranges between 0 and the effective porosity of the aquifer.

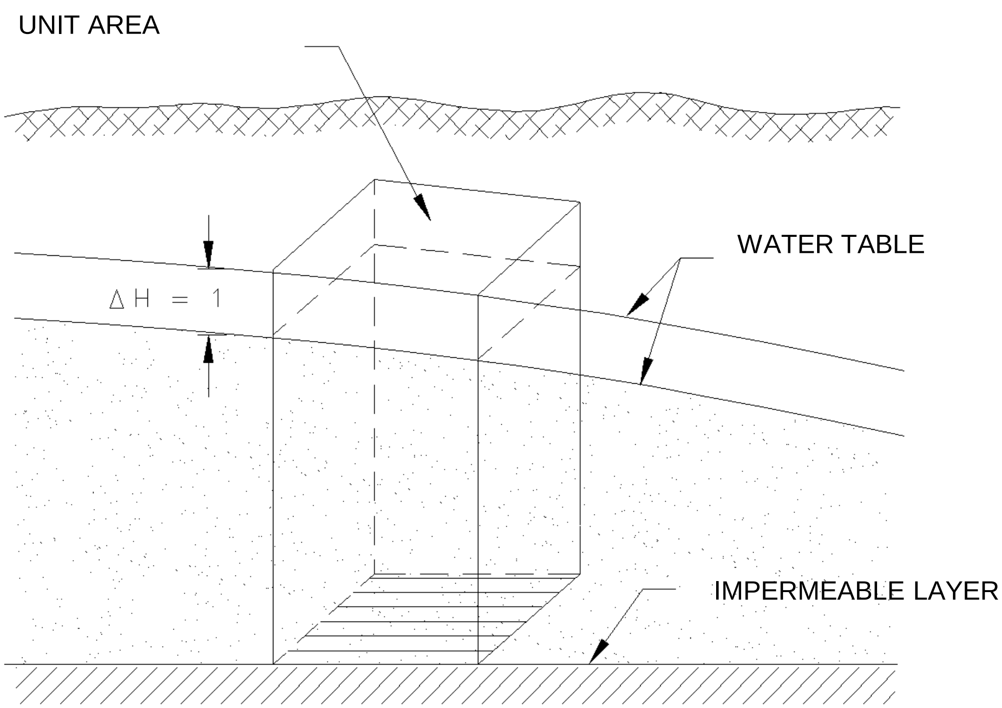


Confinded aquifer:

- value of storativity for confined aquifer: from 10^{-4} (for loose rock) to 10^{-6} (rock).

Approximately 40% of this value is due to compressibility of water and 60% due to the compressibility of the porous media.

$$S = \frac{\Delta V_v}{A \;\Delta h} \qquad [-]$$

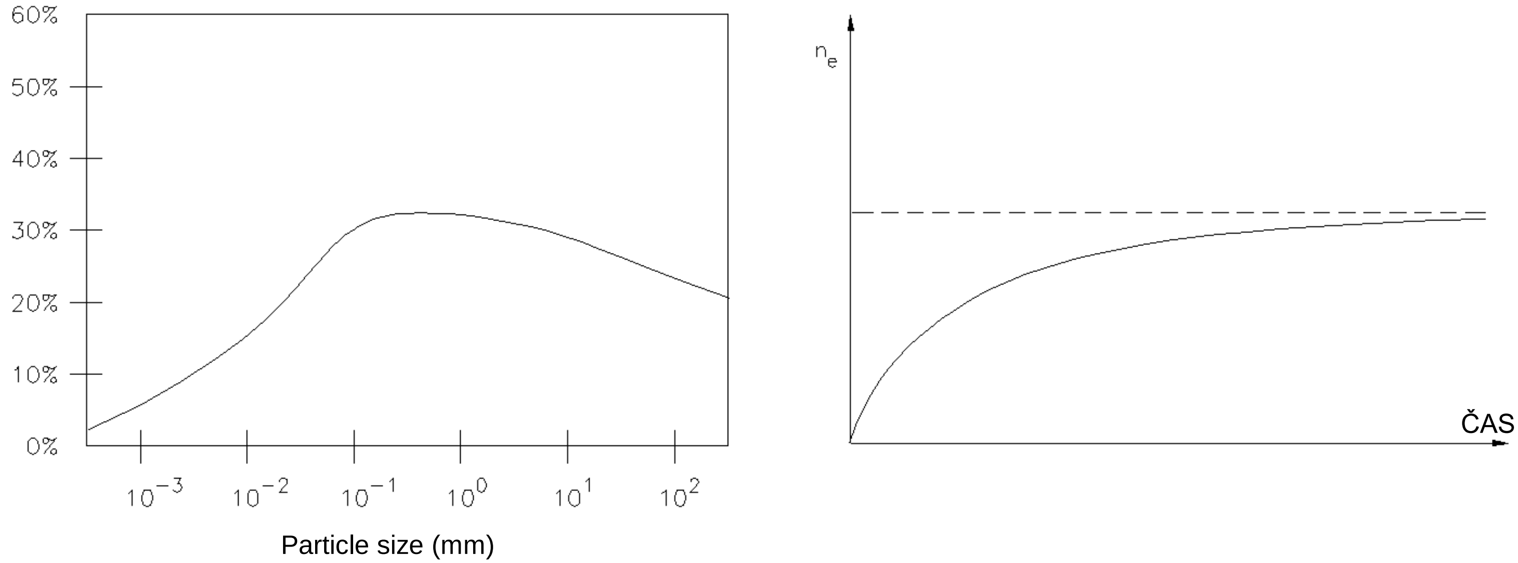


Storativity in unconfined aquifer is equal to efective porosity: n_e – tj. 0,1 – 0,3.

Porosity depends on particle size distribution.

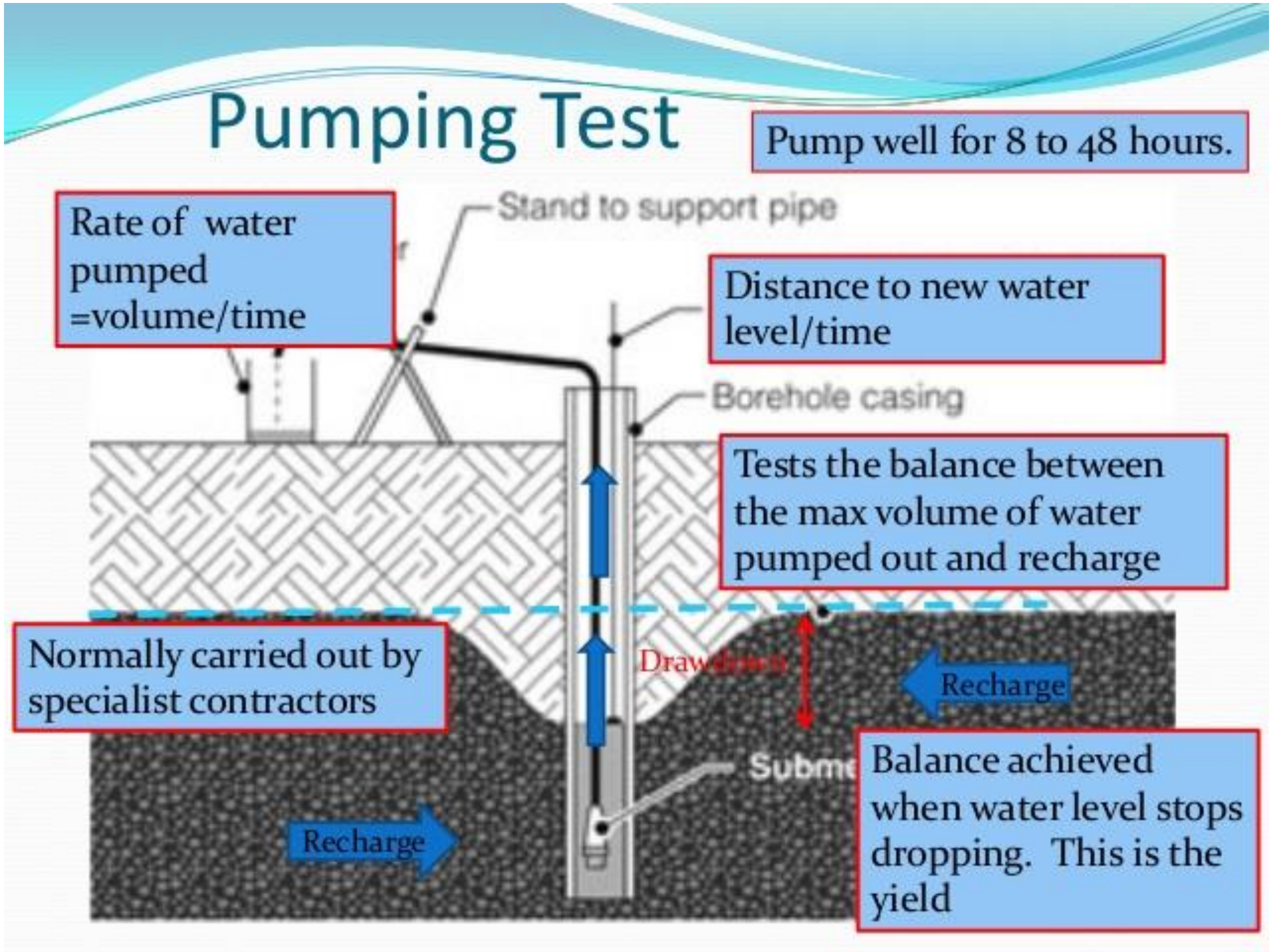
Porosity: sand: 0,05 - 0,15

fine or coarse sand 0,19 – 0,30.



Storativity of the unconfined aquifer is much higher than storativity of the confined aquifer. When the water level decrease there is much more water released from the unconfined aquifer than the confined aquifer. To produce the same pressure reduction, a large amount of water should be drained in the unconfined aquifer while a small amount of water is required for the confined aquifer.

Storativity coefficient is measured by pumping tests.



For solving 3D water flow the specific storativity S_0 and hydraulic conductivity is used.

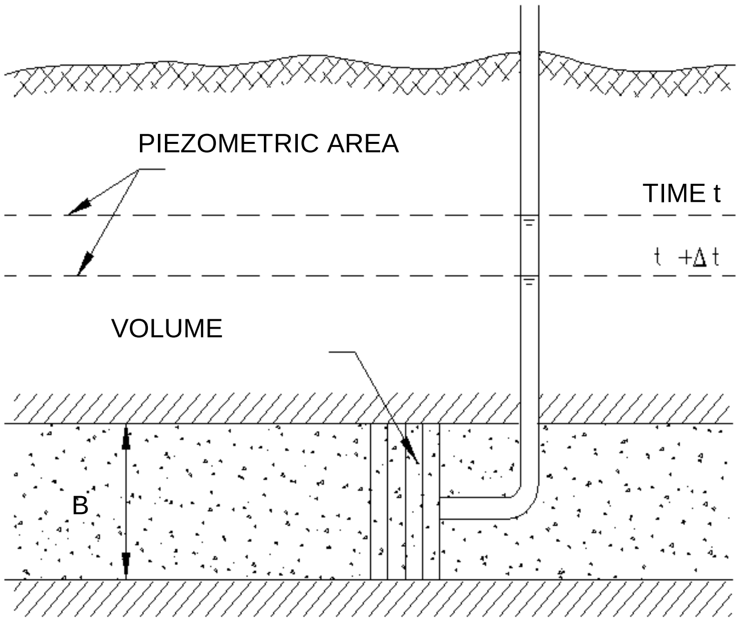
$$S_0 = \frac{\Delta V_v}{V \ \Delta H}$$

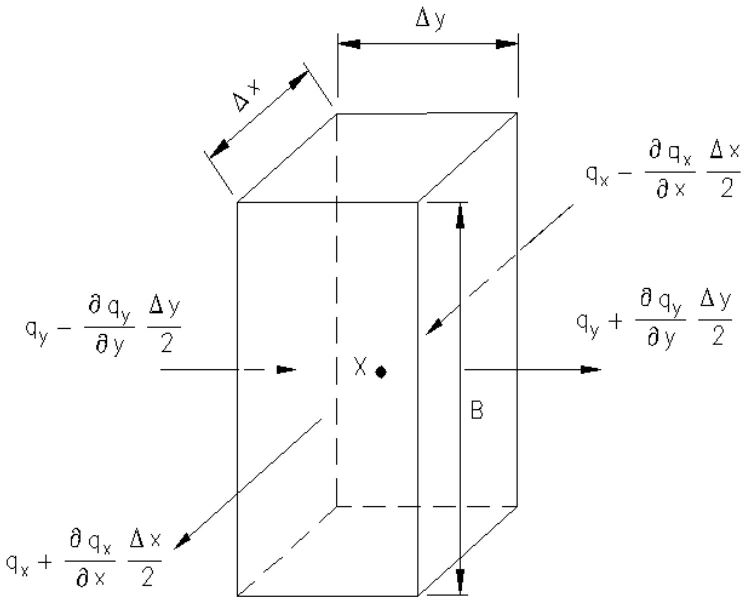
$$S_0 = \frac{S}{B}$$

B … width of aquifer

EQUATION OF 2D UNSTEADY WATER FLOW IN AQUIFERS

1. CONFINED AQUIFER, CONSTANT LAYER





Equation:

$$\frac{\partial}{\partial x}\left(T\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial H}{\partial y}\right)=S\frac{\partial H}{\partial t}$$

When discharge or recharge of water occurs: the equation on the left side is:
N …. outflow rate or inflow rate

It can be the function of x and y and time.

Variations of equations for different aquifers:

Non-homogeneous and anisotropic aquifer:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_y \frac{\partial H}{\partial y}\right)+N(x,y,t)=S\ \frac{\partial H}{\partial t}$$

Non-homogeneous and isotropic aquifer:

$$\frac{\partial}{\partial x}\left(T \frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(T \frac{\partial H}{\partial y}\right)+N(x,y,t)=S\ \frac{\partial H}{\partial t}$$

Homogeneous and isotropic aquifer:

$$T\left(\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}\right)+N(x,y,t)=S\ \frac{\partial H}{\partial t}$$

Laplace's equation

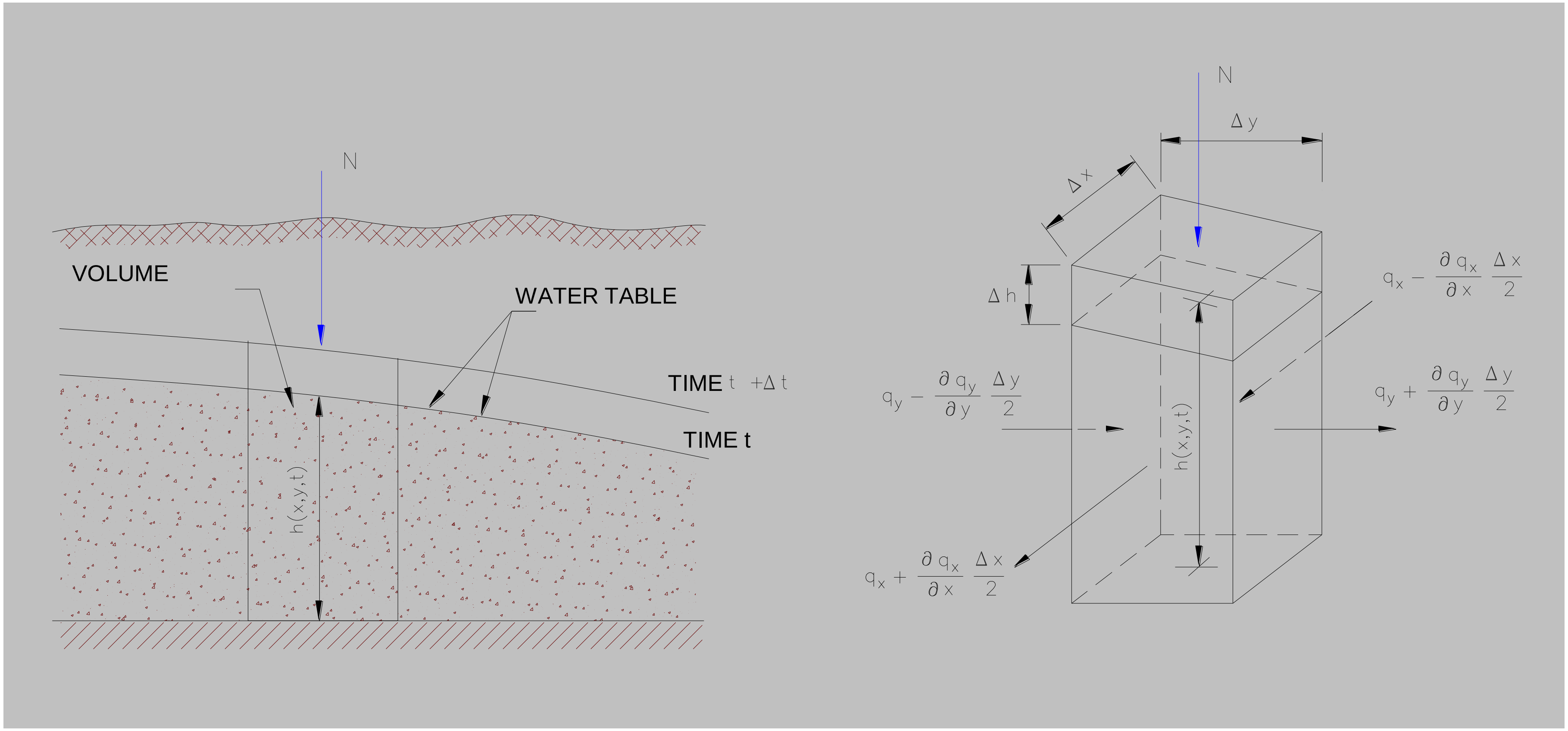
- for steady state water flow, homogeneous and isotropic aquifers

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

Notes:

- **confined aquifer** – hydraulic head: marked as H or Ø
- **unconfined aquifer -** hydraulic head: marked as h – hydraulic head is described by ground water table, function of h (x,y) is shape of water level and hydraulic head

SPATIAL WATER FLOW OF UNCONFINED AQUIFER



Equation for non-homogeneous and anisotropic:

$$\frac{\partial}{\partial x}\left(K_x\, h \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\, h \frac{\partial h}{\partial y}\right) + N = S\ \frac{\partial h}{\partial t}$$

Equation for homogeneous and isotropic aquifer – Boussinesq equation:

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x} \right)+\frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y} \right)+\frac{N}{K}=\frac{S}{K}\ \frac{\partial h}{\partial t}$$

Non-homogeneous, isotropic aquifer:

$$\frac{\partial}{\partial x}\left((h-f)\frac{\partial h}{\partial x} \right)+\frac{\partial}{\partial y}\left((h-f)\frac{\partial h}{\partial y} \right)+\frac{N}{K}=\frac{S}{K}\ \frac{\partial h}{\partial t}$$

Stationary water flow equation for homogeneous and isotropic aquifer with sources:

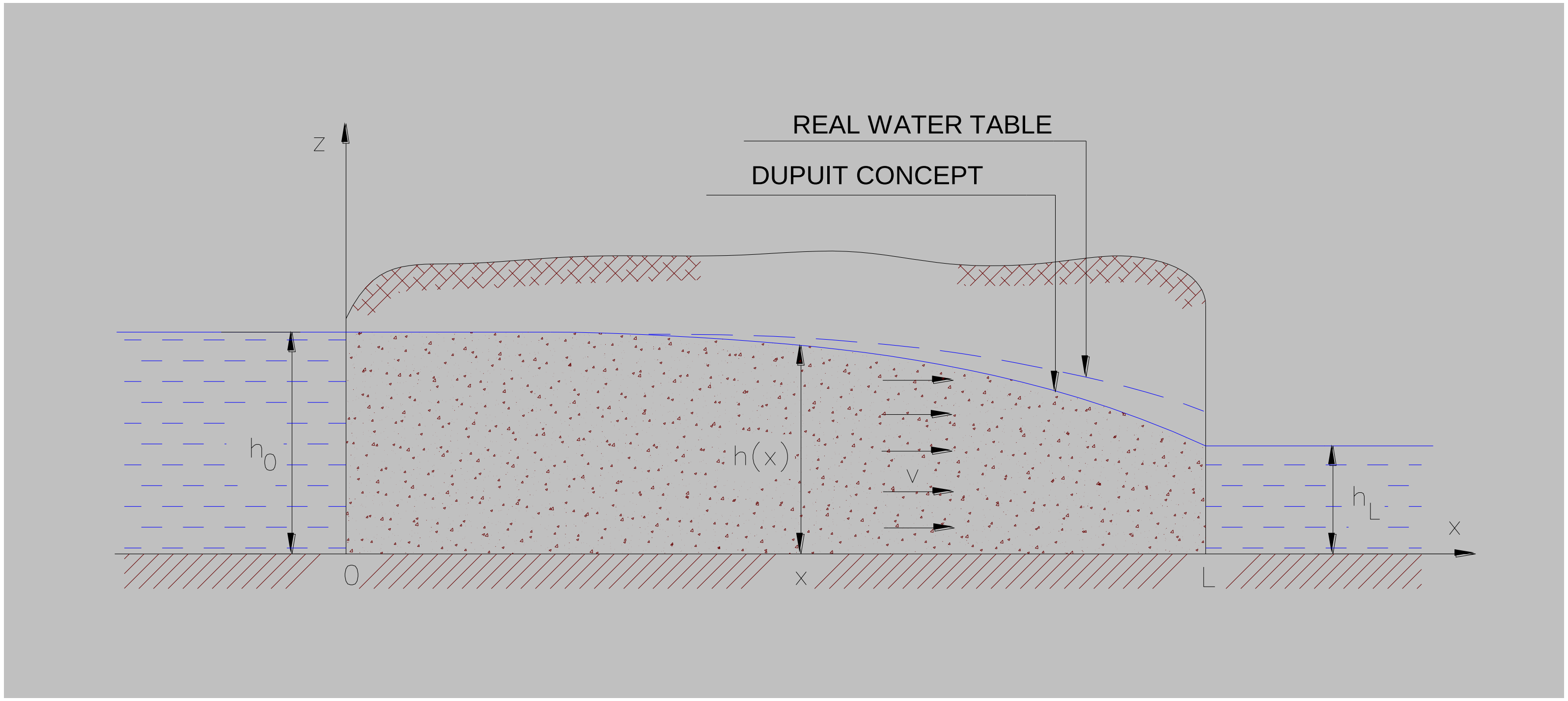
$$\frac{1}{2}\frac{\partial^2 h^2}{\partial x^2}+\frac{1}{2}\frac{\partial^2 h^2}{\partial y^2}+\frac{N}{K}=0$$

Inflow, outflow:

$$\frac{\partial^2 h^2}{\partial x^2}+\frac{\partial^2 h^2}{\partial y^2}=0$$

EXAMPLES OF DUPUIT CONCEPT – EXAMPLE 1

WATER FLOW IN SOIL WITH HORIZONTAL IMPERMEABLE LAYER



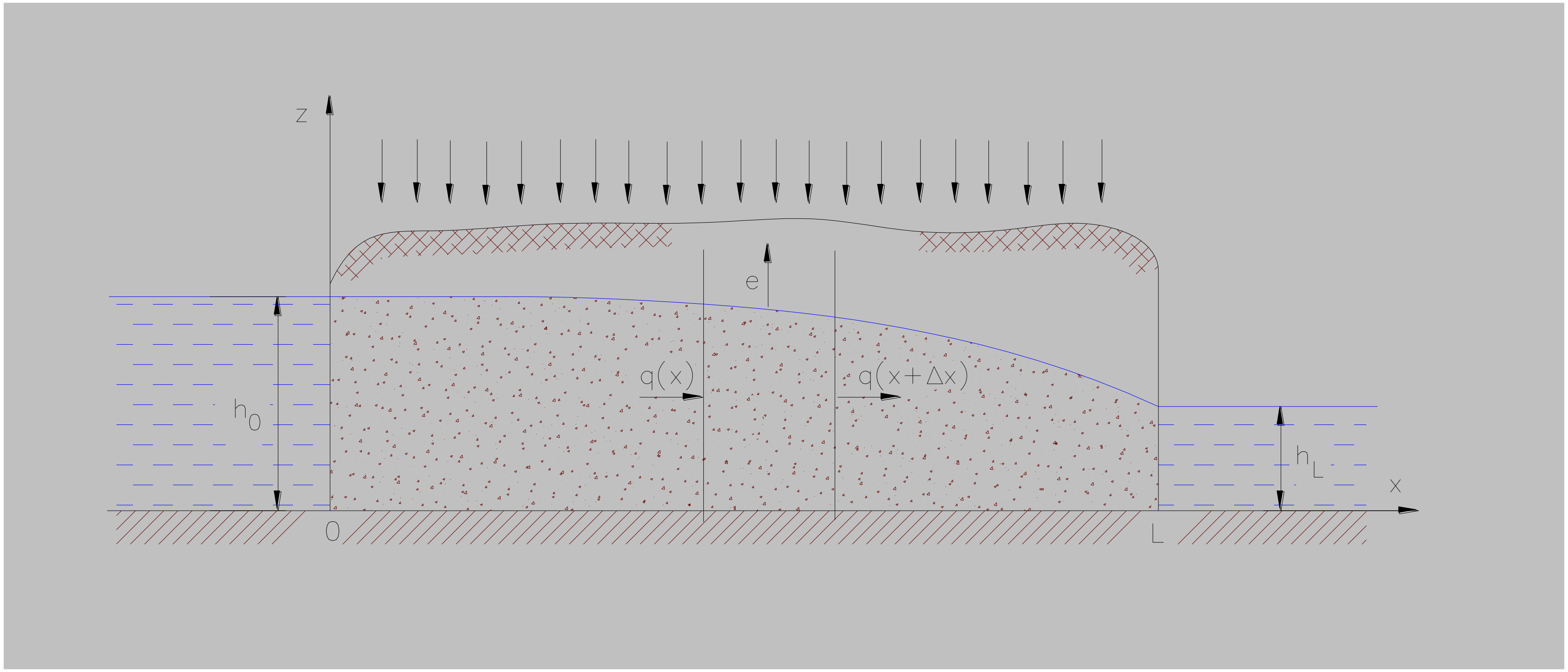
Specific flow rate:

$$q = K\,\frac{h_0^2 - h_L^2}{2L}$$

Hydraulic head:

$$h^2(x) = h_0^2 + \left(h_L^2 - h_0^2\right)\frac{x}{L}$$

WATER FLOW IN UNCONFINED AQUIFER WITH INFILTRATION OR EVAPORATION – EXAMPLE 2



Specific flow rate:

$$q_x = -\left(ex - \frac{eL}{2} + \frac{h_L^2 - h_0^2}{2L} K \right)$$

Hydraulic head:

$$h^2(x) = \frac{e}{K} x^2 + \left(\frac{h_L^2 - h_0^2}{L} - \frac{e}{K} L \right) x + h_0^2$$

THANK YOU FOR YOU ATTENTION