Groundwater hydraulics 6
Radial flow in wells

Martin Šanda, B673
martin.sanda@fsv.cvut.cz
Martina Sobotková, B609
martina.sobotkova@fsv.cvut.cz

Dept. Landscape Water Conservation

WELL

- **a vertical device used to collect water**
- **it is a type of underground construction**
- **a well is considered as water equipment**
- **it is necessary to follow certain regulations and laws when it is constructed**
- **but also when pumping water afterwards**

Wells built before 1955: you do not need a permit for water abstraction.

A well built after 1955: a water withdrawal permit is required.

A well of any age that is inoperable or unused and used solely for ornamental purposes: no water withdrawal permit is required.

WELL TYPES

- complete well
- incomplete well

ACCORDING TO THE COLLECTOR OPENING METHOD

- hydraulically perfect well
- hydraulically imperfect well
- fictitious well

ACCORDING TO THE FUNCTION OF THE WELL

- needle well
- injection well
- pumping well
- drilled well
- large diameter well – dug or drilled

ACCORDING TO THE TECHNICAL DESIGN

SYMETRICAL FLOW

1 D FLOW – a flow that is similar in all parallel planes (e.g. seepage of dams)

SYMETRICAL FLOW – is flow which is similar in all planes passing through a given line - axis of symmetry and vector of velocity at any point and time lies in the plane defined by this point and the line. So the flow is solved in one plane – flowing near by wells.

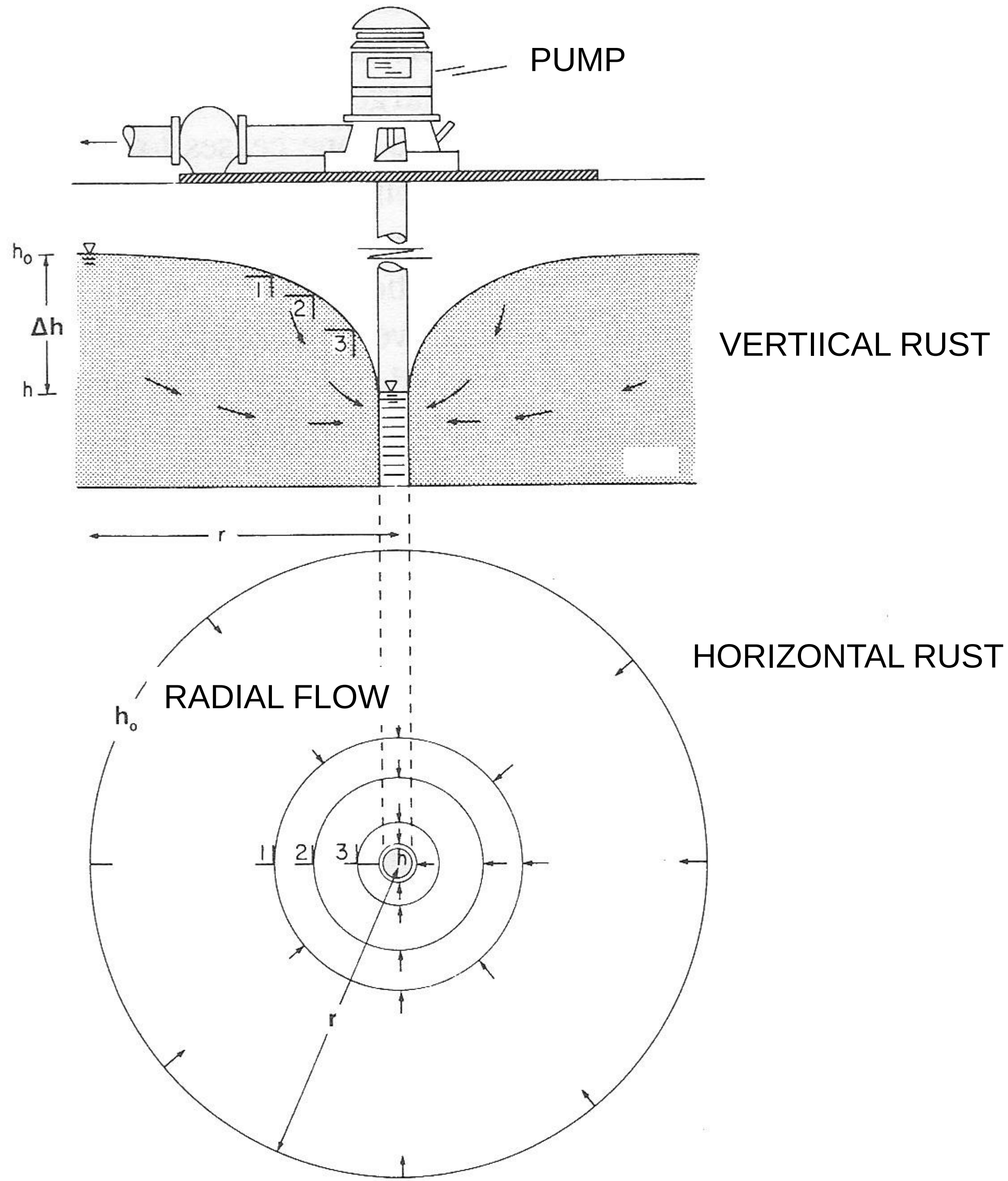
Cylindric coordinates: r,z,ϑ.

The water flow around wells is actually three-dimensional – solving equations for describing the flow is very difficult – so the hydraulic approach is applied. Due to the symmetry of the flow around the well the number of independent variables can be reduced.

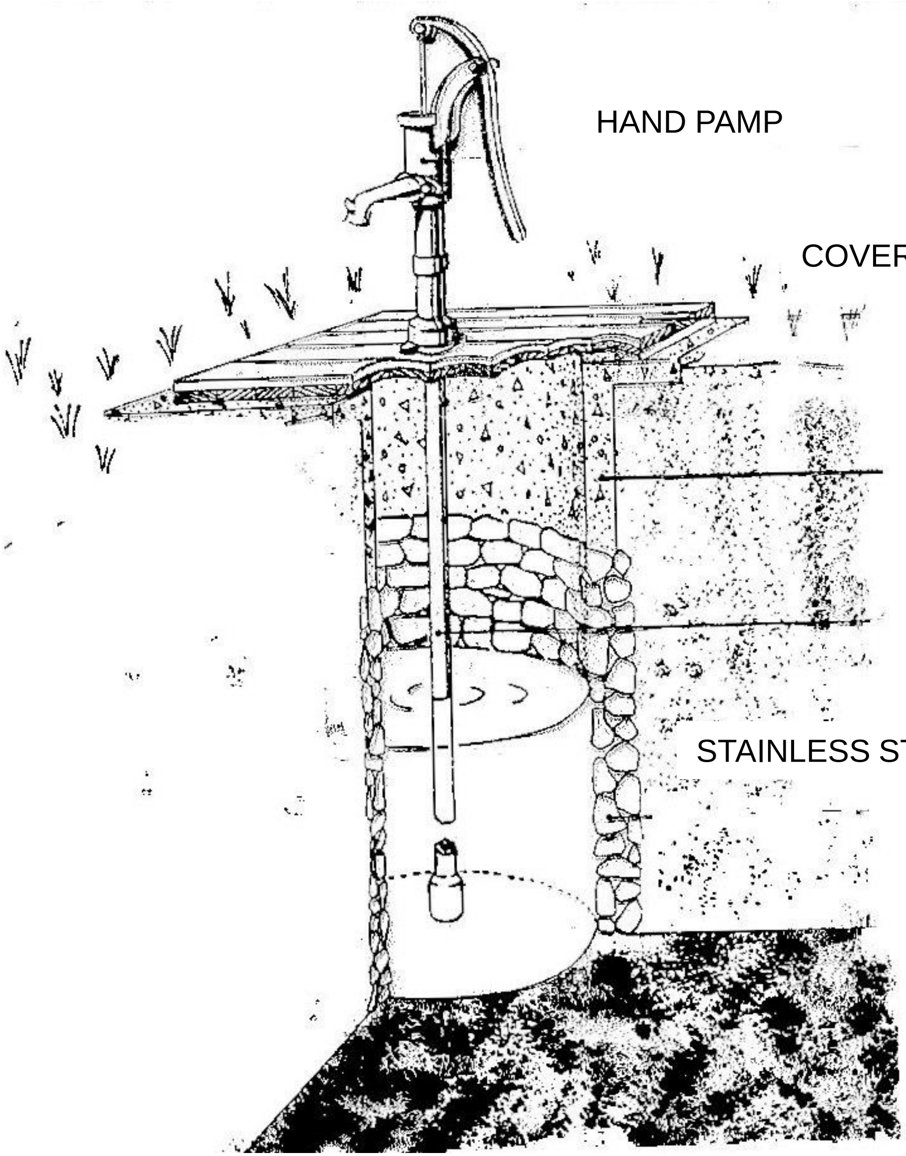
Dupuit´s assumption

- *hydraulic height*
- *hydraulic gradient on vertical constant*

Therefore, in the final equation, the function describing the groundwater level depends only on one spatial variable – r.


PUMP
VERTIICAL RUST
HORIZONTAL RUST
RADIAL FLOW
h_o
Δh
h
r
1
2
3

WELL TYPE – digging wells

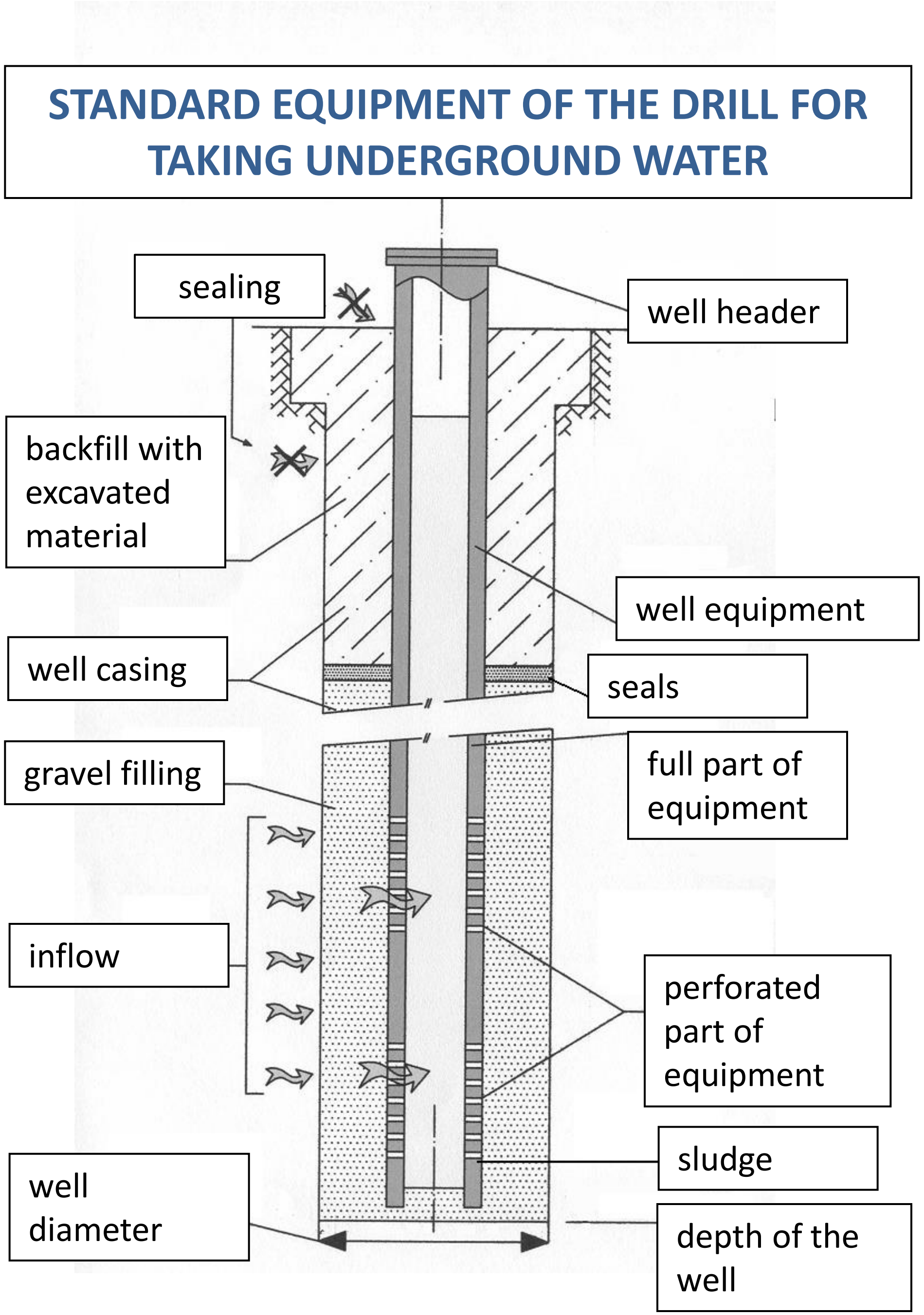


Advantage:

large accumulation

Disadvantage:

- water is taken from the first horizon of water level – possible contamination of water (from surface)
- fluctuations of water supply due to the intensity of rain



WELLS *are used for pumping groundwater from aquifers*

Well dividing by function:

- pumping wells
- sink wells

Dividing by method and depth of installation:

- complete wells
- incomplete wells

Dividing by pressure conditions:

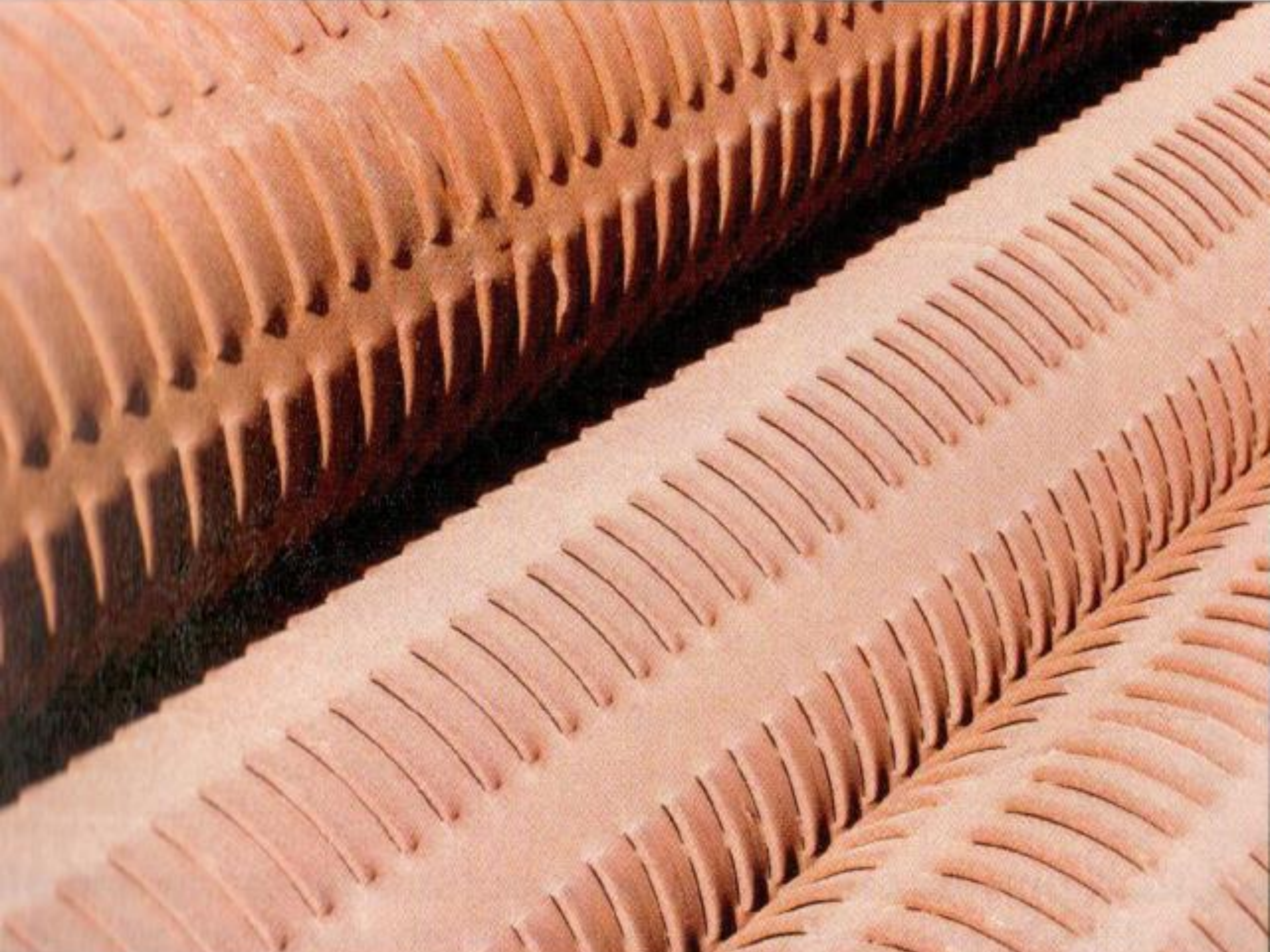
- classical
- artesian

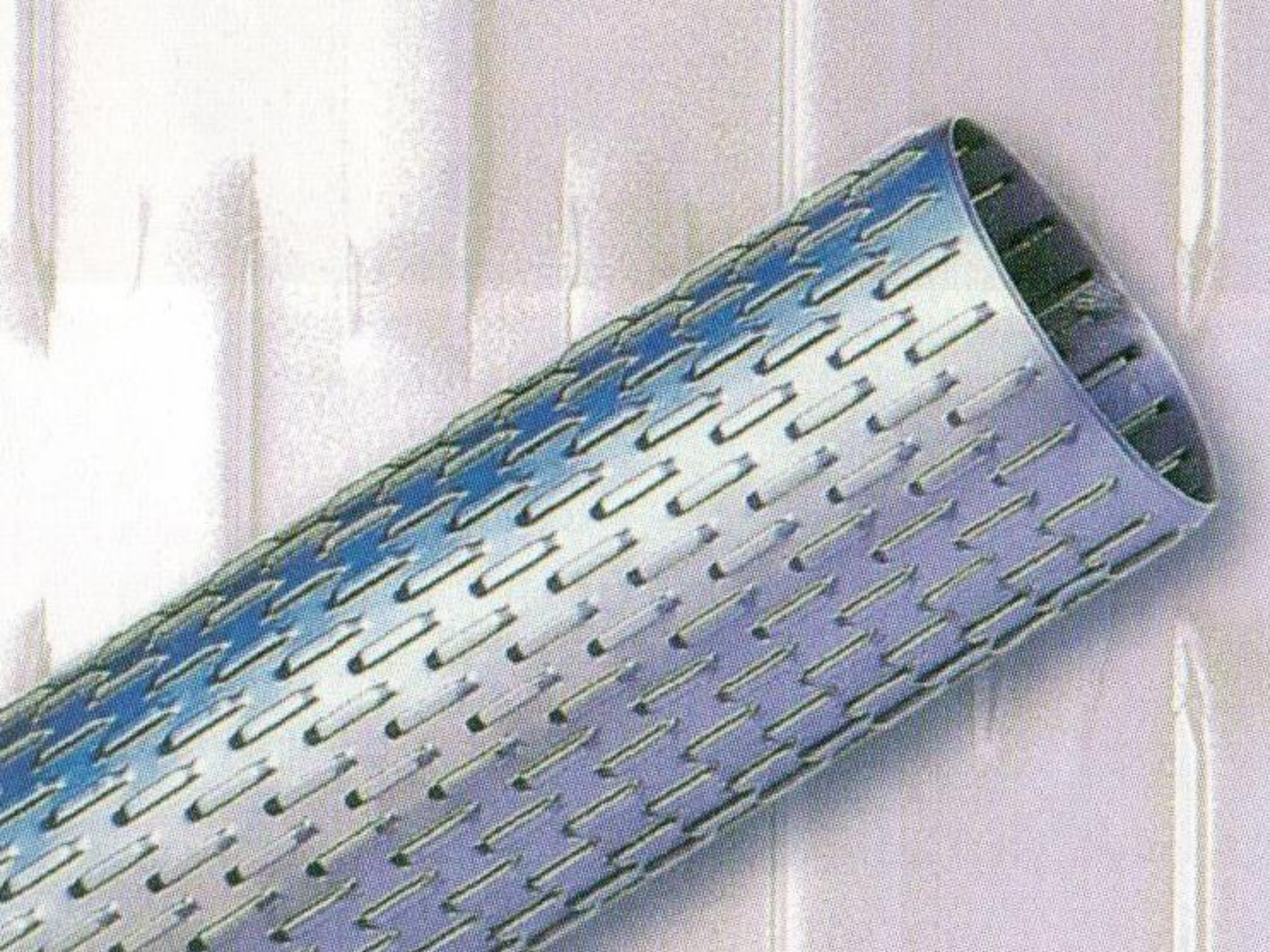
When the ground water is pumped – the water level is decreased and a depression cone is formed.
Infinite aquifer - depression cone is increasing - the flow is unsteady.

Simple solution – we assume a quasi-steady state when changes in the depression cone are insignificant.

Range of wells – distance from the center of the well, where the water level is almost equal to the water level before pumping.

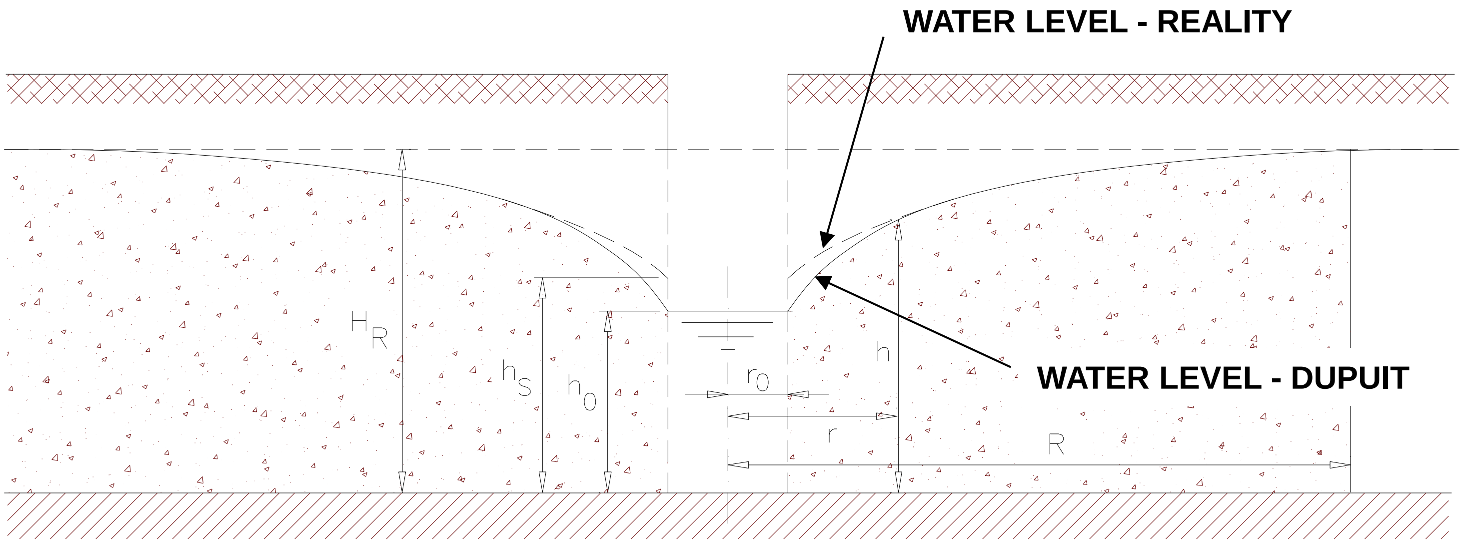
$$R = 3000z\sqrt{K}$$





ASSUMPTIONS OF WATER FLOW AROUND WELLS

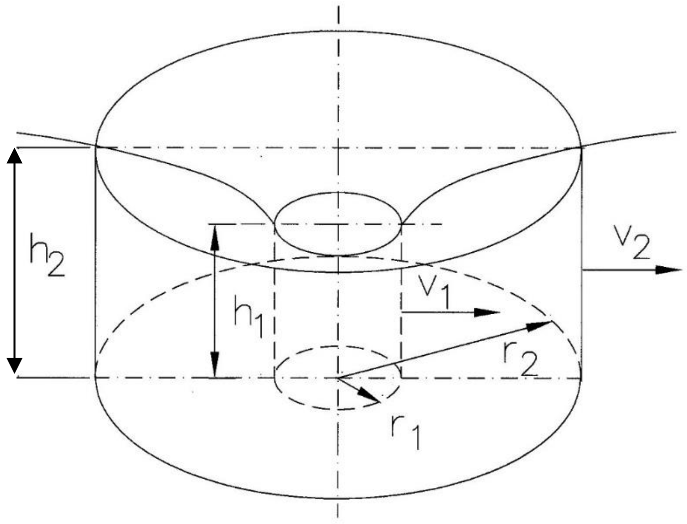
- validity of Darcy law
- validity of Dupuit´s assumption
- looses on well´s walls is neglected
- horizontal impermeable layer
- confined aquifer with constant head



FLOW AROUND FULL WELL

Steady state: water level in well h_0, steady state water level HPV H_R in distance R (range of well).
Water flow into the well is rotationally symmetrical with the axis of symmetry identical to the well axis.
Looking for the function *h (r)* – it describes the shape of the water level.

FLOW IN COMPLETE WELLS



•The flow of water through any cylindrical surface of radius $\mathbf{r_0 \leq r \leq R}$ is constant and can be expressed as:

$$Q = 2\pi r h(r) v(r)$$

$$Q = 2\pi r h(r)\left(-K \frac{dh}{dr}(r)\right) \qquad Q = 2\pi r q(r)$$

After including boundary conditions $r = r_0$, $h = h_0$ a $r = R$, $h = H_R$ we obtain equation expressing of water level and equation for determination of taken amount of water:

$$h^2(r) = H_R^2 - \frac{Q}{\pi K} \ln \frac{r}{R}$$

$$Q = \frac{\pi K \left(H_R^2 - h_0^2\right)}{\ln \frac{r_0}{R}}$$

Q value is negative - **pumping well** (flow is going upstream -r)
Q value is positive - **infiltration well** (flow is going in the direction +r).

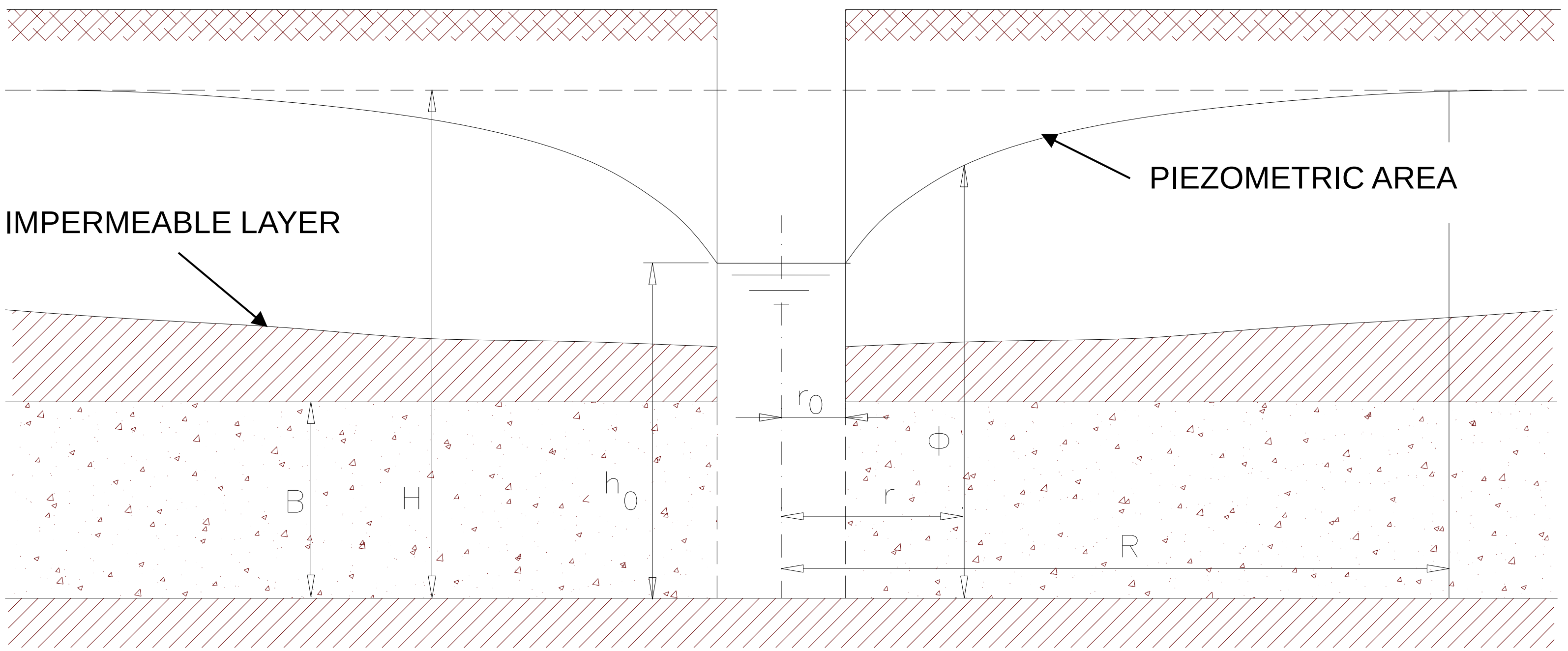
Girinsky potential – for solving wells
The derivation in any direction is the specific flow q in this direction. The flow through the cylindrical surface of radius r can then be expressed as:

$$Q = 2\pi r q(r) \qquad Q = 2\pi r \frac{dG}{dr}(r)$$

Po integraci a zavedení okrajové podmínky pro r = R ve tvaru G(r) = G(R) dostaneme rovnici:

$$G(r) = G(R) + \frac{Q}{2\pi} \ln \frac{r}{R}$$

FLOW AROUND ARTESIAN WELLS



Water flow through the cylindrical surface with a radius of $r_0 \leq r \leq R$ and B is constat and can be expressed:

$$Q = 2\pi r B\left(-K\frac{d\phi}{dr}(r)\right) \qquad -\frac{Q}{2\pi BK}\frac{1}{r}dr = d\phi$$

After introducing the boundary condition (it expresses the piezometric height) at a distance corresponding to the well radius (r = R Φ = H). We get the expression - it is possible to determine the piezometric height at any distance from the center:

$$\phi(r) = H - \frac{Q}{2\pi K B} \ln \frac{r}{R}$$

After including the second boundary condition (expressing the position of water level in the well (r = r_0 Φ = h_0), we obtain the equation:

$$Q = 2\pi K B \frac{H - h_0}{\ln \frac{r_0}{R}}$$

Girinsky´s potential:

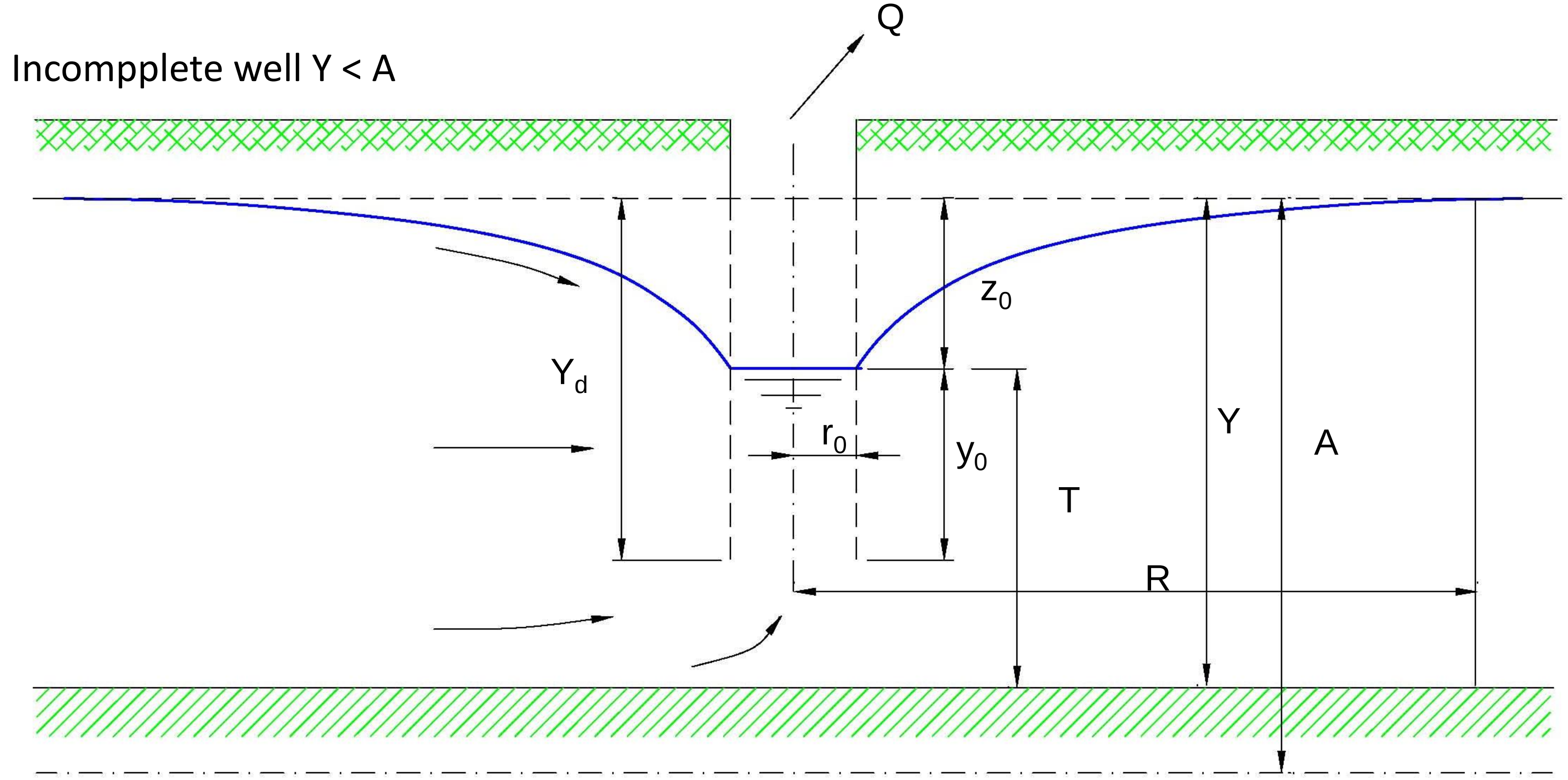
$$G(r_0) = G(R) + \frac{Q}{2\pi} \ln \frac{r_0}{R}$$

where G(r_0) and G(R) is described from definition for confined aquifer:

$$G(r_0) = K B \left(\frac{B}{2} - h_0 \right) \qquad G(R) = K B \left(\frac{B}{2} - H \right)$$

FLOW FOR INCOMPLETE WELLS

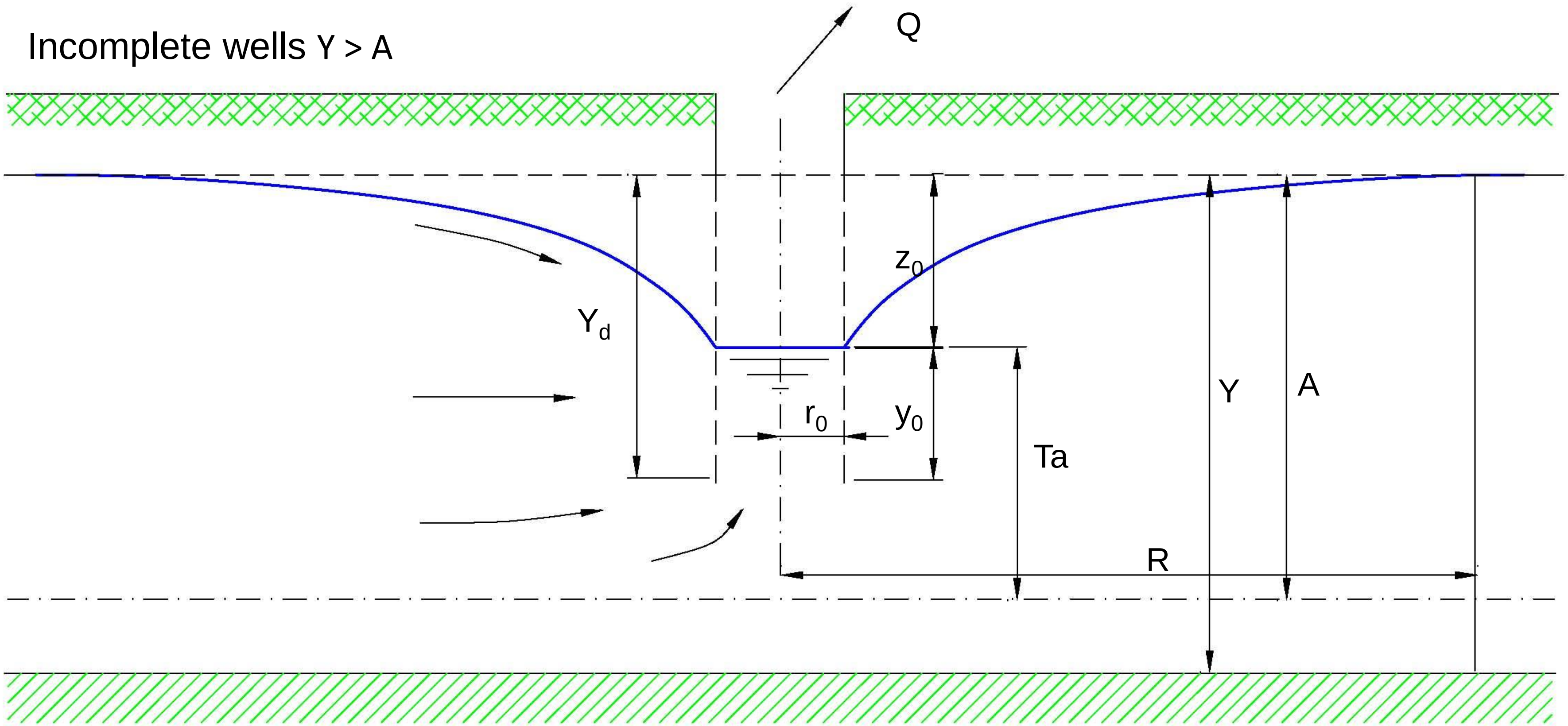
Active depth A – part of aquifer where the water is flowing into the well.
The water level decrease is larger for incomplete wells than for the complete wells at the pumping.



Forchheimer:

$$Y < A \quad Q = \pi K \frac{Y^2 - T^2}{\ln \frac{R}{r_0}} \sqrt{\frac{y_0 + 0.5 r_0}{T}} \sqrt[4]{\frac{2T - y_0}{T}}$$

Incomplete wells Y > A

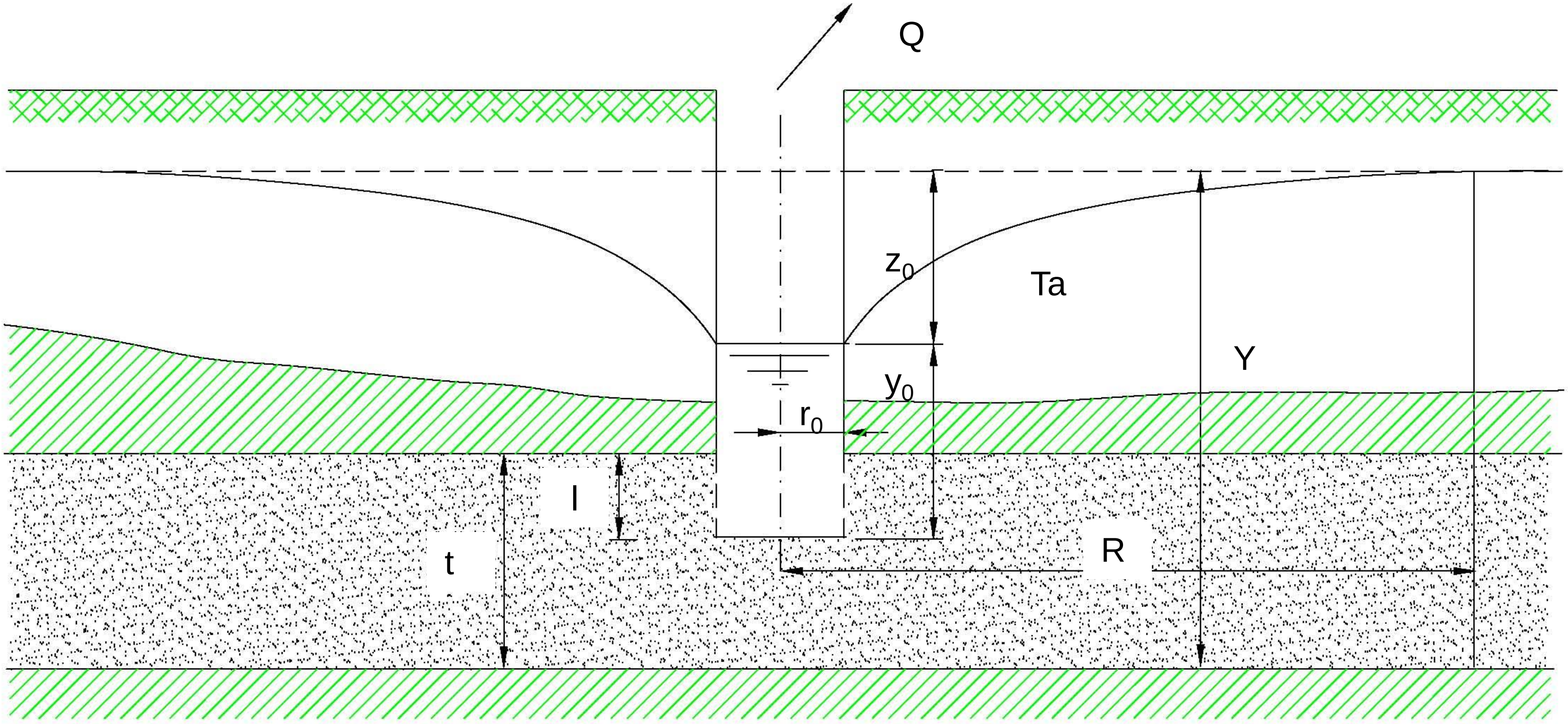


$$Y > A \quad Q = \pi K \frac{A^2 - T_a{}^2}{\ln \frac{R}{r_0}} \sqrt{\frac{y_0 + 0.5 r_0}{T_a}} \sqrt[4]{\frac{2T_a - y_0}{T_a}}$$

Active depth:

z_0/y_d	0.10	0.20	0.30	0.40	0.50	0.80
A/y_d	1.15	1.30	1.60	1.65	1.70	1.85

ARTESIAN WELL



Čertousov:

$$Q = \frac{2\pi K l z_0}{\ln \frac{R}{r_0}} \left[1 + \frac{5}{t} \sqrt{l r_0} \cos\left(\frac{\pi}{2} \frac{l}{t} \right) \right]$$

Principles of pumping

(capture zone)

shape and extent of the influence

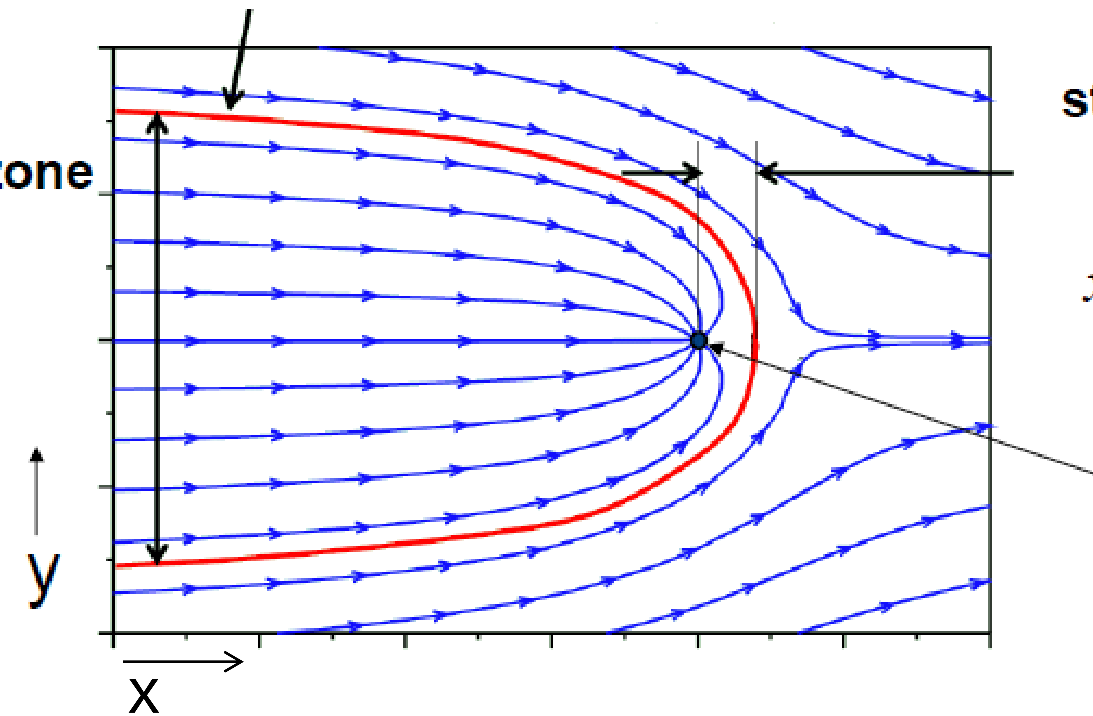
$$\frac{y}{x} = \pm \tan\left(\frac{2\pi T i y}{Q}\right)$$

shape and extent of the area influenced by pumping in the general groundwater flow relates to:

- **aquifer transmisivity T (m²/s)**
- **hydraulic gradient i (-)**
- **pumping intensity Q (m³/s)**

with of the zone

$$w = \frac{Q}{Ti}$$



stagnation point

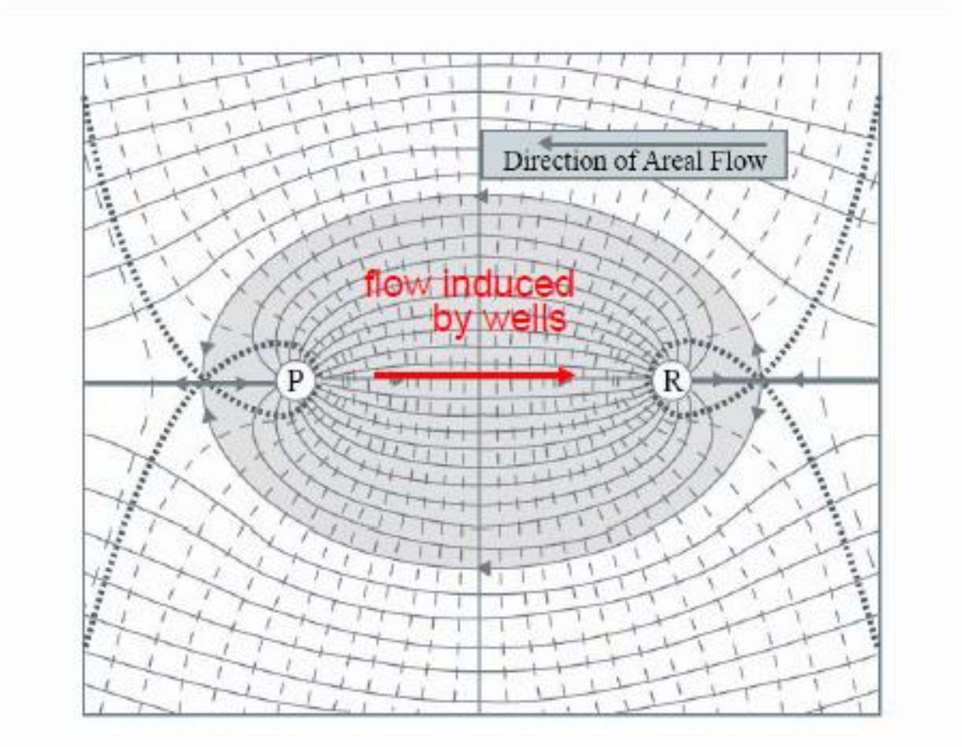
$$x = \frac{Q}{2\pi T i}$$

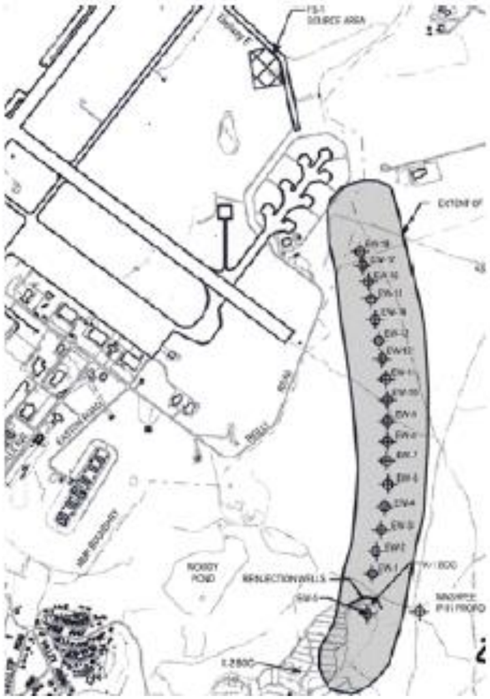
well

Principles of pumping
couples of infiltration and extraction wells

R - exctraction well
P – infiltration well

set of infiltration and extraction wells, or their combination can create dynamic protection of the groundwater and direct flow of contamination (e.g. invert the flow)





Optimalization of pumping to reach goals

- mathematical models – can help increase effectivity of pumping, evaluation of scenarios of i/e wells combination and rate of decontamination respecting properties of the environment
- effectivity of the borehole depends on well designet and situated screen and proper sand/gravel filter (appropriate grain sizes), hydraulic "complete" well at the bedrock, pumping test
- disadvantage is in relatively low effectivity, i.e. long time to remediation is accomplished – economical and practical aspects discrepancy