Groundwater hydraulics 7
Systems of wells

Martin Šanda, B673

martin.sanda@fsv.cvut.cz

Martina Sobotková, B609

martina.sobotkova@fsv.cvut.cz

Dept. Landscape Water Conservation

SYSTEM OF WELLS

SYSTEM OF WELLS

- group of wells which interact to each other
- distance of one well overlaps the other well
- for solving the system of wells – we use Girinsky´s potential

Girinsky´s potential – based on the principle of superposition
When two functions are general solutions of a linear partial differential equation then each linear combination of these functions is a solution of the given equation.

Example: We solve the groundwater flow in the unconfined aquifer with N numbers of wells with radius of r_{01}, r_{02}, ... r_{0N} and maximum distance of R_1, R_2, ...R_N. We withdrawn the amount of water from wells Q_1, Q_2, ... Q_N and the height of the water level out of the distance of wells is H.

After using of the principle of superposition we obtain the equation where the Girinsky´s potential is determined in any point of the system (x,y):

$$G(x,y) = -\frac{KH^2}{2} + \sum_{i=1}^{N} \frac{Q_i}{2\pi} \ln \frac{r_i}{R_i}$$

Where r_i is the distance of point with coordinates x,y from the centre axis of wells. If the distance is longer than R_i we get $\frac{R_i}{R_i}$.

Unconfined aquifer – position of the level in any location of the system:

$$h^2(x,y) = H^2 - \sum_{i=1}^{N} \frac{Q_i}{\pi K} \ln \frac{r_i}{R_i}$$

Confined aquifer – position of the piezometric level in any location of the system:

$$\phi(x,y) = H - \sum_{i=1}^{N} \frac{Q_i}{2\pi KB} \ln \frac{r_i}{R_i}$$

In the systeme we might have either pumping wells or infiltration wells.

Pumping wells: $Q < 0$

Infiltration wells: $Q > 0$

SOLUTION OF WELLS CLOSE TO IMPERMEABLE LAYERS OR STREAM, RIVER – METHOD OF IMAGINARY WELLS

In previous examples we assumed that the wells are located in the endless aquifer.

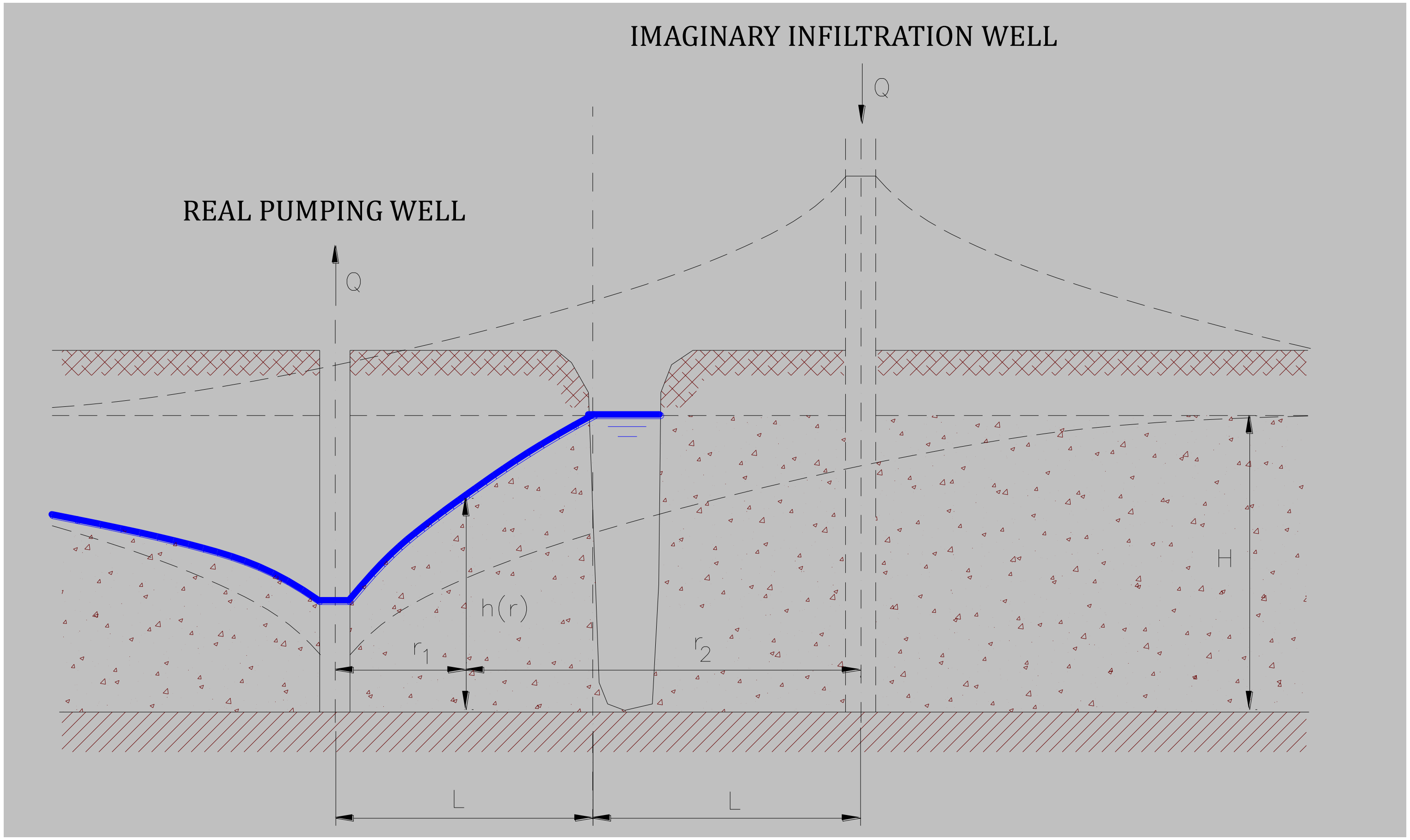
For now we will solve the problem: the well is located close to impermeable wall or stream or river – **METHOD OF IMAGINARY WELLS (SUPERPOSITION APPLICATION)**

In case that the boundary can be replaced by the line the solution is done by the method of imaginary wells. The effect of the boundary we replace by the imaginary well which is the same radius and maximum distance and we pump (infiltrate) the same amount of water as from the real well.

The height of the ground water level of the real well is solved by the system of real and imaginary wells. The boundary condition at the actual area have to correspond to the type of the boundary.

Using this method, Q has always a positive value.
Whether a pumping or infiltration well is used we express by the sign +/- in front of the part of the given well.

WELL CLOSE TO RIVER

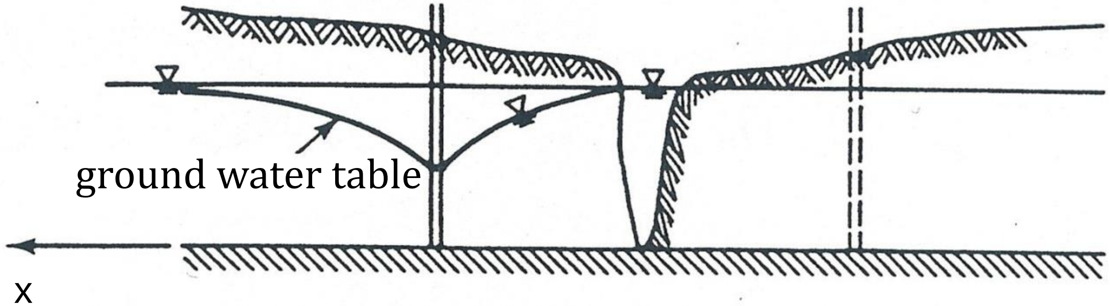


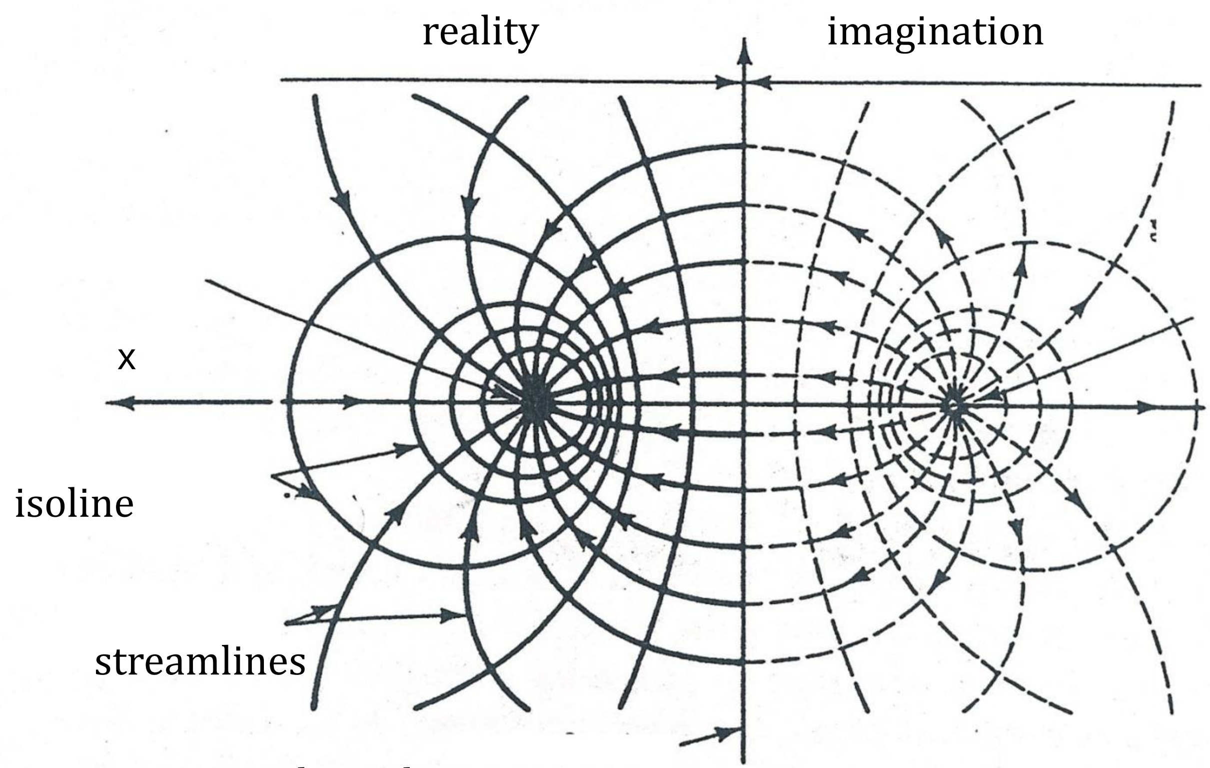
The height of water level in the real well we obtain from the eqation:

$$h^2(x,y) = H^2 + \frac{Q}{\pi K} \ln \frac{r_1}{r_2}$$

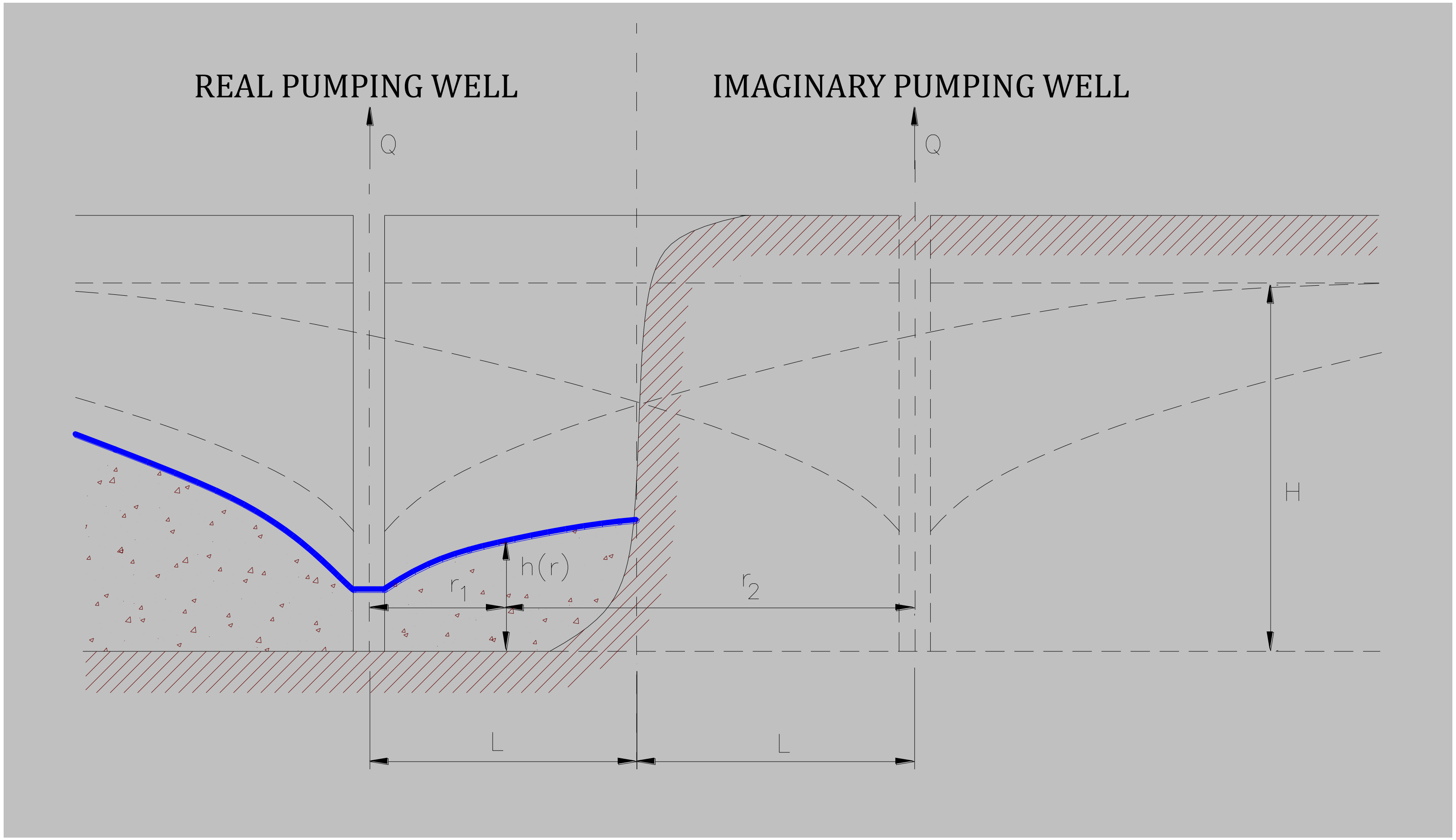
The boundary condition is fulfilled when $\mathbf{r_1 = r_2}$ and $\mathbf{h = H}$. If the distance of the well from the stream is greater than **R / 2** the water level in the well is at same height as if the well would have been situated in the endless aquifer.

STREAMLINES - WELL CLOSE TO STREAM





WELL CLOSE TO IMPERNEABLE WALL



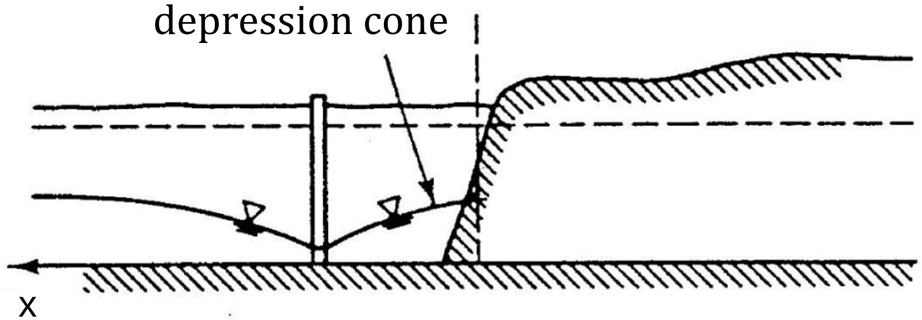
The height of water level around the real well we obtain from the eqaution:

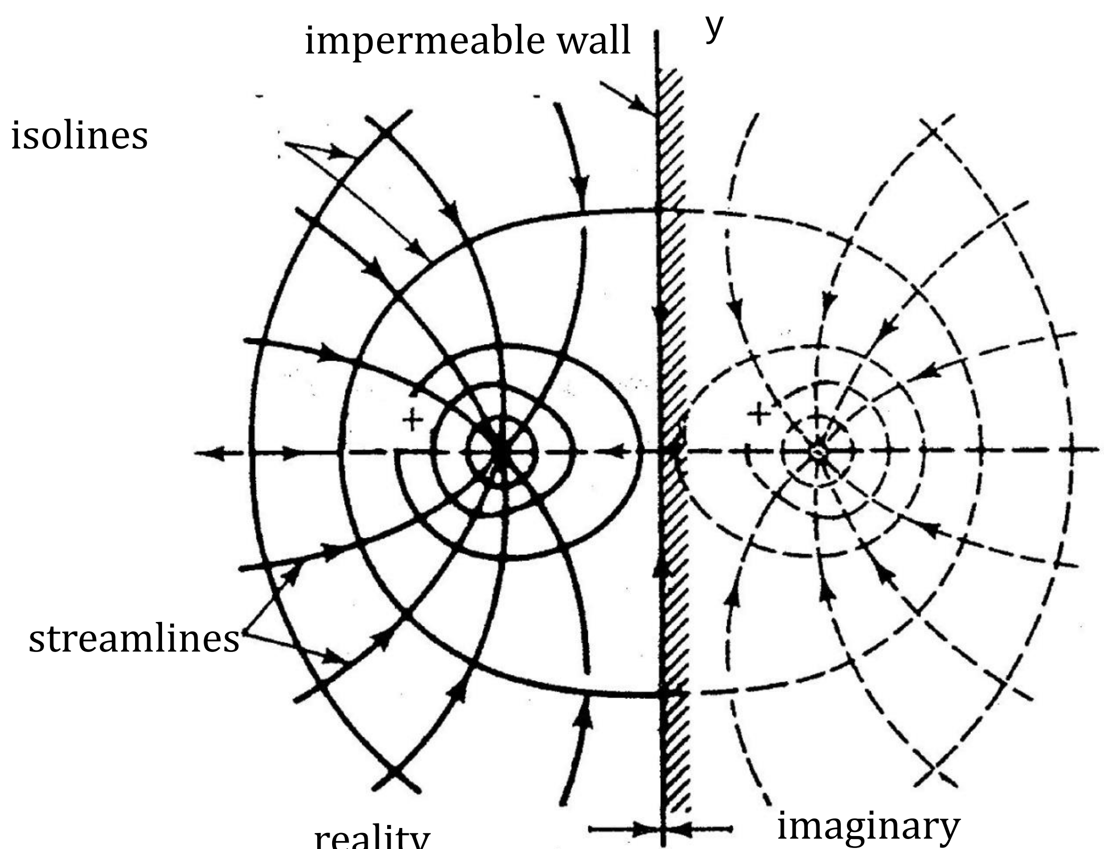
$$h^2(x,y) = H^2 + \frac{Q}{\pi K} \ln \frac{r_1 r_2}{R^2}$$

Boundary condition for the impermeable wall is 0 flow – verification: Girinsky´s potential:

$$q_i(x,y) = \frac{\partial G}{\partial r_i} = -\frac{Q}{2\pi r_i}$$

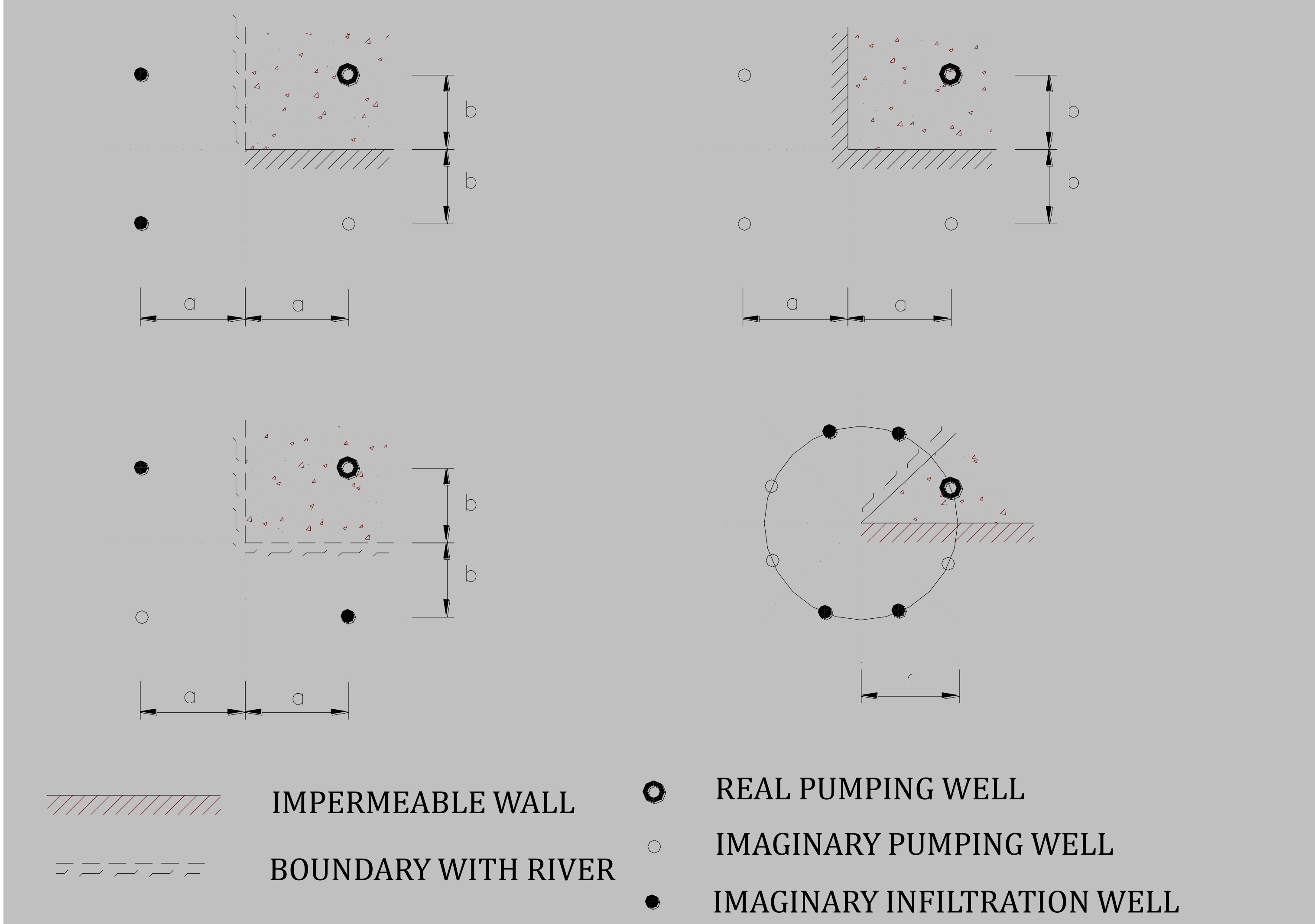
STREAMLINES - WELL CLOSE TO IMPERMEABLE WALL





Method of the fictitious wells is used in areas which are bounded by straight lines forming an angel of < 180 °.

Examples:

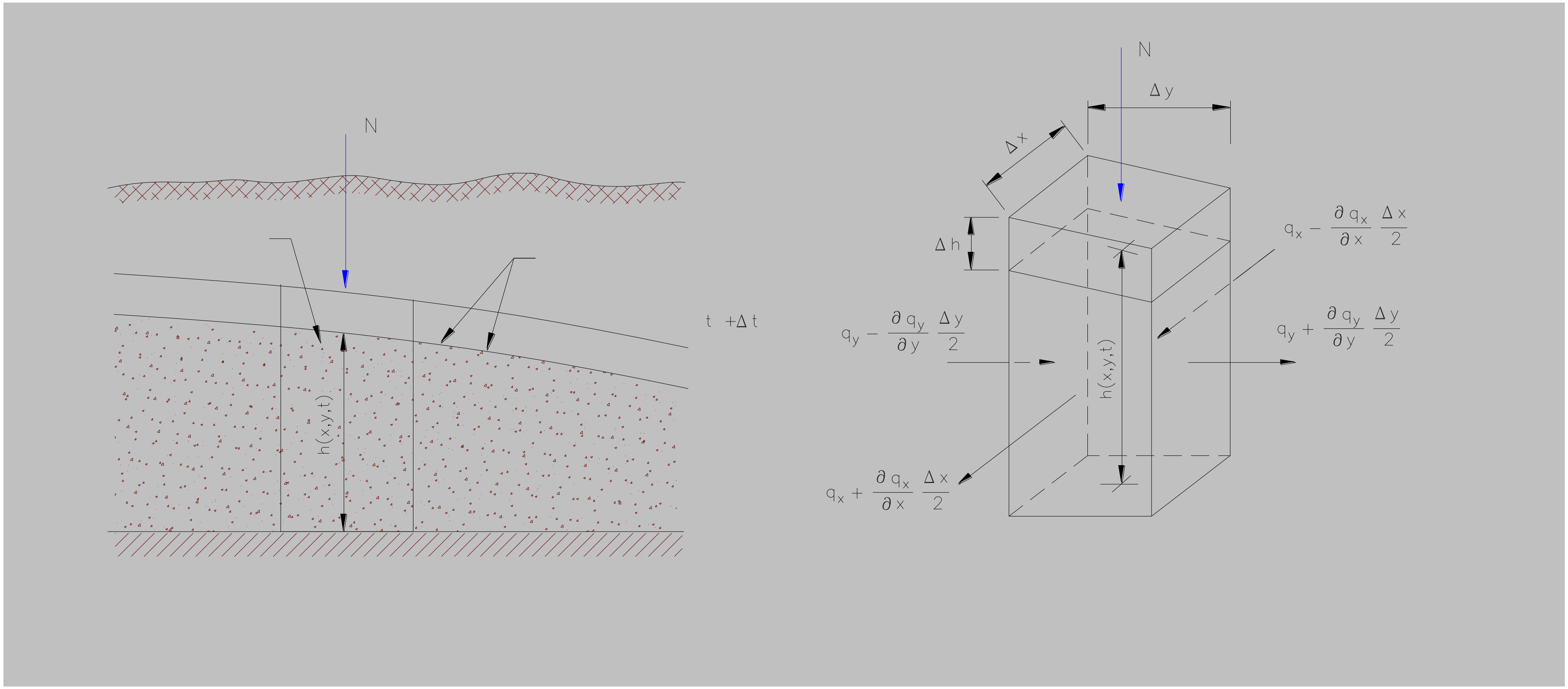


UNSTEADY FLOW IN UNCONFINED AQUIFER

It is described by Boussinesq´s equation.

Assumptions:

- Darcy law
- Dupuit´s assumption
- storativity is constant
- above ground water level – the flow is vertical and it is function of time



Boussinesq´s equation:

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y} \right) + \frac{N}{K} = \frac{S}{K}\frac{\partial h}{\partial t}$$

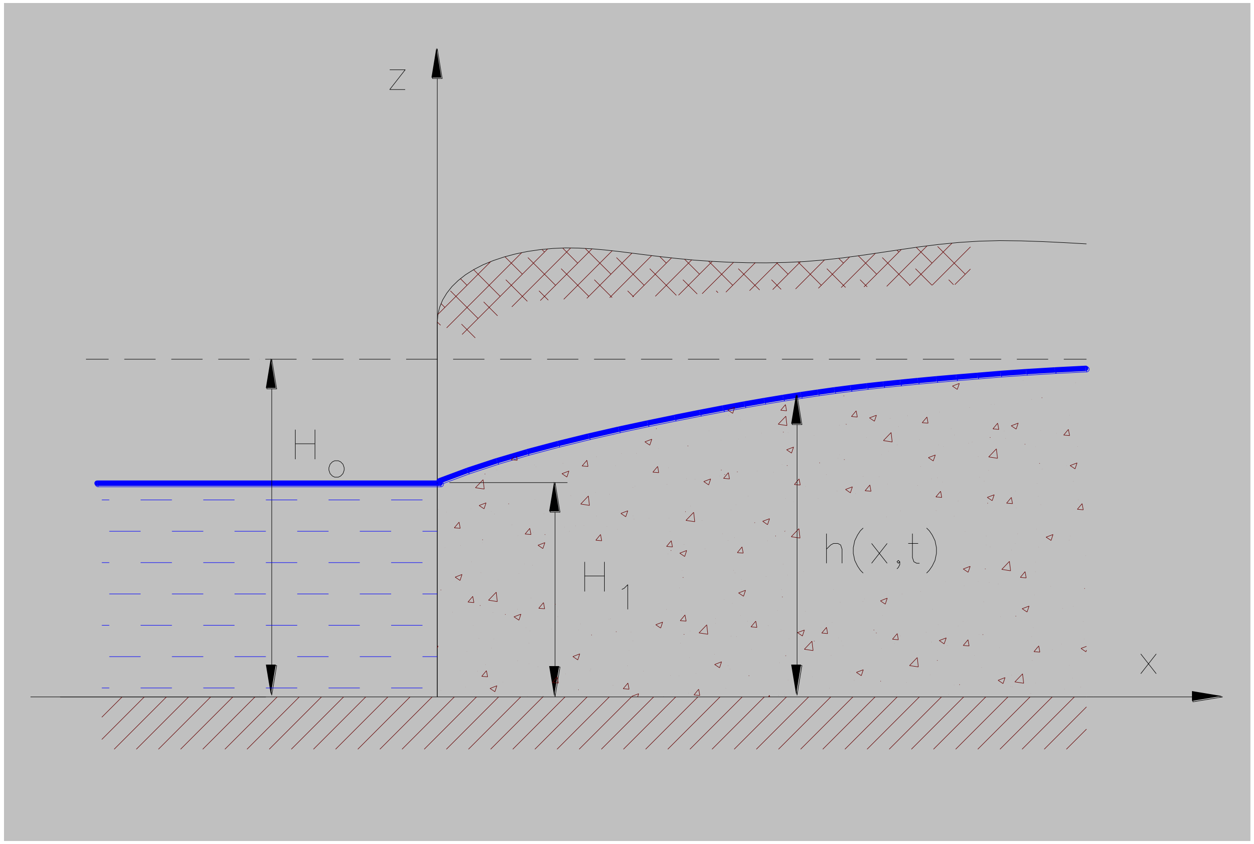
$$\frac{\partial h}{\partial t} = \frac{K}{2S}\left(\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} \right) + \frac{N}{S}$$

with using of the Girinsky potential:

$$S\frac{\partial h}{\partial t} + \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} - N = 0$$

Equations can be used to solve problems of the unsteady flow in the unconfined aquifer with changes in groundwater level over time. It occurs due to fluctuations in the water level on boundary or due to infiltration. After applying boundary and initial conditions, we get the function h (x, t), by which we calculate the position of the water level at any point and time. Because equations are not linear - linearization is required.

UNSTEADY 1D GROUND WATER FLOW



Ground water flow is described by: **Boussinesq´s equation** – solution by linearization:

$$\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2}$$

where $D = Kh_s/S$.

Infiltration is not included so: *u (x,t) = h (x,t).*

Initial condition $h(x,0) = H_0$ pro $x > 0$

Boundary condition: $h(0,t) = H_1$ pro $t > 0$

General solution of equation is defined:

$$h = C_2 + C_1\, erf(\eta), \qquad kde\ \eta = \frac{x}{2\sqrt{Dt}}$$

Where function **erf** is error function which is defined by:

$$erf(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} \exp(-\xi^2)\, d\xi$$

The values of function is tabularize.

It is agreed: when $\eta=0$, $erf(0)=0$ and $\eta \rightarrow \propto$, $erf(\propto)=1$.

After applying the initial conditions:

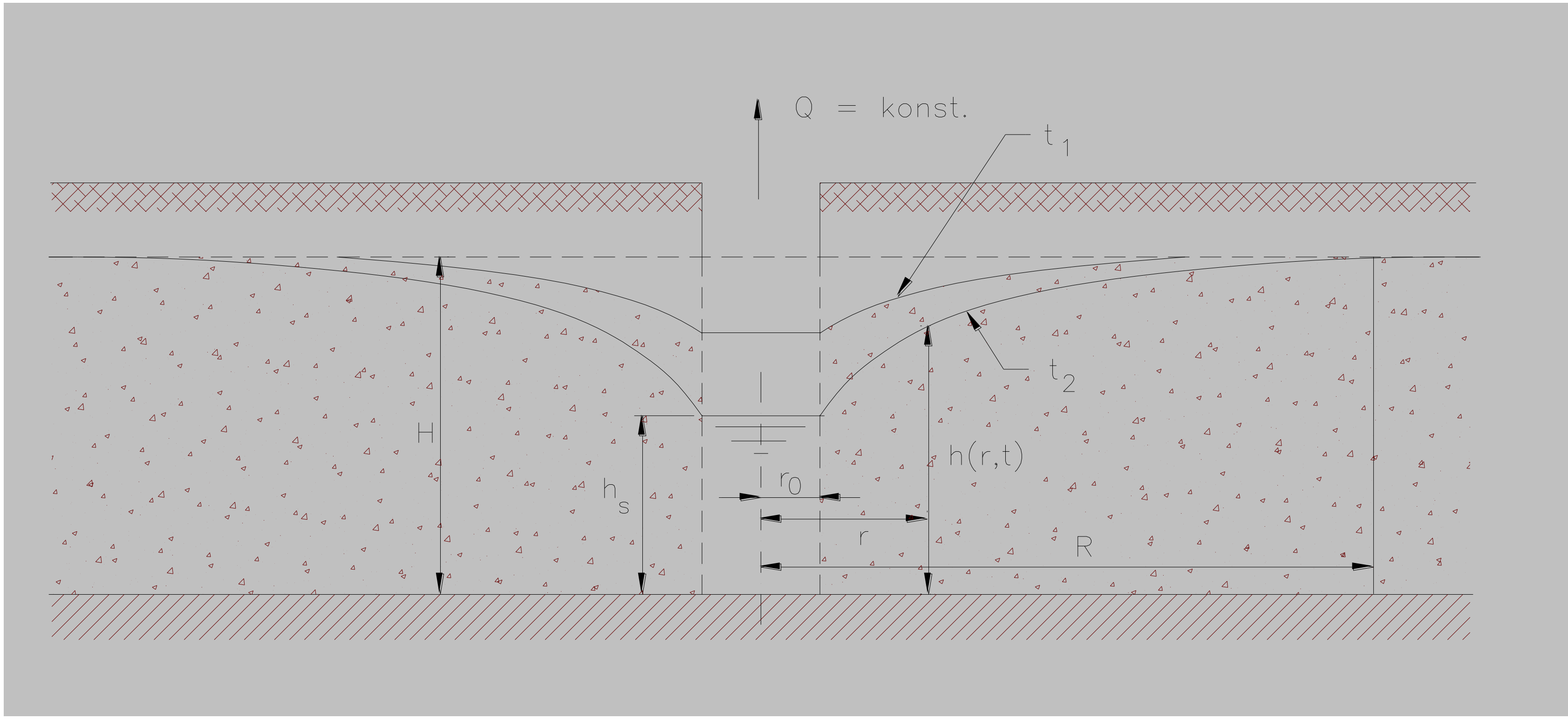
$$H_0 = C_2 + C_1$$

And boundary conditions: $H_1 = C_2$

The water level at different time and point we defined by:

$$h(x,t) = H_1 + (H_0 - H_1)\, erf\left(\frac{x}{2\sqrt{Dt}}\right)$$

FLOW INTO THE WELL WITH CONSTANT PUMPING



Initial condition: $u(r,t) = u_0$

$$S\frac{\partial h}{\partial t} = Kh\frac{\partial^2 h}{\partial r^2} + \frac{Kh}{r}\frac{\partial h}{\partial r} + K\left(\frac{\partial h}{\partial r}\right)^2 + P$$

For function is: $u_0 = H$ or $u_0 = H^2$
where H is water level before pumping.

Solution of equation:

$$u(r,t) = u_0 - \frac{u_1}{2} \int_{\frac{r^2}{4Dt}}^{\infty} \frac{\exp(-x)}{x} dx$$

Integral is called well´s function – W. Equation is defined:

$$u(r,t) = u_0 - \frac{u_1}{2} W\left(\frac{r^2}{4Dt}\right)$$

When ***u(r,t) = h(r,t)*** we obtain for decreasing the water level "*s*" a new equation:Theis´s equation:

$$s(r,t) = \frac{Q}{4\pi K h_S} W\left(\frac{r^2}{4Dt}\right)$$

THEIS EQUATION

We can replace the well´s function by logarithmic function when the value is lower than 0,03:

$$W\left(\frac{r^2}{4Dt}\right) \approx \ln \frac{2.25Dt}{r^2}$$

We get a simpification for equation of the water level:

$$s(r,t) = \frac{Q}{4\pi K h_S} \ln \frac{2.25Dt}{r^2}$$

If we used a substitution $u(r,t) = h^2(r,t)$ than the equation is defined:

$$s(r,t) = \frac{Q}{2\pi K} \ln \frac{2.25Dt}{r^2}$$

FLOW INTO THE WELL WITH CONSTANT DECREASE OF WATER TABLE

Example: searching for the water table after rapid decrease of water level.

When t = 0 than the water table h_0 is constant h_0 . At t1 the water level decrease to h_1. The water level is kept by the variable subscritption.

Initial condition:

$$u(r,0) = u_0 \qquad r \geq r_0$$

Boundary condition:

$$u(r_0, t) = u_1 \qquad t \geq 0$$

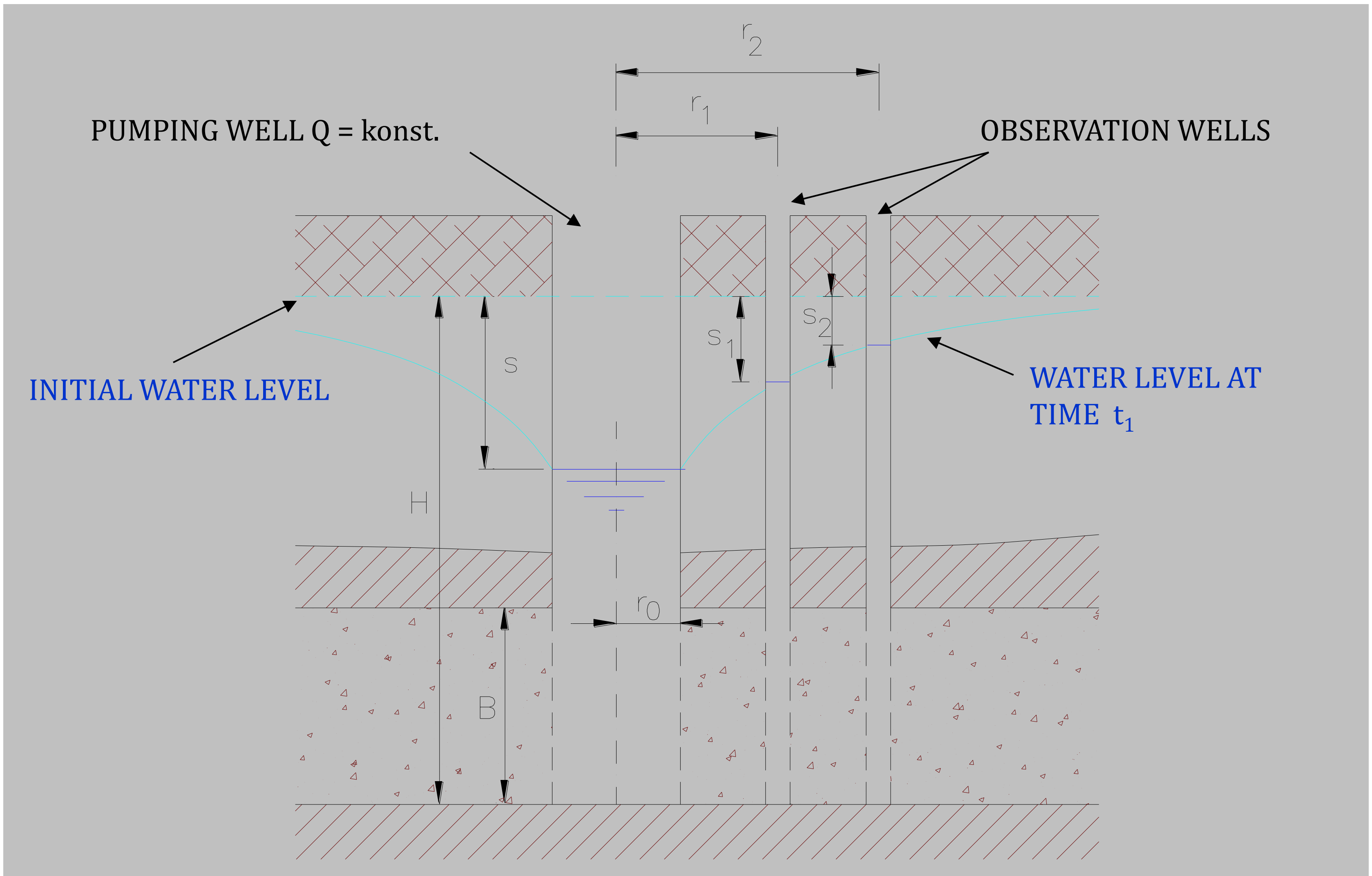
Solution is done by equation:

$$u(r,t) = u_0 + (u_1 - u_0) \int_{\frac{r-r_0}{\sqrt{4Dt}}}^{\infty} \exp(-x^2) dx$$

Two ways of pumping tests – determine the hydraulic conductivity and storativity.

PUMPING TEST FOR UNSTEADY WATER FLOW

The most common way: pumping constant amount of water from the well while the water level is measured in monitoring wells. A decrease of the water level over time is marked.



Notes of pumping test

The evaluation is performed by using graphical - analytical methods. Comparison of measured drop in the water level with type curves is made. The curves were derived on the basis of the theory of water flow for the specific pumping test.

EVALUATION OF PUMPING TESTS DOWNLOAD BY THEISE

In a homogeneous isotropic confined aquifer with a constant height B.
A pumping test of constant yield is carried out.

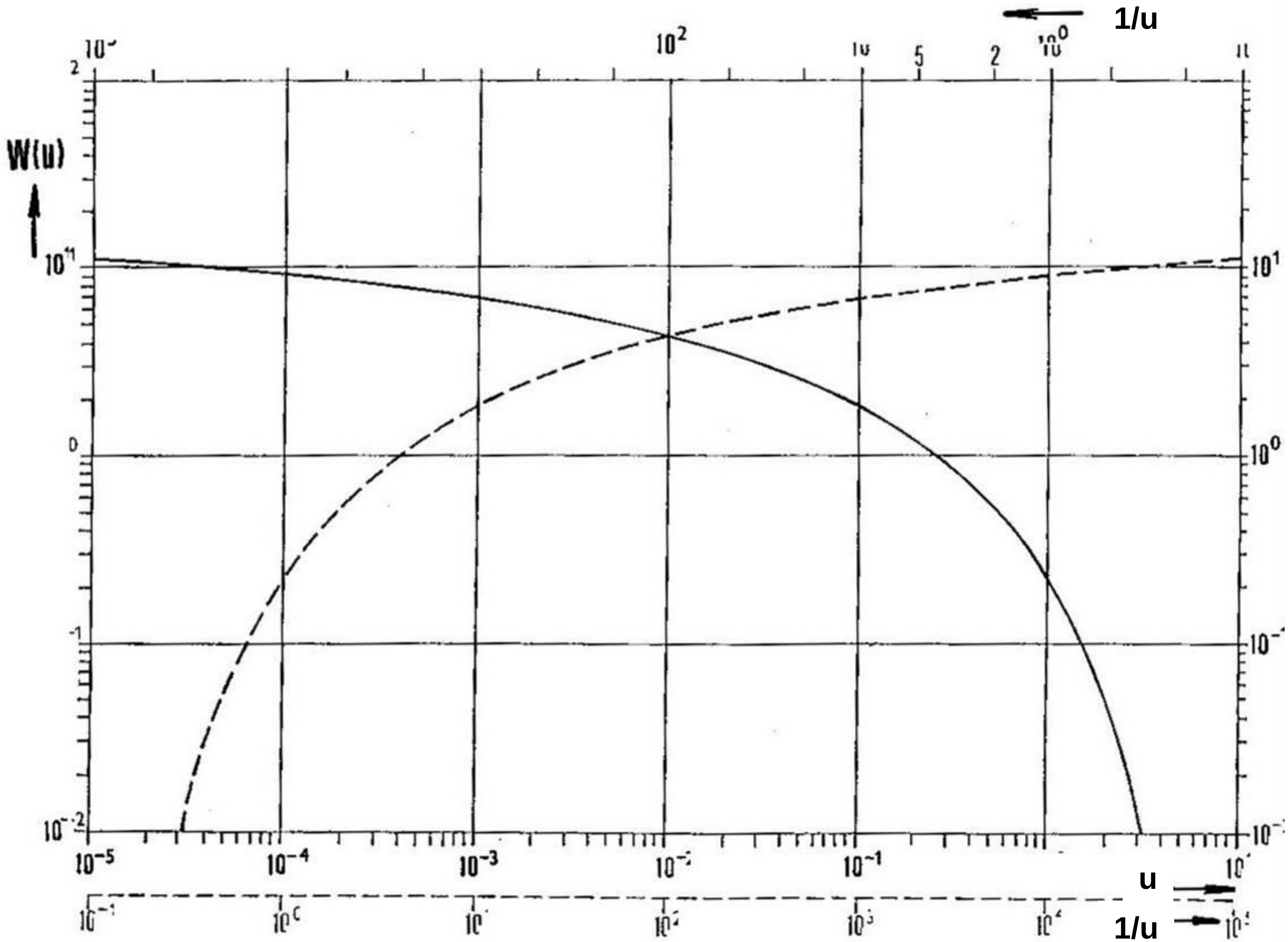
- complete pumping well - r_0,
- constant pumping amount of water Q – water level H
- at a distance r_1 - observation well - water level is measured over time

Theis´s equation:

$$s(r,t) = \frac{Q}{4\pi T} W(u) \ , \qquad u = \frac{r^2 S}{4T\,t}$$

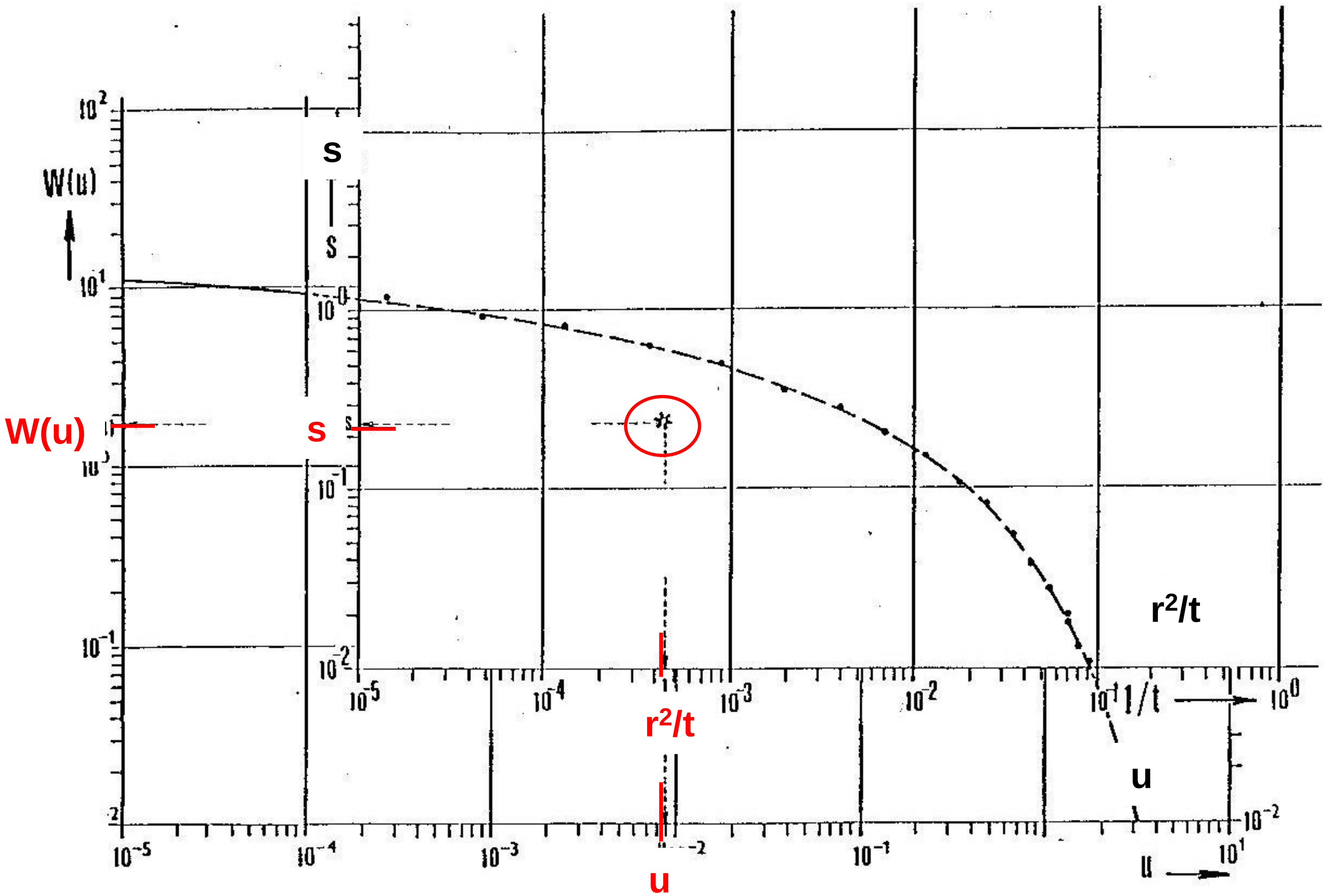
The shape of the well function *W (u)* is shown on a type curve.

Graph of function W(u)



The evaluation: plotting decrease of the water level dependent on the time in monitoring well.

It is best to draw this on transparent paper and then place this curve on the graph of the well function so that the measured points are interlaced with the well.



Then we subtract the coordinates on both graphs for any point.

Then we subtract the coordinates on both graphs for any point and validation of equation is:

$$s(r,t)=\frac{Q}{4\pi T}W(u)\ ,\qquad u=\frac{r^2 S}{4Tt}$$

we subtract the readings and we can calculate the K and S values from:

$$K=\frac{Q}{4\pi B}\frac{W(u)}{s}\qquad T=KB$$

$$S=\frac{4uTt}{r^2}$$

THANK YOU FOR YOUR ATTENTION