Groundwater hydraulics 8
Numerical modelling

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Flow equation

Based on Darcy’s equation and Dupuit assumptions (planar horizontal flow)

- **For transient nonhomogeneous anisotropic environment**

$K_{x,y,z}$=fce(x,y,z)

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right)=S_0 \frac{\partial H}{\partial t}$$

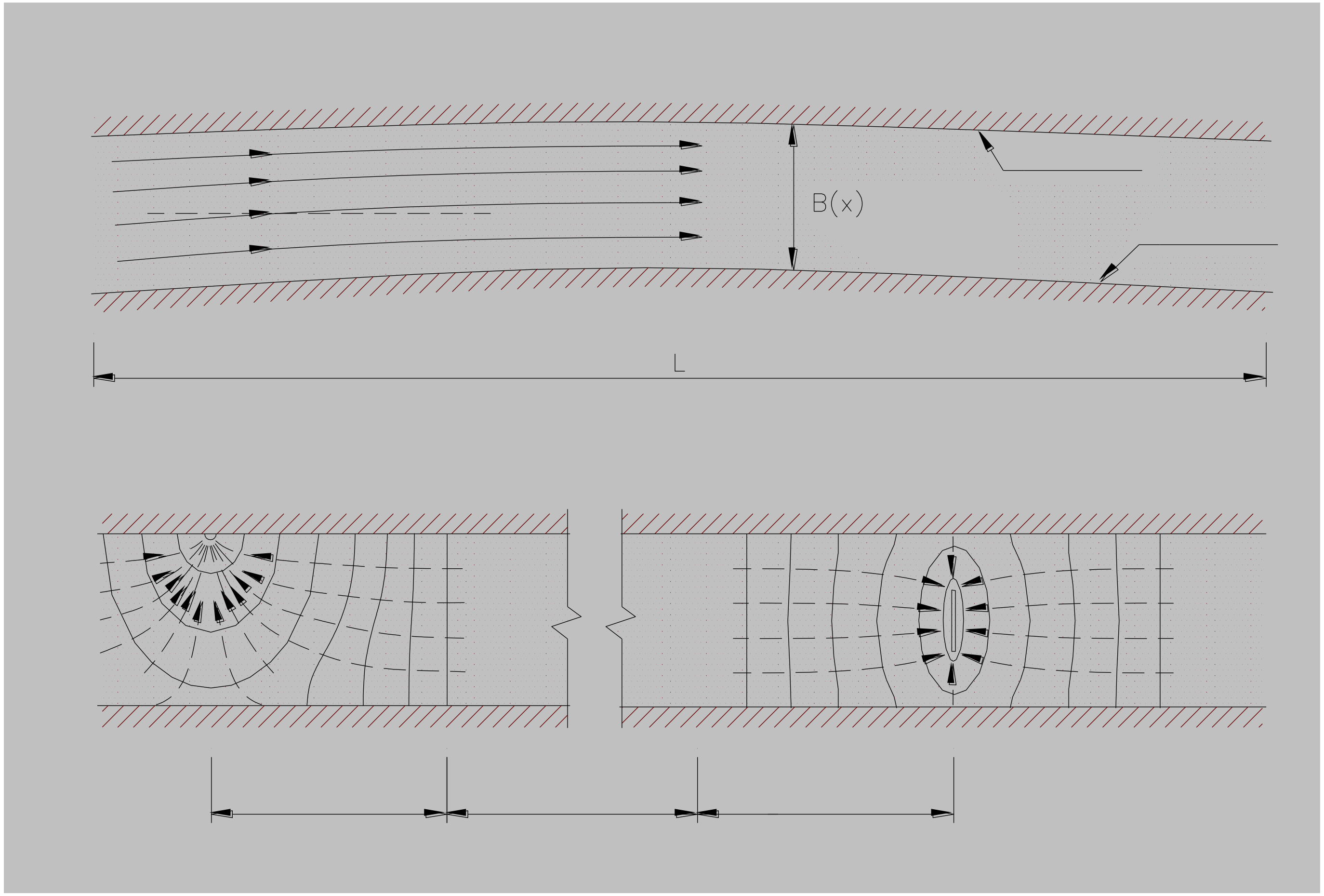
- **For steady state flow in homogeneous isotropic environment**

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}=0$$

kde So is specific storativity (sink or source)

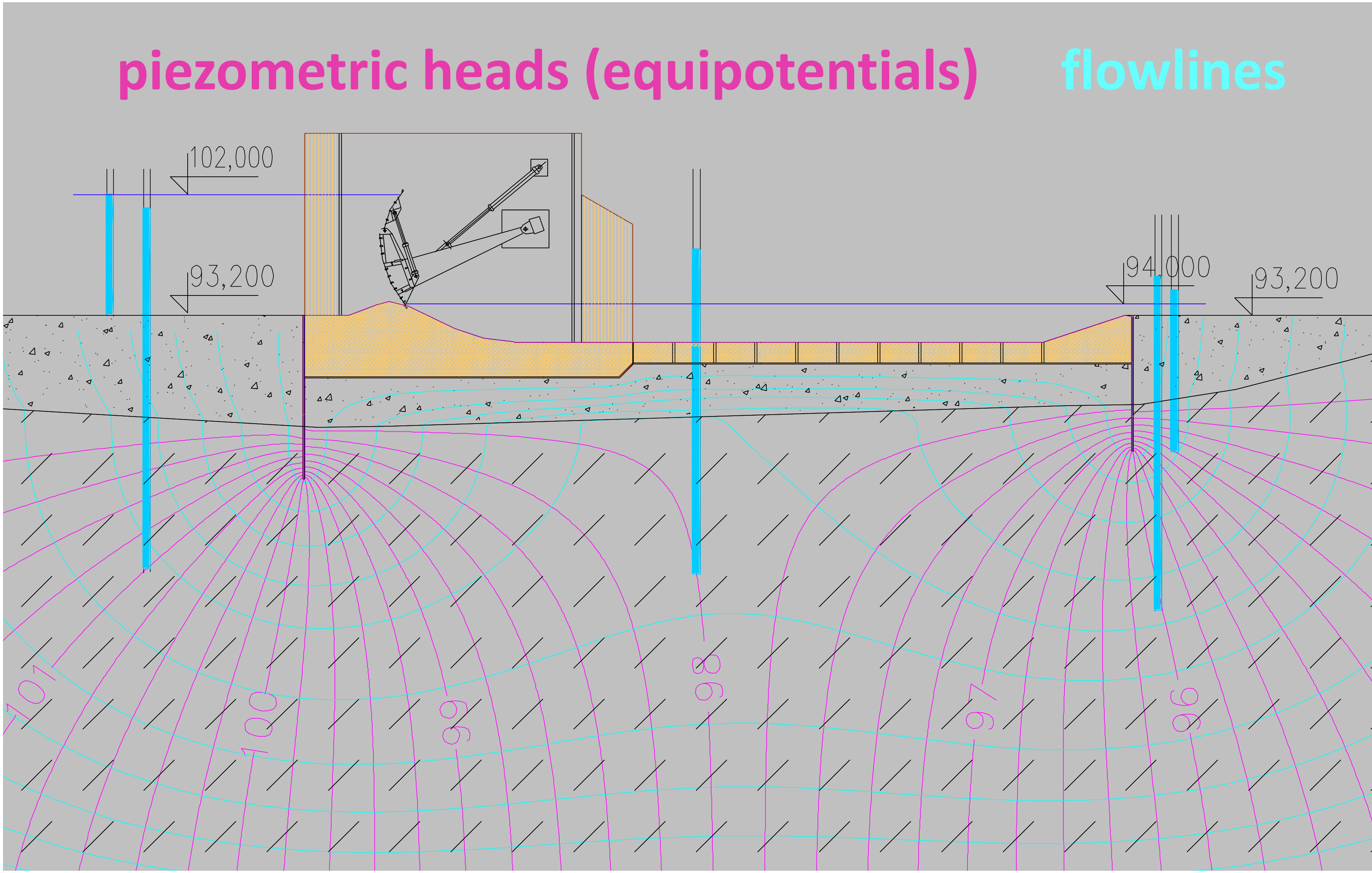
MATHEMATICAL DESCRIPTION OF 2D AREAL GROUND WATER FLOW

HYDRAULIC APPROACH – we neglect vertical parts of flow rate and we assume that the flow occurs in horizontal directions only.



By Valentová, J - Hydraulika podzemní vody

Groundwater under river construction (segment weir)



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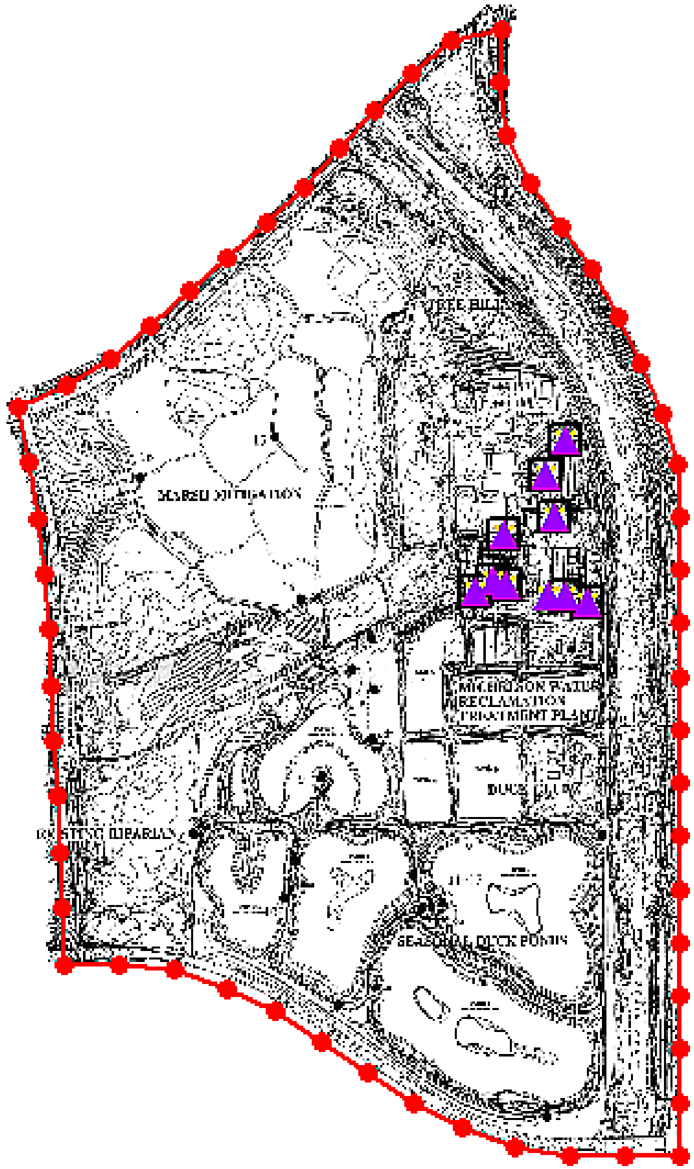
Conceptual model



- Map import
- Definition of boundary conditions
- Definition of inputs and outputs

Model creation

Necessary to make conceptual model prior to mathematical discretizaton

- Boundaries and communication with them
- **Hydrological balance – long term observations, targeted research (flow measurements), groundwater table monitoring, literature data**
- Analysis of hydraulic properties based on hydrogeological survey
- Stratigraphy of drilling activity – layering, geometry, bedrock

Ways to solve flow – initial and boundary conditions

In order to solve we must supply such conditions

Solution is given in the time and spatial frame

INITIAL CONDITIONS

Initial condition gives distribution of unknown (e.g. pressure head/groundwater table) within the domain of solution

BOUNDARY CONDITIONS

Boundary conditions define interaction of the system in domain with the outer world. Two common types (other just combination of these)

Pressure (Dirichlet) condition – known suction (or moisture) at the boundary of the domain

Flow (Neumann) condition – known flow (or no flow) perpendicular to the boundary of the domain

modified from Vogel, T – Podpovrchová hydrologie

Groundwater model inputs

- **Geometry** – shape, dimensions, thickness of layers
- **Hydraulic properties** – hydraulic conductivity, transmissivity, storativity, *dispersivity, retardation absorbtion for chemistry*
- **Boundary conditions** – contact with the outer environment – water table, inflow, infiltration/evaporation, pumping/recharge, *concentration* initial condition
- **Initial conditions** – water table in the area of model (plane)

Common software

Finite differences

MODFLOW (1983-2005) – USGeological Survey - 3-D aquifers (Fortran, freeware) - **FDM**

MODPATH – addition for pathlines

MT3D, RT3D (1990, 1998) - 3-D transport addition for MODFLOW (MT3D for advection dispersion eq, RT3D reactions modulel)

Finite elements

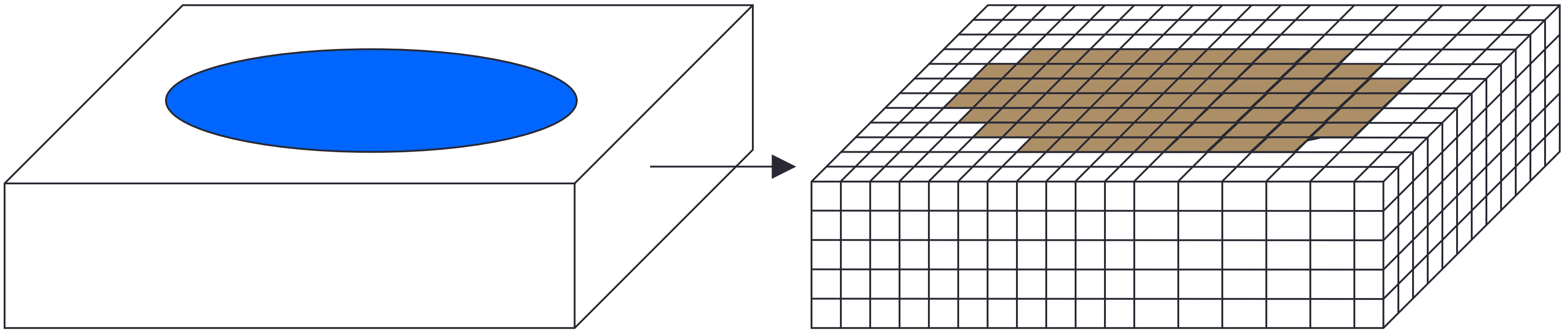
FEMWATER, FEFLOW -**FEM**

Commerical pre and post- processors

GMS, Groundwater Vistas, Visual Modflow, Argus ...

Ways to solve flow in – finite element method (FDM)

Discretization, simplification



Ways to solve flow – finite element method (FEM)

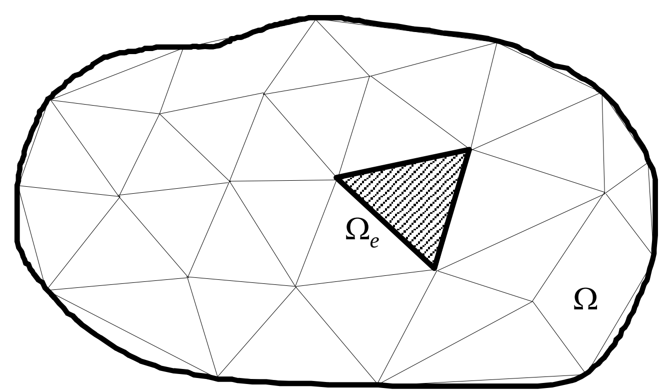
Area discretized (in time and space).

FEM suits better the non-linear equations

FDM (Finite Difference Method) is used for Laplace eq. – groundwater

In 1D – FEM and FDM almost same. FEM aims better the nonregular boundary and localization of boundary conditions, sinks and sources

FEM discretization of the domain



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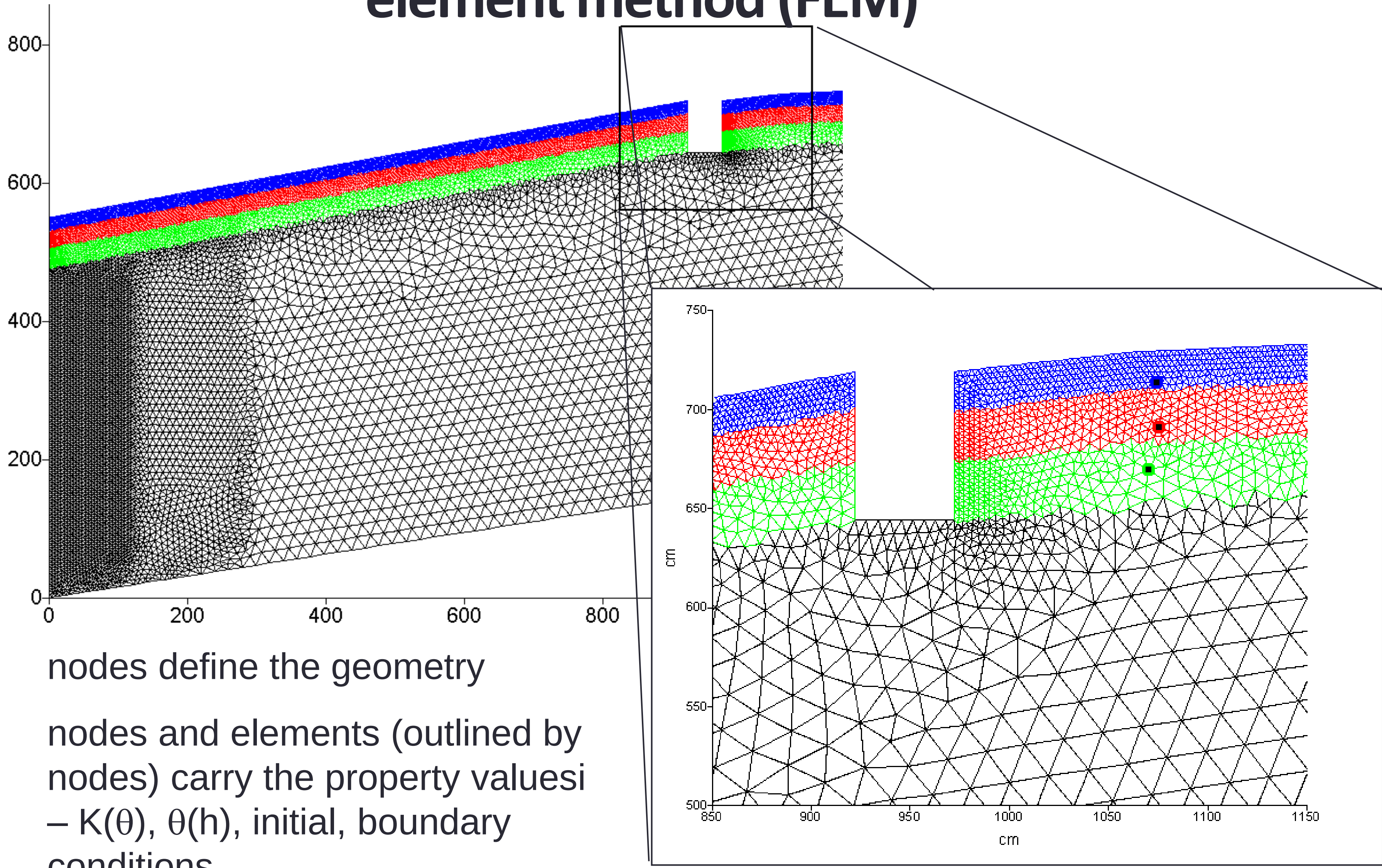
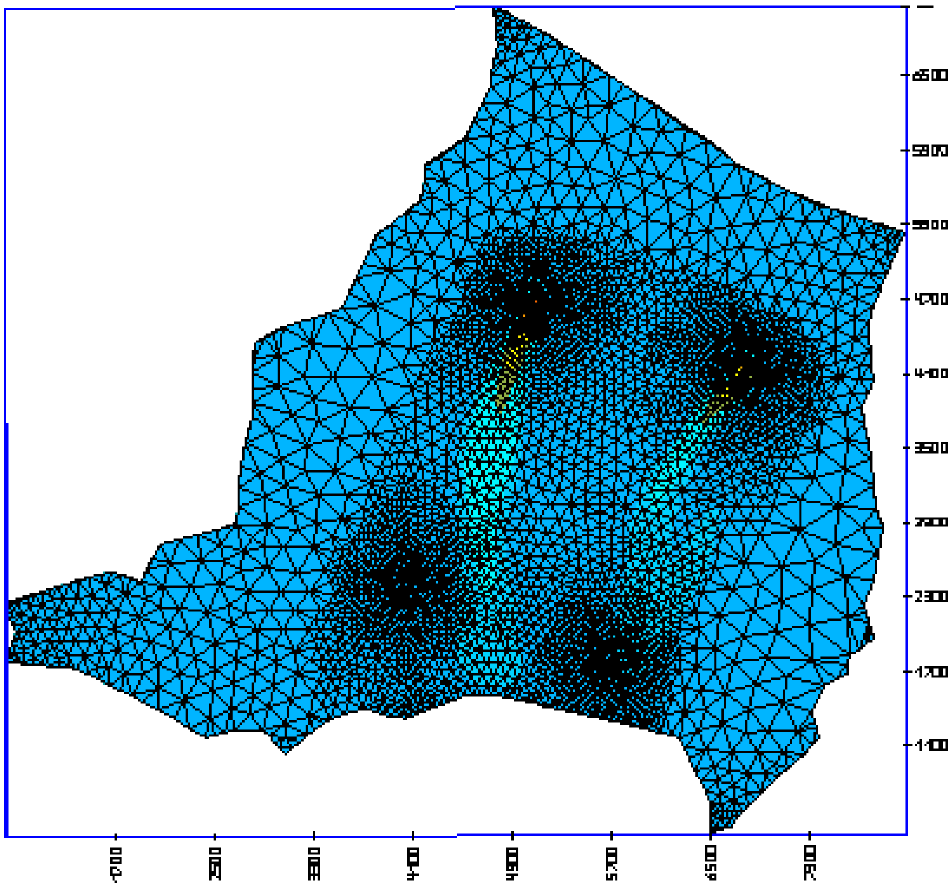
Ways to solve flow in unsaturated zone – finite element method (FEM)



nodes define the geometry

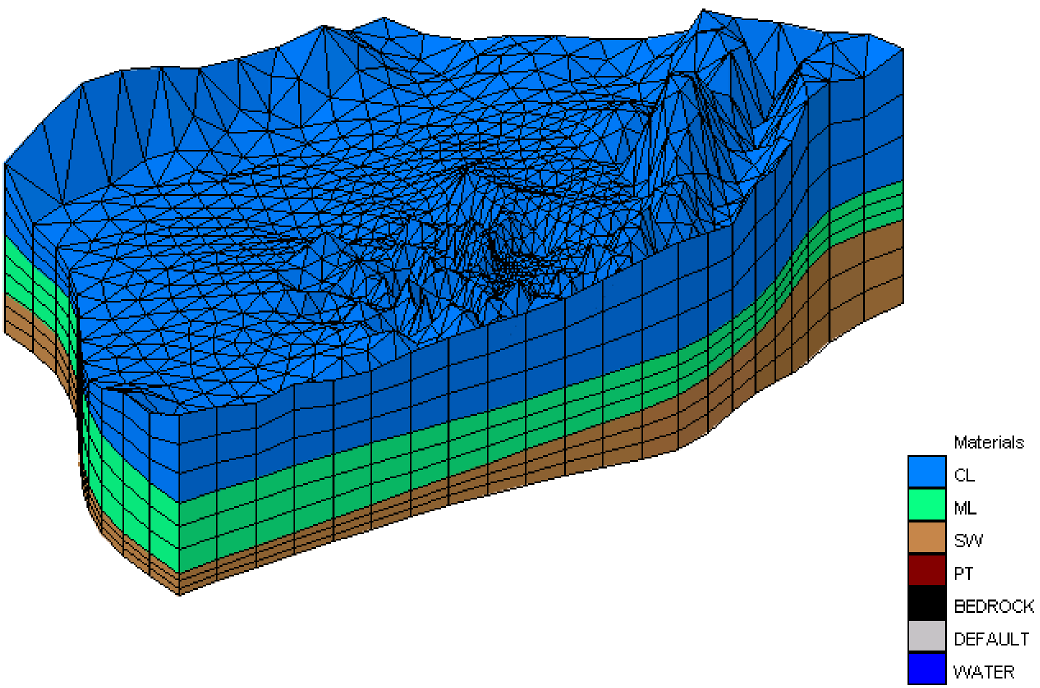
nodes and elements (outlined by nodes) carry the property valuesi – K(θ), θ(h), initial, boundary conditions

FEFLOW, FEMWATER - finite elements



3D mesh generators

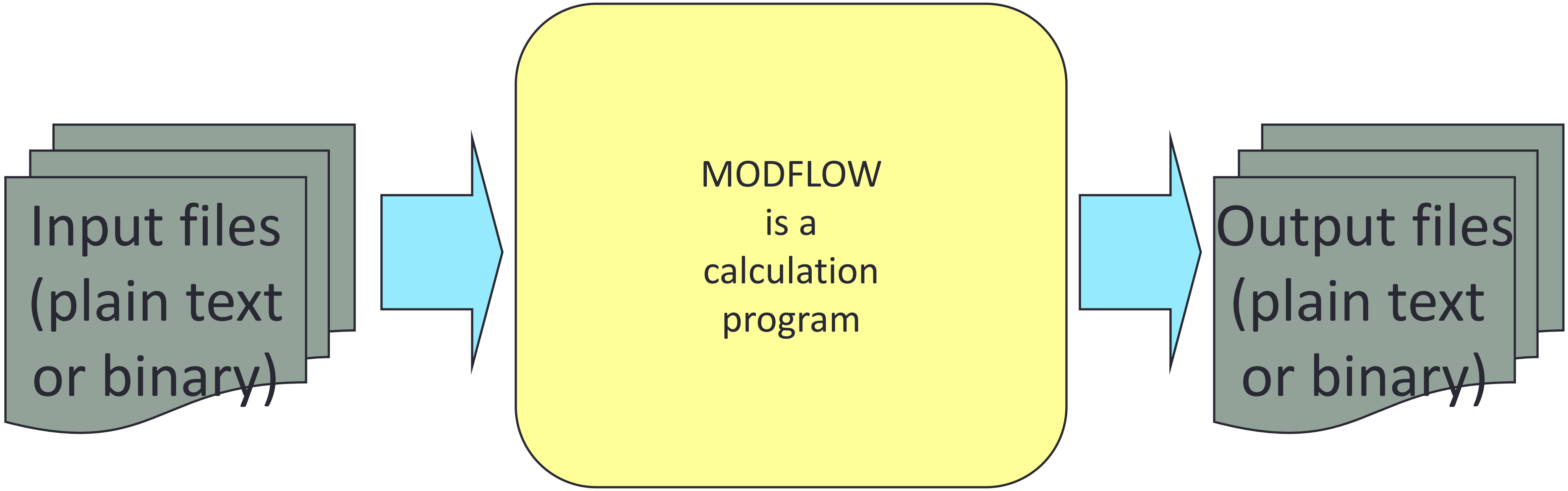
- Definition of material properties
- Use of DMT
- Arbitrary intensification of network unlike finite differences



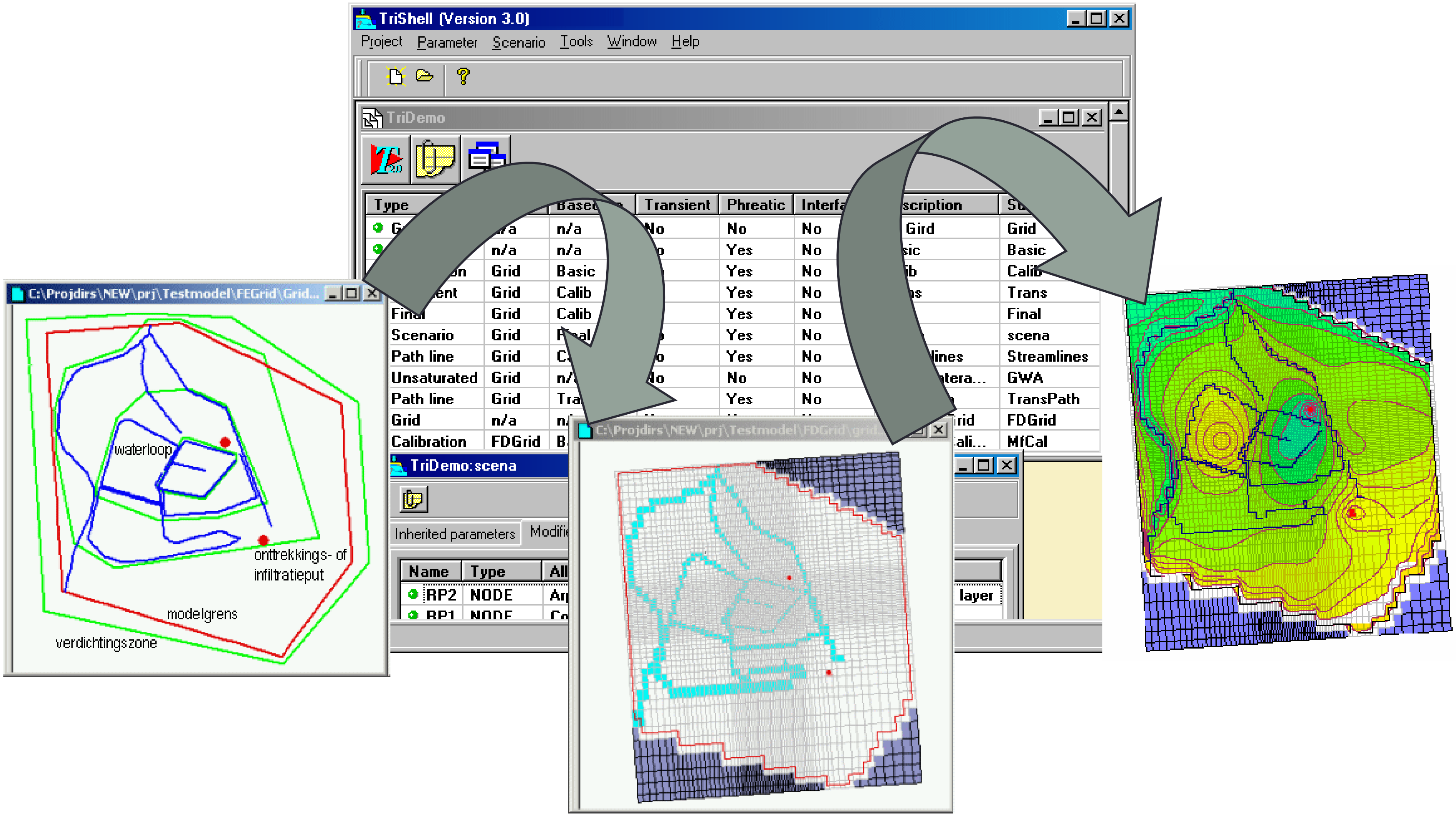
MODFLOW - features

- MODFLOW (US Geological Survey model since 1988) most versatile and accepted groundwater flow models
- Suitable for heterogeneous regions
- Allows vertical flow
- Allows geometrically flexible networks for calculation acceleration
- Applied for thousands of tasks worldwide

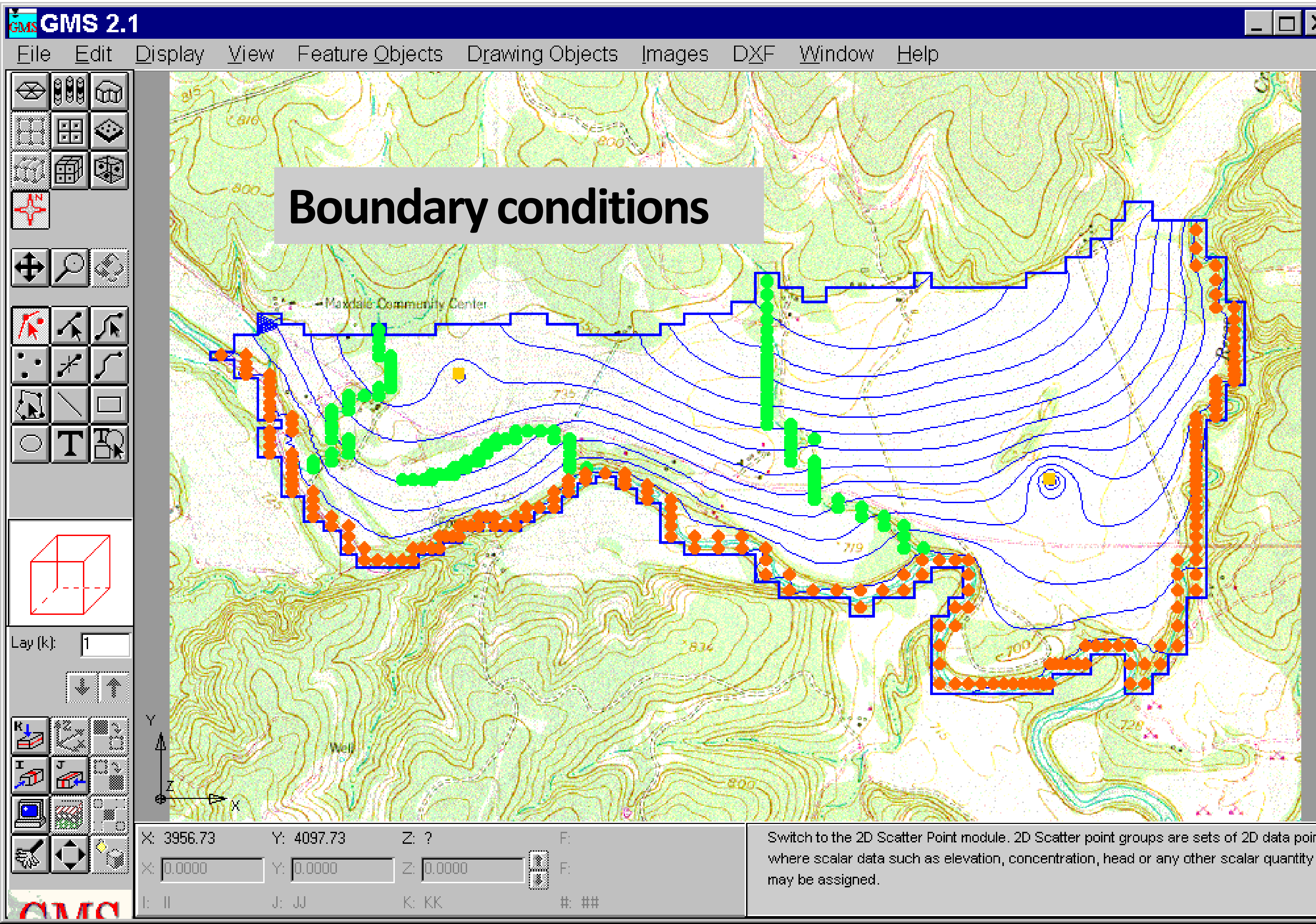
What is MODFLOW?

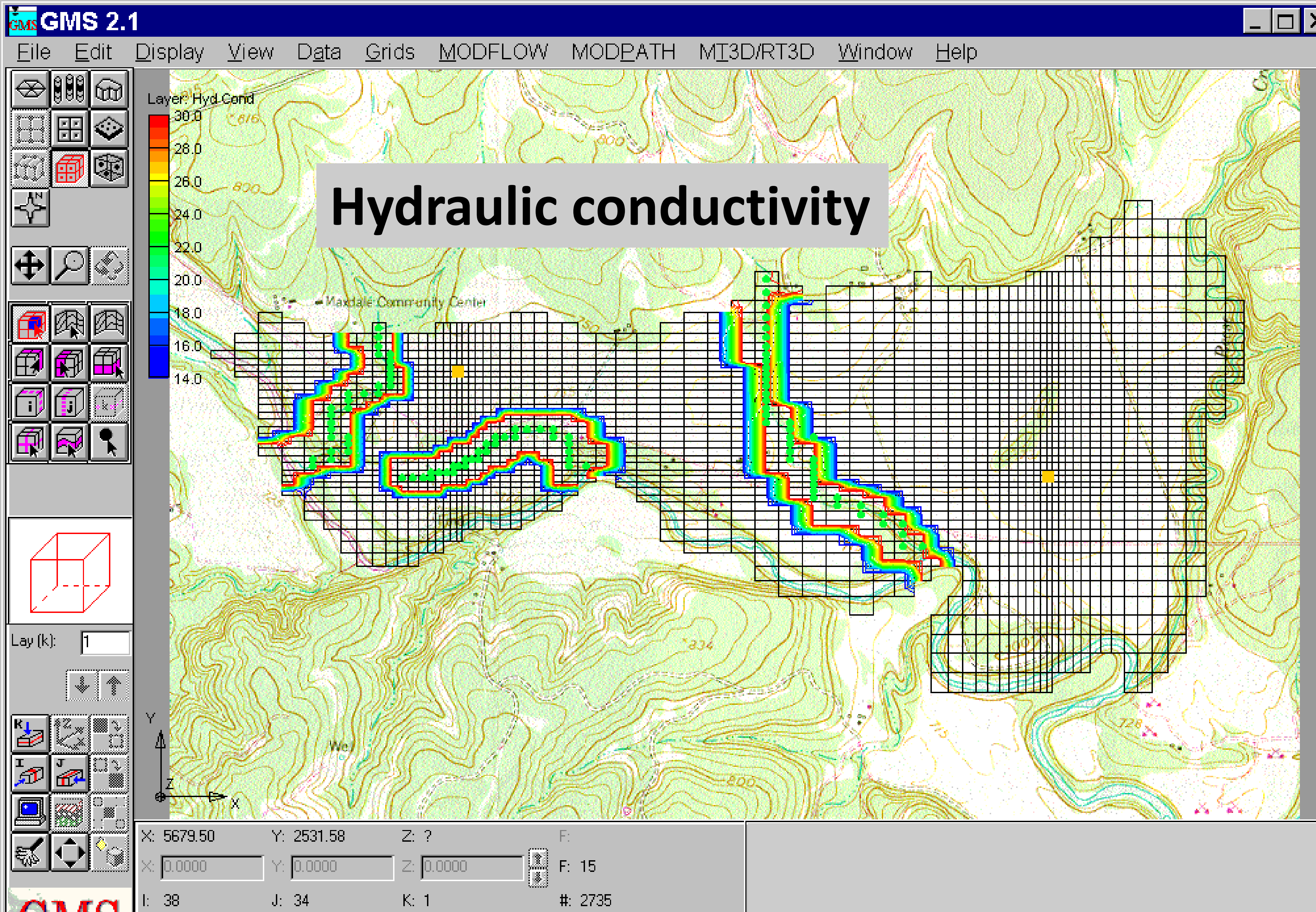


Often use MODFLOW through a (Graphical) User Interface



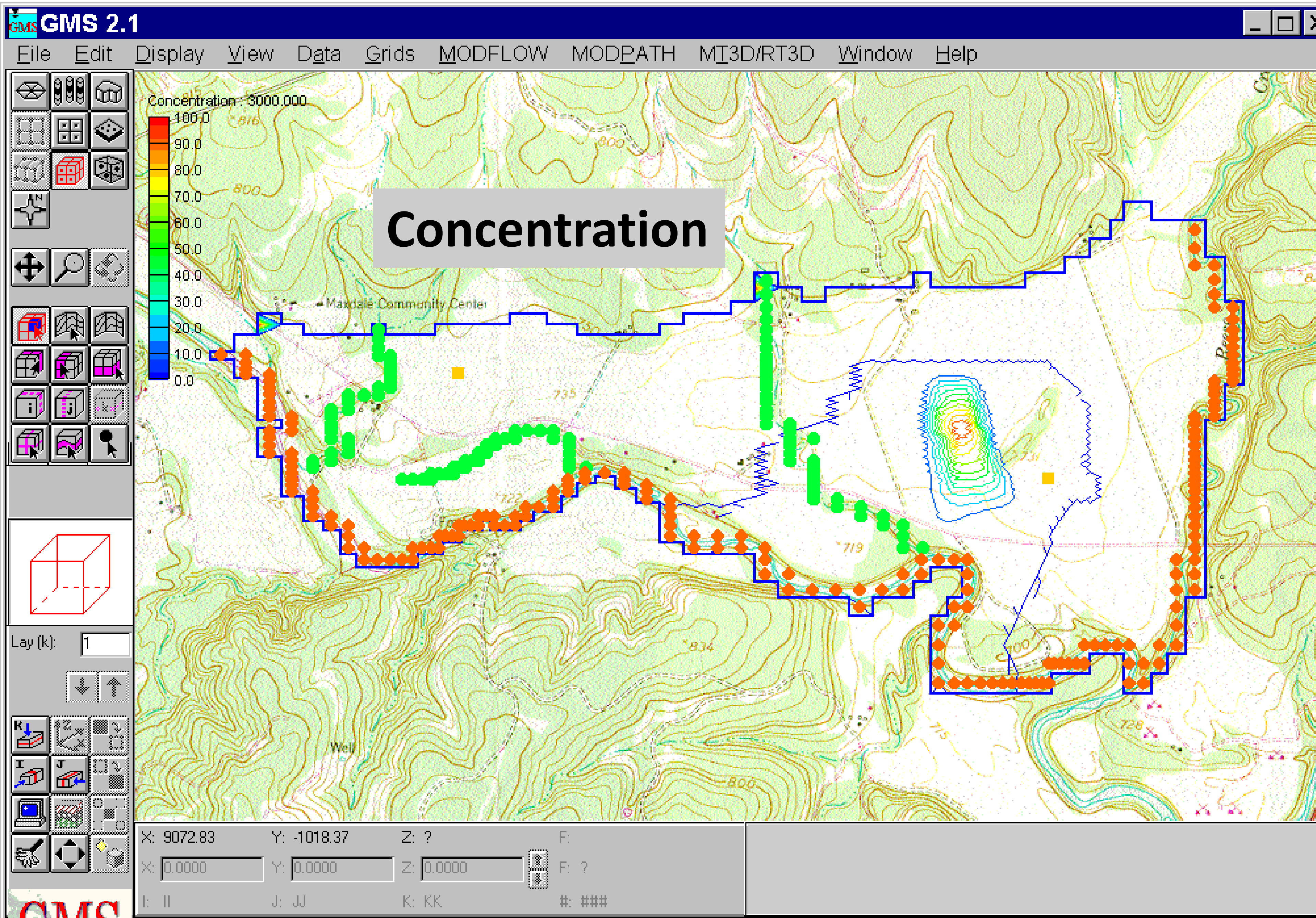
Boundary conditions





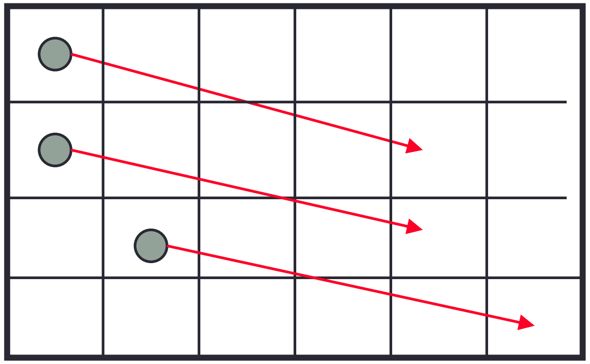
GMS – use of GIS for data preparation

Concentration

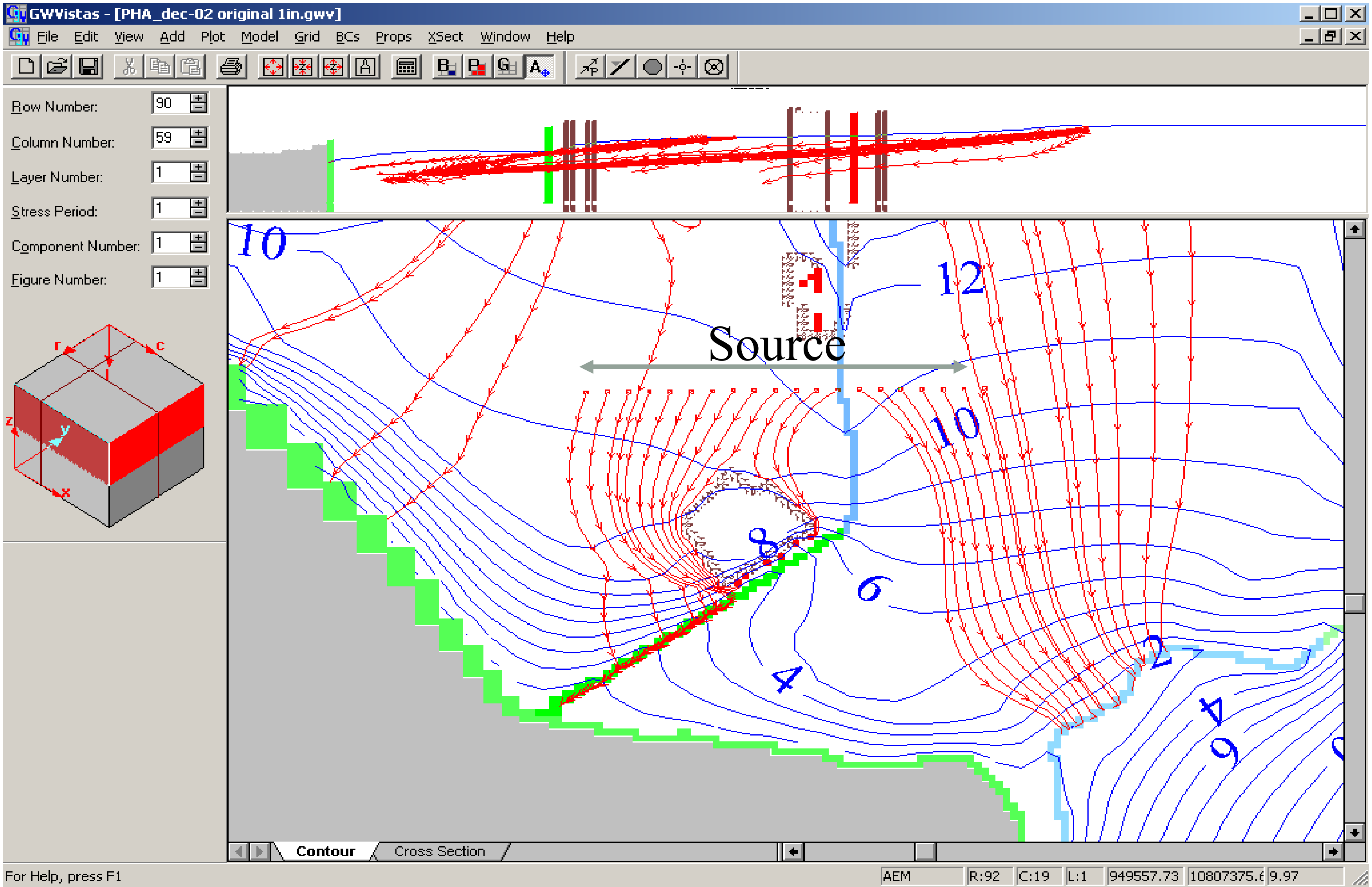


MODPATH

- MODFLOW addition for forward and backward tracking based on flow velocities V_x a V_y.
- Particles can be placed into the areas with assumed source of contamination or look for places of water inflows into the well and plot isochrones (time zones of travel)



MODPATH



GWVistas - [PHA_dec-02 original 1in.gwv]
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Row Number: 90
Column Number: 59
Layer Number: 1
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Component Number: 1
Figure Number: 1
Source
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6
4
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AEM
R:92
C:19
L:1
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verification, calibration

Verification is confirmation, that model tool (e.g. MODFLOW) gives right results as compared with analytical solution (number of tasks help to debug the software itself

Calibration is proces of tuning up real world with model by adjusting model parameters for best fit

- Optimization could be “automatic”, **inverse modelling,** e.g. knowing points of water table in the area and finding values of hydraulic conductivity (minimization of functiona add-ons as PEST, UCODE)

validation, sensitivity analysis

Validation is process of confirmation of the calibrated model by comparison its performance on independent dataset from the same area (typically different time interval)

Sensitivity analysis is process of parameter change to observe behavior of the model

This attitude leads to information, which parameters are sensitive and which not, finding vulnerability and dependence of model on given inputs, which influence model precision

Ways to solve flow - solvers

TOTAL BALANCE OF WATER

Mass must be conserved – often balance is the goal of solution. It is integration of continuity eq. through the domain during the given time period balancing inflow, outflow, sinks and sources.

ATTITUDE TOWARDS SOLUTION

Nowadays – numeric modelling (capacity form dominant)

Iterative solutions – advanced matrix solvers. Time consuming task of finite element solutions, parallel matrix solutions

IBM p690 Cluster JUMP

http://www.fz-juelich.de/jsc/nic/en/

modified from Vogel, T – Podpovrchová hydrologie

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