143SRPP
Stream Revitalization: Principles & Practices

LECTURE 3

Fluvial Geomorphology

Fluvial Processes:
Open Channel Hydraulics; Fluid Forces, Flow Resistance

Winter 2019 Semester

7 October 2019

ČVUT ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE
CTU in Prague - Faculty of Civil Engineering
The Department of Landscape Water Conservation

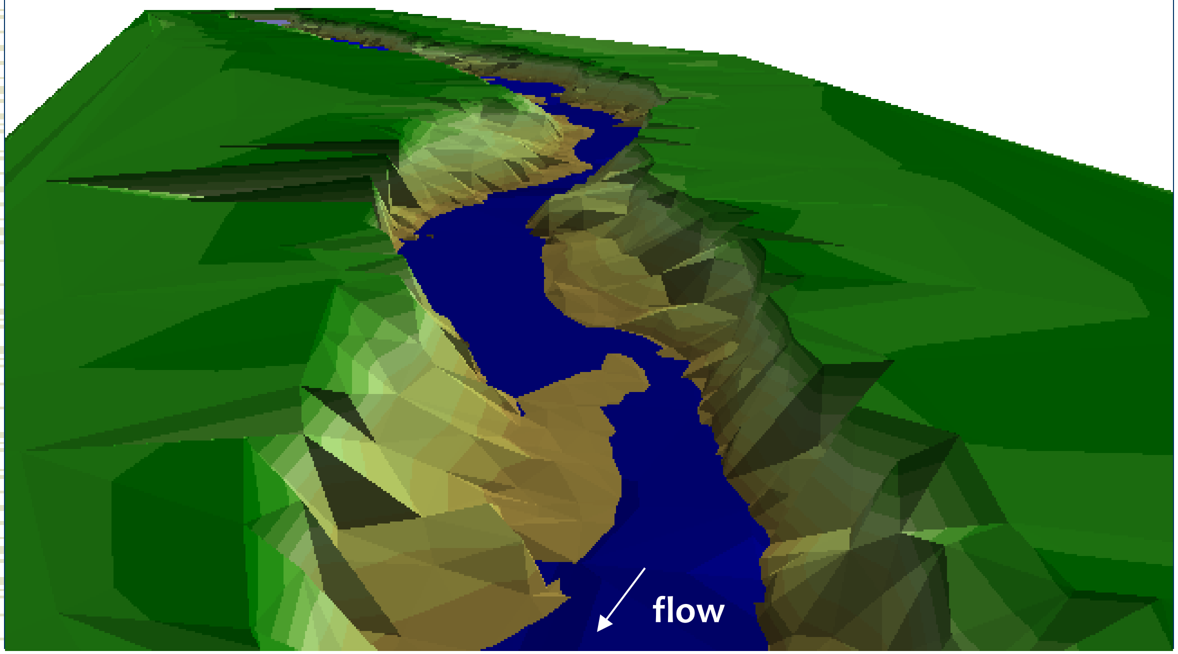
Open Channel Hydraulics

- Open channel: one in which the stream is not completely enclosed by solid boundaries.
- It has a **free surface** subjected only to atmospheric pressure (free-surface flow).
- The flow is only caused by gravity, a force component along the slope of the channel (gravity flow).
- Examples of open channels: natural streams and rivers, artificial canals and sewers, pipelines flowing not completely full.
- Steady/unsteady flow: Steady = all properties of flow at every point remain constant with respect to time. Unsteady = flow properties change with time, Q(t), A(t).

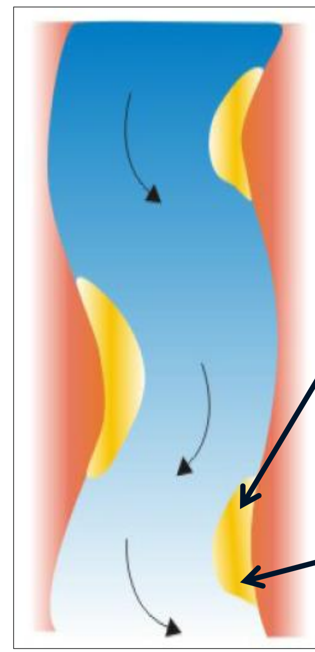
Fluvial Hydraulics

Three-Dimensional View of Stream Channel

Stream Hydraulics at Low-Flow Stages



Point Bar Sediment Sorting



Course Sediment Gravel-sand mixture

↓

Fining of sediment particle size

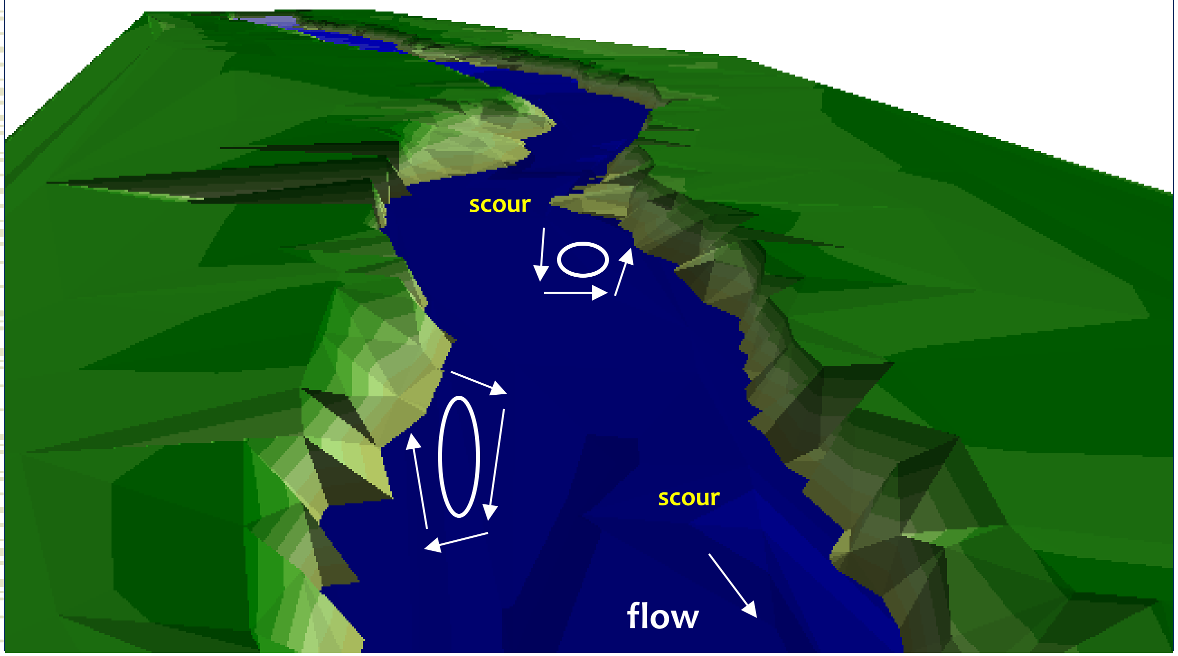
↓

Fine Sediment Sand

Fluvial Hydraulics

Three-Dimensional View of Stream Channel

Stream Hydraulics at High-Flow Stages



HYDRAULICS / SEDIMENT

1. **Flow structures influenced by bed and bank roughness, and channel curvature.**
2. **Flow structures consist of separated flow forming hydraulic recirculation zones.**
3. **Flow structure in main channel consisting of areas of accelerating flow (*scour*) and decelerating flow (*deposition*).**
4. **Sediment deposition occurs in areas where hydraulic recirculation zones occur in the channel.**

Fluvial Hydraulics

Channel Bed Patterns: Sediment Sorting from Hydraulic Forces

High- and low-gradient streams vs headwater channels
Effects of in-channel roughness elements.

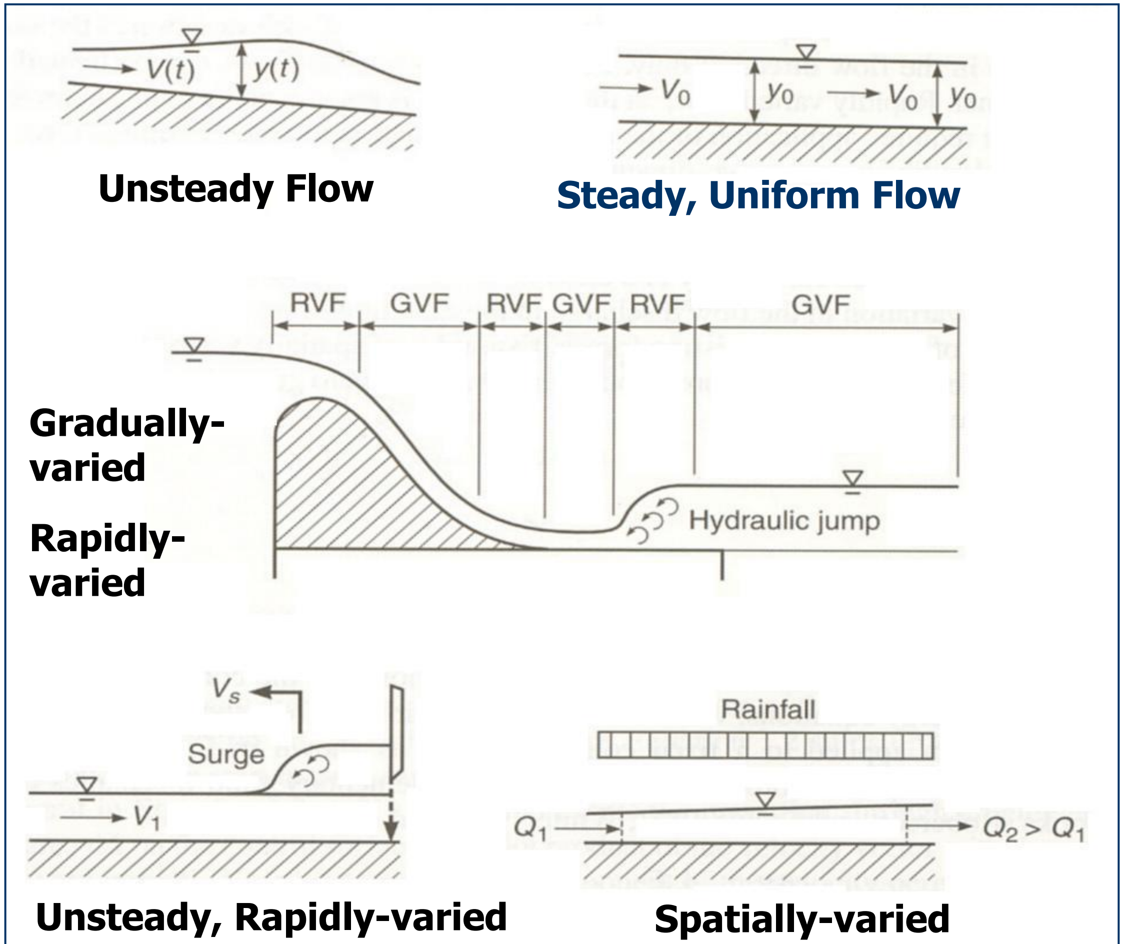
versus

Sorting of soil substrates dependent on channel size and slope, bed soil/sediment size, roughness elements (rocks, boulders, large woody debris), and bank vegetation.

Open Channel Hydraulics: Flow Types

- *Flow Types with respect to* **time**:
  - **Steady**: all properties of flow (i.e., velocity, flow depth and cross-sectional area) at every point remain constant in time (e.g., flow of water at constant discharge rate in a canal).
  - **Unsteady**: properties of flow at a point change with time (e.g., flood in a river).
- *Flow types with respect to* **space**:
  - **Uniform:** no change in flow characteristics (i.e., depth, slope, discharge) with distance along the channel (e.g., flow of water in a long straight canal of constant cross-section).
  - **Varied**: flow characteristics change with respect to location at any instant in time (e.g., river flow through pools and riffles).

Open Channel Hydraulics: Flow Types

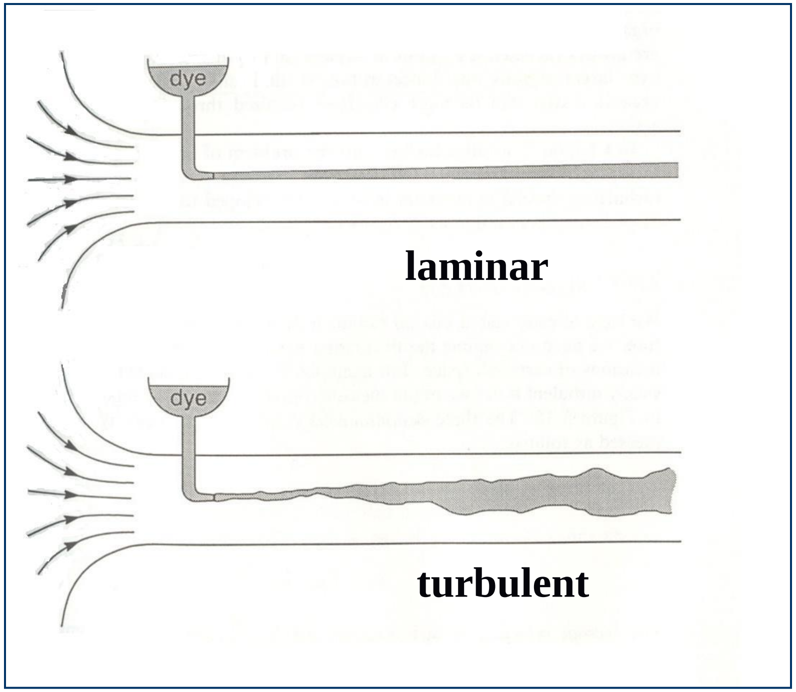


Illustrations of flow types

Open Channel Hydraulics
T. Sturm, 2002
Figure 1.1

Fluvial Hydraulics

- **Laminar vs Turbulent Flow:** general lateral fluctuations of molecular motion of along flow path.
  - **Laminar**: flow with smooth appearance, devoid of intense mixing phenomena and eddies
  - **Turbulent**: flow characterized by mixing action throughout the flow field



Reynolds Number (**Re**): defined by a ratio of inertial forces to viscous forces.

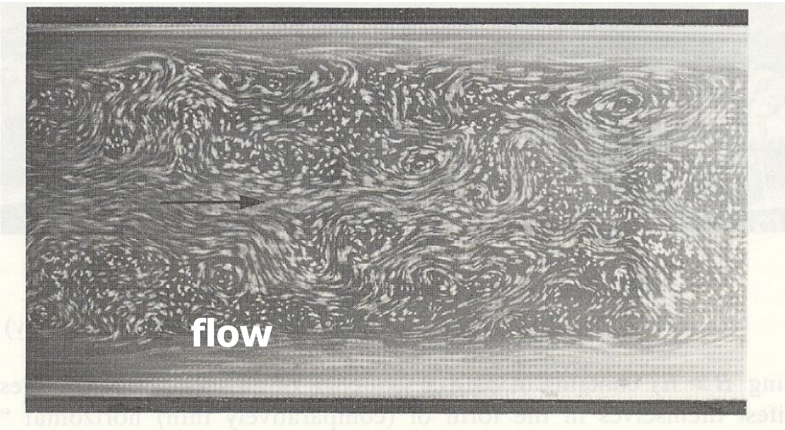
- **Pipe flow = DV/ν**
- **Open Channel = $4R_hV/\nu$**

R_h = D/4 for circular XS

Fluvial Hydraulics

Turbulent Flow

Most sediment-laden flows in rivers are characterized by irregular velocity fluctuations indicating turbulence.



Plan View:
Turbulence structure in a flume

Yalin (1992)

Dimensionless Ratio

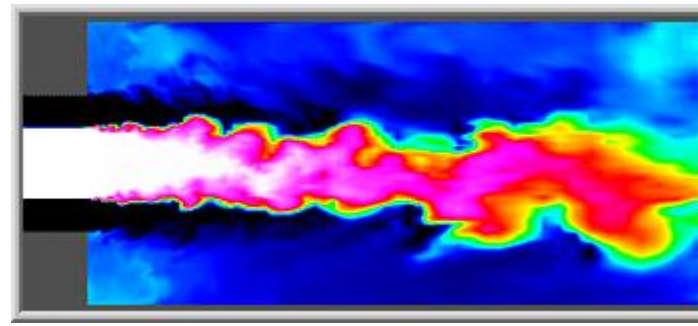
Reynolds Number (**R**, Re)

$$Re = V \cdot R_h / \nu$$

V = Average channel velocity
R_h = Hydraulic Radius (A/P_w)
A = Area; P_w = Wetted perimeter
ν = Kinematic viscosity

Re < 500 Laminar
Re > 500 Turbulent

Fluid Hydraulics: Turbulent Flow



- Newton's viscosity equation:

  $\tau = \mu$ du/dy is *only valid for laminar flow*.
- In *turbulent flow* the velocity at any point fluctuates in both magnitude and direction.
- Turbulent fluctuations result from a multitude of small micro-eddies; thus there is a continuous mixing of water molecules, with a consequent transfer of momentum.
- Turbulent fluid shear stress: $\tau = \eta$ du/dy $\Rightarrow \tau = \rho(\nu+\varepsilon)$ du/dy ;

  where $\nu \cong 0$ ~ small) and $\varepsilon = /^2$ du/dy
  and / is eddy mixing length scale

Fluid Hydraulics: Turbulent Flow

Instantaneous velocity components:

Reynolds Averaging

downstream (u) $u_{1t} = \overline{U}_1 + u'_1$

lateral (v) $u_{2t} = \overline{U}_2 + u'_2$

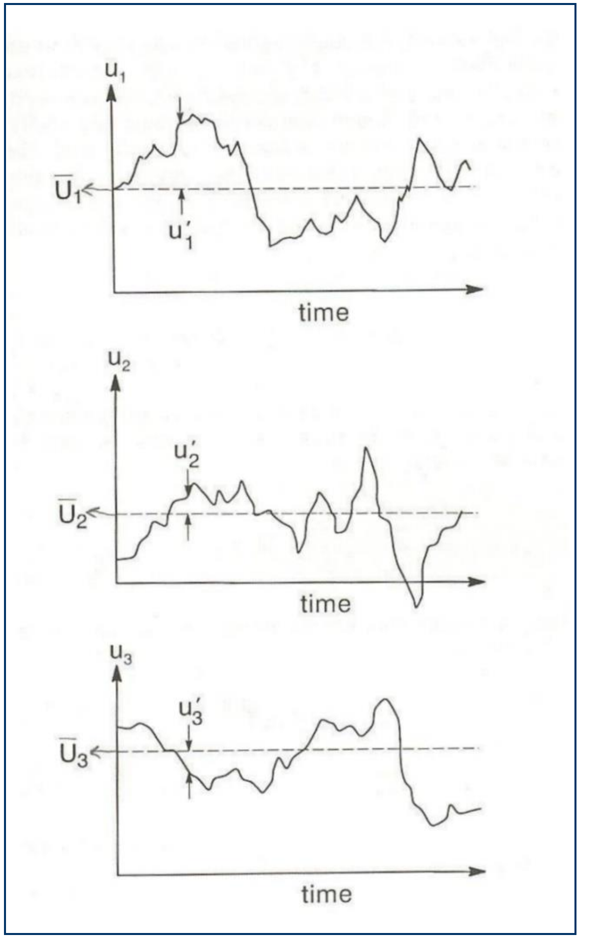
vertical (w) $u_{3t} = \overline{U}_3 + u'_3$

$$\overline{u} = \frac{1}{t}\int_0^t u_t \, dt$$ **[*time averaging*]**

$$\boxed{Q = \int_A \overline{u}\, dA = V \cdot A}$$

volume flow rate

turbulent flow



Fluid Hydraulics: Turbulent Flow

Reviewing Velocity Symbols:

u_t (e.g., u_{1t}; u_{2t} ; u_{3t}) = instantaneous velocity component at a point

u' (e.g., u'_1 ; u'_2 ; u'_3) = fluctuating velocity component at a point

$\overline{u}$ (e.g., $\overline{U}_1$; $\overline{U}_2$; $\overline{U}_3$) = average velocity component at a point

V = mean velocity across a section perpendicular to flow in the downstream direction

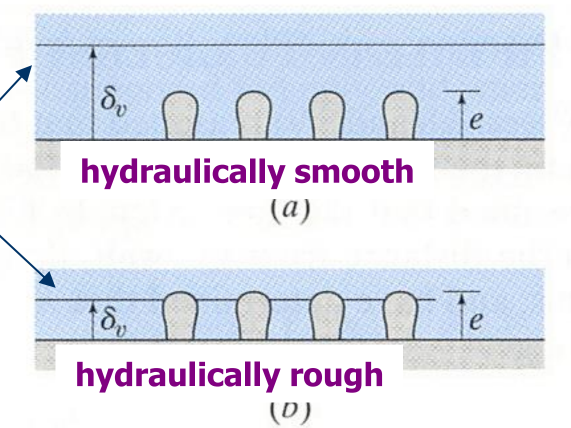
Note: u_{1t} = $\boldsymbol{u}$ **downstream point vector**
u_{2t} = $\boldsymbol{v}$ **cross-sectional point vector**
u_{3t} = $\boldsymbol{w}$ **vertical point vector**

Open Channel Hydraulics: Properties

Reviewing Key Properties:

1. Free surface (open to atmospheric pressure);
2. Gravity-driven (no internal pressure, i.e., pipe flow);
3. Reynolds Number (**Re**) large enough that frictional resistance factor is invariant with **Re** (Moody Chart), *therefore,* flow is hydraulically rough in channels, not always but in general.

δ_c = viscous sublayer depth



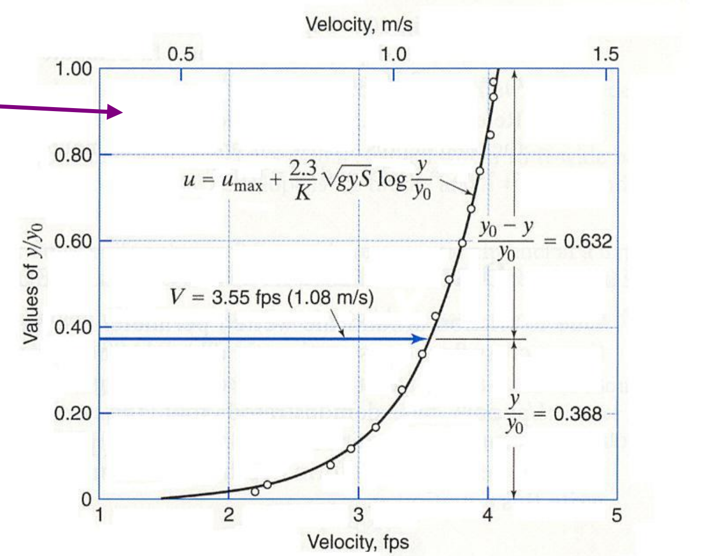
River2D Model
Note:

$\mathbf{e \approx k_s}$

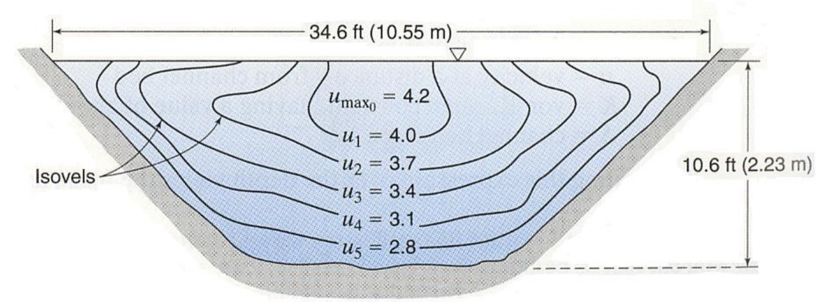
Open Channel Hydraulics: Properties

- Universal logarithmic velocity distribution law (applies law of the wall)
- Viscous sublayer exists

$$u = V + \frac{1}{\kappa}\sqrt{g y_0 S}\left(1 + 2.3\log\frac{y}{y_0}\right)$$



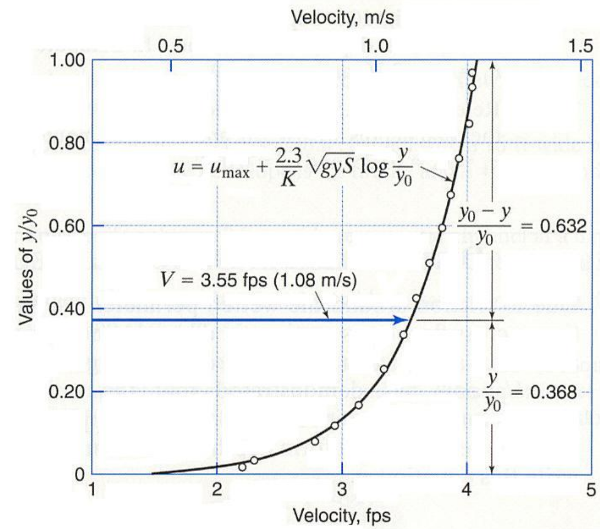
- Mean V is equal to the local velocity u at a distance of $0.632y_0$ beneath the water surface.

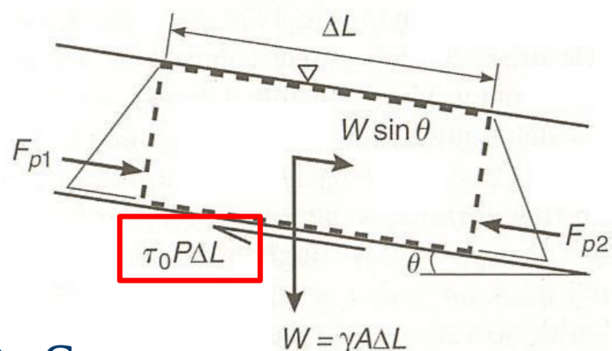


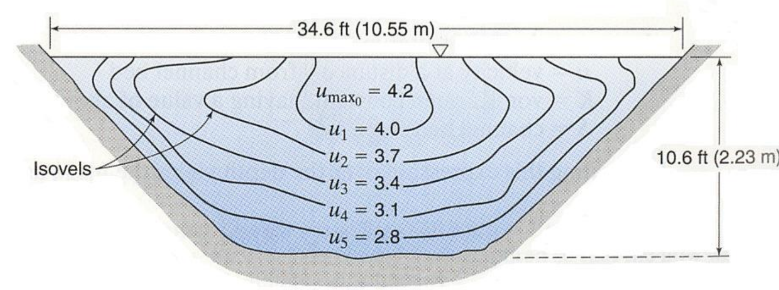
Fluvial Processes: Hydraulics

Bed shear stress (τ_0) is the "resistance" force, as a function of the velocity difference along flow "layers" near the bed.

If the bed shear stress (force) is greater than the forces holding streambed sediment in place, sediment will move, and will be sorted by differential velocities in the channel.





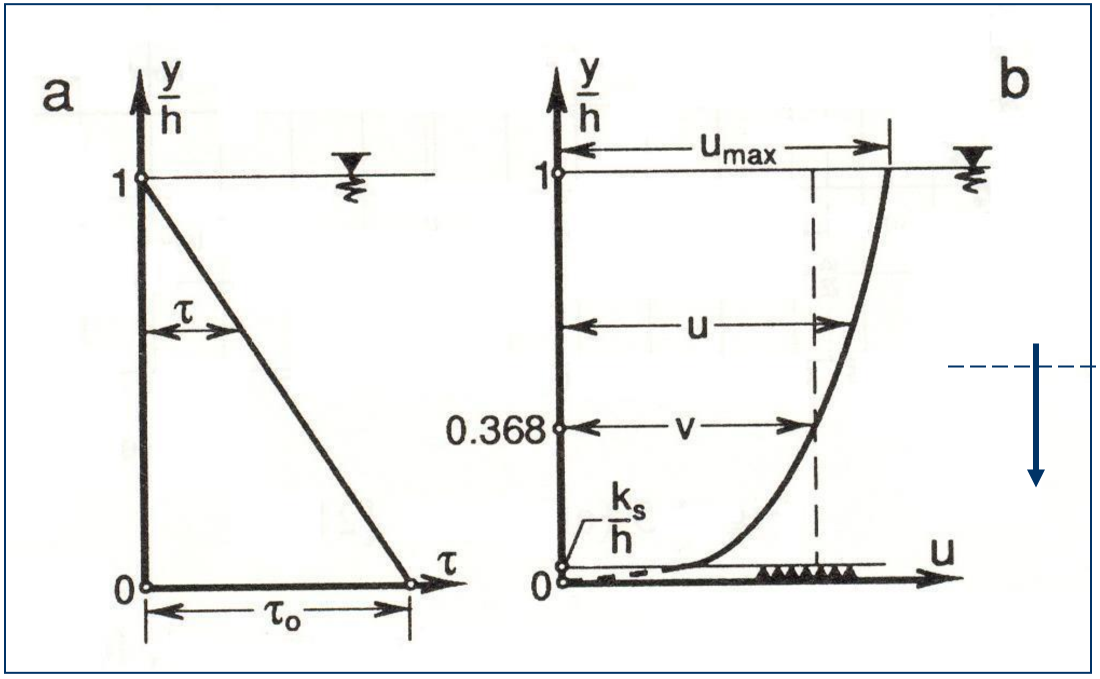


$$\tau_0 = \gamma \cdot R_h \cdot S$$

Fluvial Processes: Hydraulics

Turbulent Flow: Logarithmic Velocity Profiles

$$\tau = \tau_0(1 - y/h); \quad \text{where } \tau_0 = \rho g R S$$



ρ = mass density
g = gravity
R = hydraulic radius
S = channel slope

streamwise momentum, viscous flux of momentum (du/dy) is directed downward towards the bed.

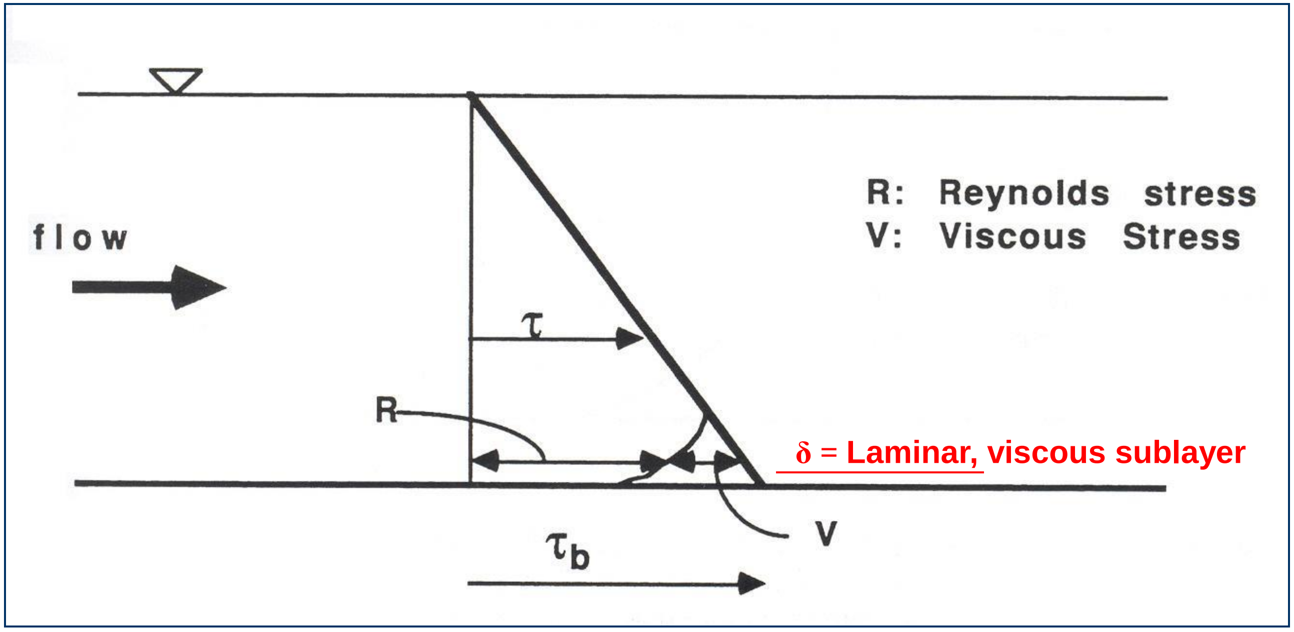
Yalin (1992)

Fluvial Processes: Hydraulics

Turbulent Flow Reynolds Equations – Total Stress Tensor

$$\mathrm{T}_{ij} = \mu \frac{d\bar{u}}{dy} - \rho \cdot \overline{u' \cdot w'}$$

viscous stress   Reynolds stress



δ = Laminar, viscous sublayer

Fluvial Processes: Hydraulics

Smooth versus Rough Velocity Profiles:

Boundaries are said to be hydraulically smooth when the viscous sublayer thickness is greater than the grain size ($\delta_v >> d_s$); and hydraulically rough for the opposite ($\delta_v << d_s$).

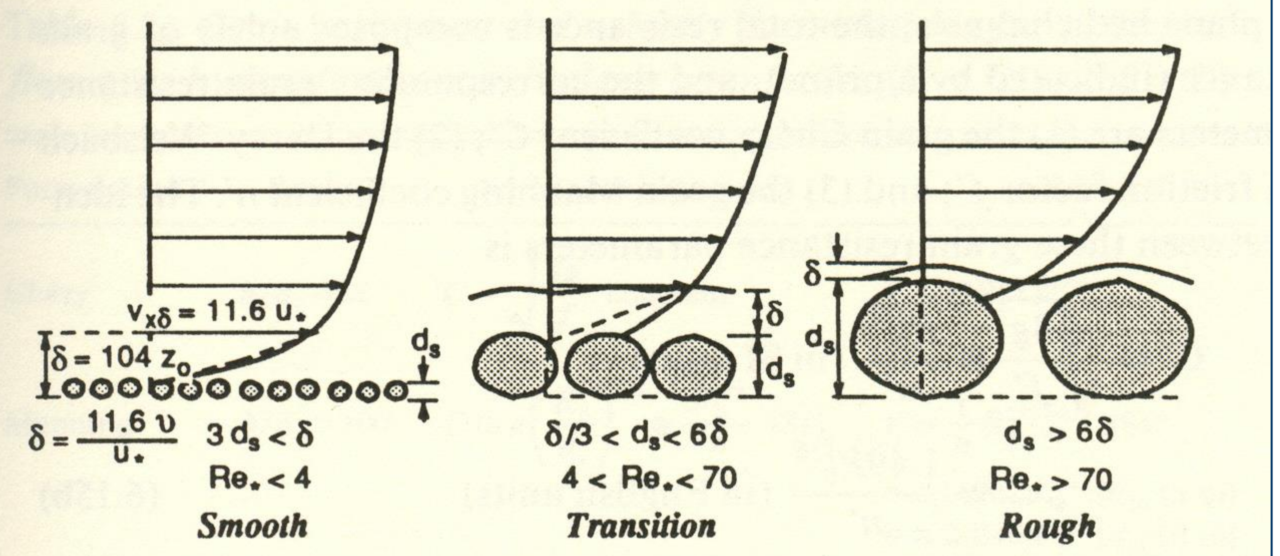


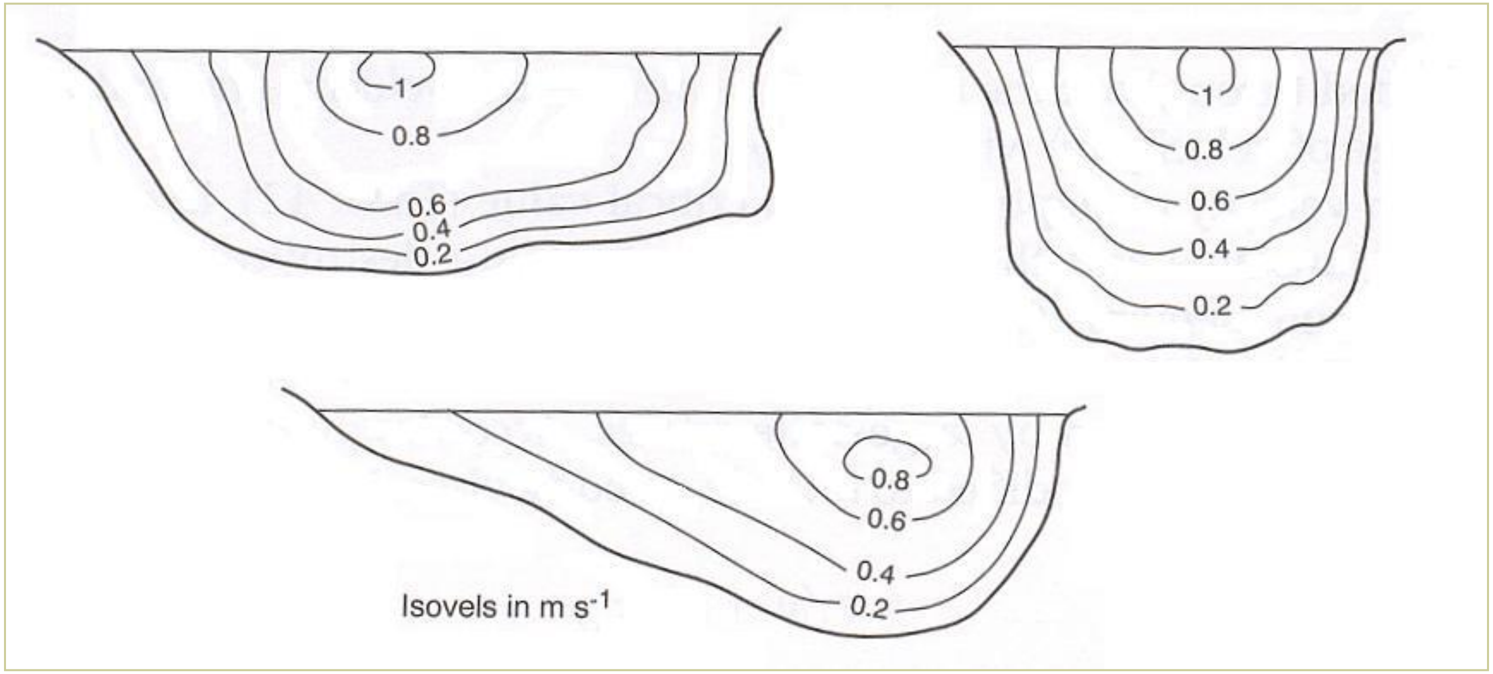
Figure 6.2. Hydraulically smooth and rough boundaries

$$Re_* = u_* \cdot d_s / \upsilon$$

Julien (1998)

Fluvial Processes: Hydraulics

Channel Resistance and Velocity Patterns:
Rivers are hydraulically complex with three-dimensional flow regimes impacted by both bed and bank roughness.



Fluvial Processes: Hydraulics

Fundamental Equations for Open Channel (Free-surface) Flow

Control volume approach: Model finite element cells

Fluid System: specific mass of a fluid within the boundaries defined by a closed surface that may change in time.

Control Volume (cv): a fixed region of space, which does not move or change shape.

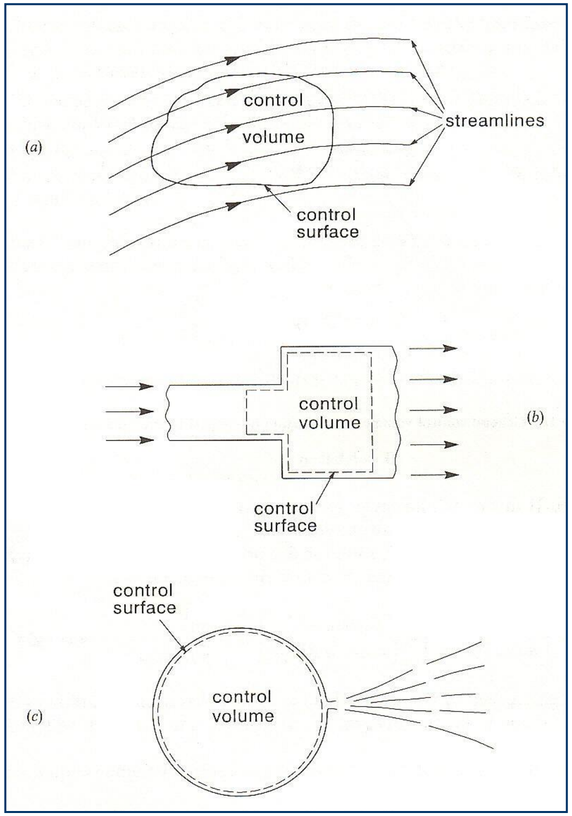
Control Surface (cs): the boundaries of the control volume.

Apply:

Conservation Laws applied across control surfaces: system properties:

Mass	[M]
Energy	[M·L^2/T^2]
Momentum	[M·L/T^2]

M = mass
L = length
T = time



Fluvial Processes: Hydraulics

Conservation of Mass:

$$dm_S/dt = dm_{CV}/dt + dm_{CV}^{out}/dt - dm_{CV}^{in}/dt$$

> $dm_S/dt = 0$ mass must be conserved

> $dm_{CV}/dt = \forall \cdot (d\rho_{CV}/dt)$ mass accumulated; changed in CV

> $dm_{CV}^{in}/dt = \rho A_1 V_1$ mean mass into the CV

> $dm_{CV}^{out}/dt = \rho A_2 V_2$ mean mass out of the CV

$$\boxed{0 = \forall \cdot (d\rho_{CV}/dt) + \rho A_2 V_2 - \rho A_1 V_1}$$

$$\boxed{V_1 A_1 = V_2 A_2}$$

Fluvial Processes: Hydraulics

Conservation of Momentum:

$$d(m\mathbf{V})_S/dt = d(m\mathbf{V})_{CV}/dt + d(m\mathbf{V})_{CV}^{out}/dt - d(m\mathbf{V})_{CV}^{in}/dt$$

Newton's Second Law (Impulse-Momentum Principle)

$$\sum \mathbf{F} = d(m\mathbf{V})_S/dt$$

Sum of the external forces on a fluid system =
rate of change of linear momentum of that fluid system

Notes:

Force (**F**) and velocity (**V**) *are vectors*

Momentum = mV [M-L/T]
Rate of Momentum = d(mV)/dt $[ML/T/T] = [ML/T^2]$
Force $[ML/T^2]$

Fluvial Processes: Hydraulics

Conservation of Momentum:

Simplifying for STEADY FLOW conditions:

$$\sum \mathbf{F} = d(m\mathbf{V})_{CV}/dt + d(m\mathbf{V})_{CV}^{out}/dt - d(m\mathbf{V})_{CV}^{in}/dt$$

= 0; no accumulation of momentum over time

$$\boxed{\sum \mathbf{F} = d(m\mathbf{V})_{CV}^{out}/dt - d(m\mathbf{V})_{CV}^{in}/dt}$$

$$d(m\mathbf{V})/dt = (dm/dt)(\mathbf{V}) = mV = \rho QV$$

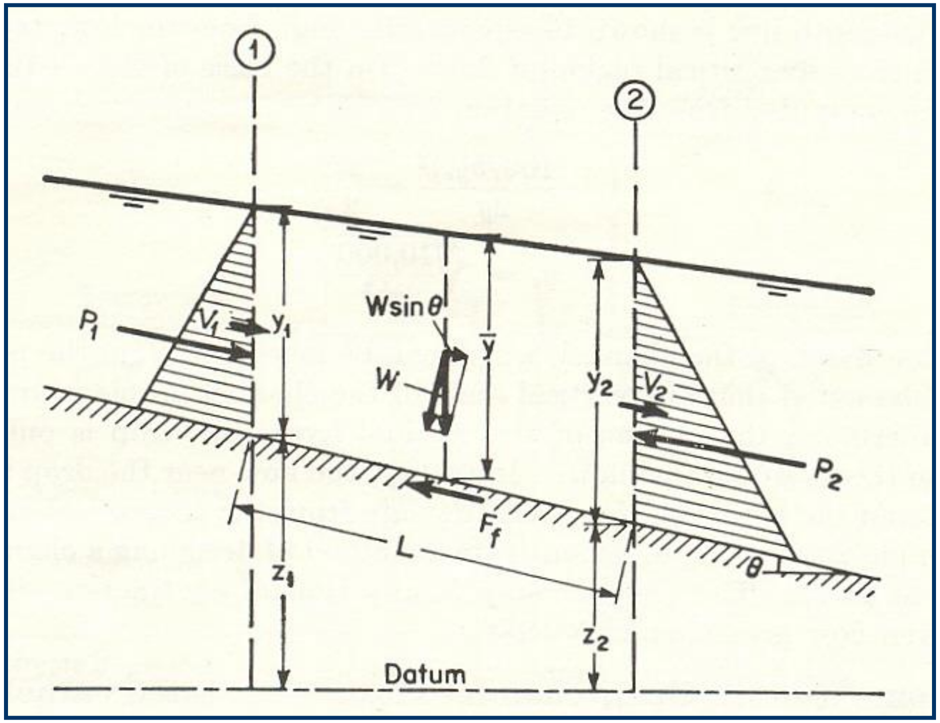
$$\boxed{\sum \mathbf{F} = \rho_2 Q_2 \mathbf{V}_2 - \rho_1 Q_1 \mathbf{V}_1 = \rho Q(\mathbf{V}_2 - \mathbf{V}_1)}$$

Momentum in Open Channel Flow

Change of momentum per unit of time in the body of water in a flowing channel is equal to the resultant of all external forces that are acting on that body:

$$\rho Q(\beta_1 \mathbf{V}_2 - \beta_2 \mathbf{V}_1) = F_{P1} - F_{P2} + W \cdot \sin\theta - F_f$$

< For uniform and gradually-varied flow



ρ = mass density
Q = discharge
V = velocity vector
β= momentum flux correction factor
P, F_p = hydrostatic pressure force
W = fluid weight
F_f = friction forces

For uniform and gradually-varied flow:

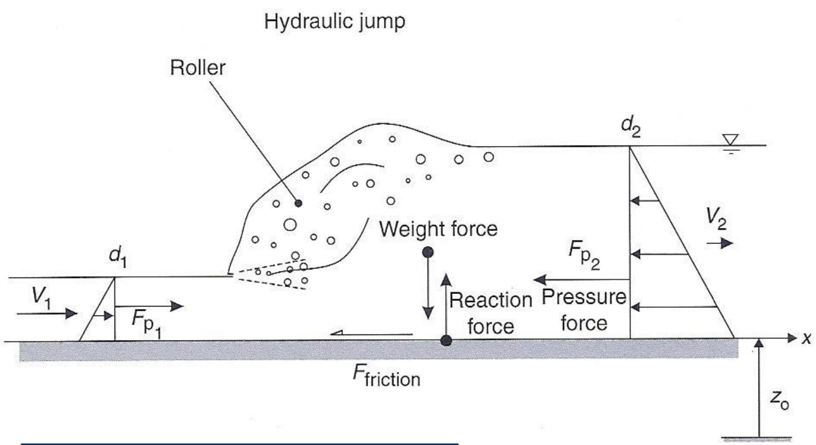
$$F_p = \gamma\, h_c\, A$$

Chow 1959

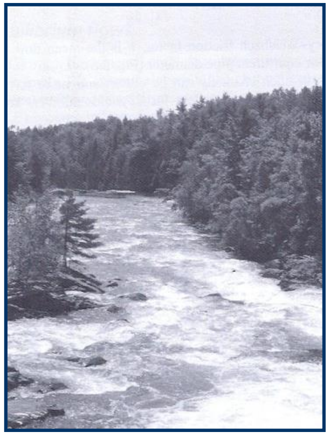
Fluvial Processes: Hydraulics

Flow Force Balance:

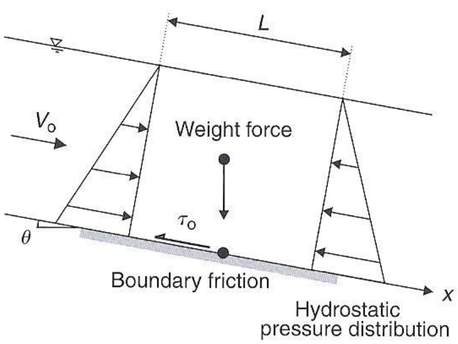
x, y, and z flow directions

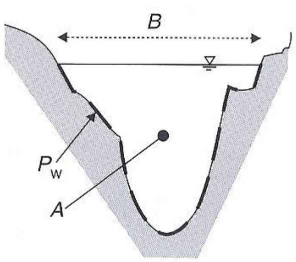


Forces:
1. hydrostatic pressure
2. ineria, momentum
3. friction, resistance
4. weight, gravity
5. shear stresses



control volume 1→ 2





Chanson (2004)

Fluvial Processes: Hydraulics

Conservation of Energy:

Basic Energy Equation expressed in terms of unit weight (Real Fluid).

$$\frac{p_1}{\gamma} + z_1 + \alpha \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \alpha \frac{V_2^2}{2g} + h_L$$

Open Channel Flow Equivalent

$$y_1 + z_1 + \alpha \frac{V_1^2}{2g} = y_2 + z_2 + \alpha \frac{V_2^2}{2g} + h_L$$

Pressure head p/γ (hydrostatic pressure) = flow depth y

Specific Energy Equation: $$y_1 + E_1 = y_2 + E_2 + h_L$$

Open Channel Hydraulics: Specific Energy

Total Energy Equation for Open Channel Flow for a flow reach, *neglecting friction loss* :

$$\boxed{z_1 + y_1 + \frac{Q^2}{2gA_1^2} = z_2 + y_2 + \frac{Q^2}{2gA_2^2}}$$

z = bottom elevation
y = flow depth
Q = discharge
g =gravity
A = cross-sectional area of flow

- **Specific energy**: for any channel cross-section, specific energy is defined as the sum of the depth and velocity head:

$$\boxed{E = y + \alpha V^2/2g}$$

α = kinetic energy flux correction factor

Open Channel Hydraulics: Specific Energy

- **Specific energy**: In rectangular channels the *flow per unit width* is: $q = Q/b$;

  and the xs average velocity: $V = Q/A = qb/by = q/y$

  and the specific energy becomes:

  $$E = y + V^2/2g$$
  $$= y + (1/2g)\, q^2/y^2$$

  If q remains constant:

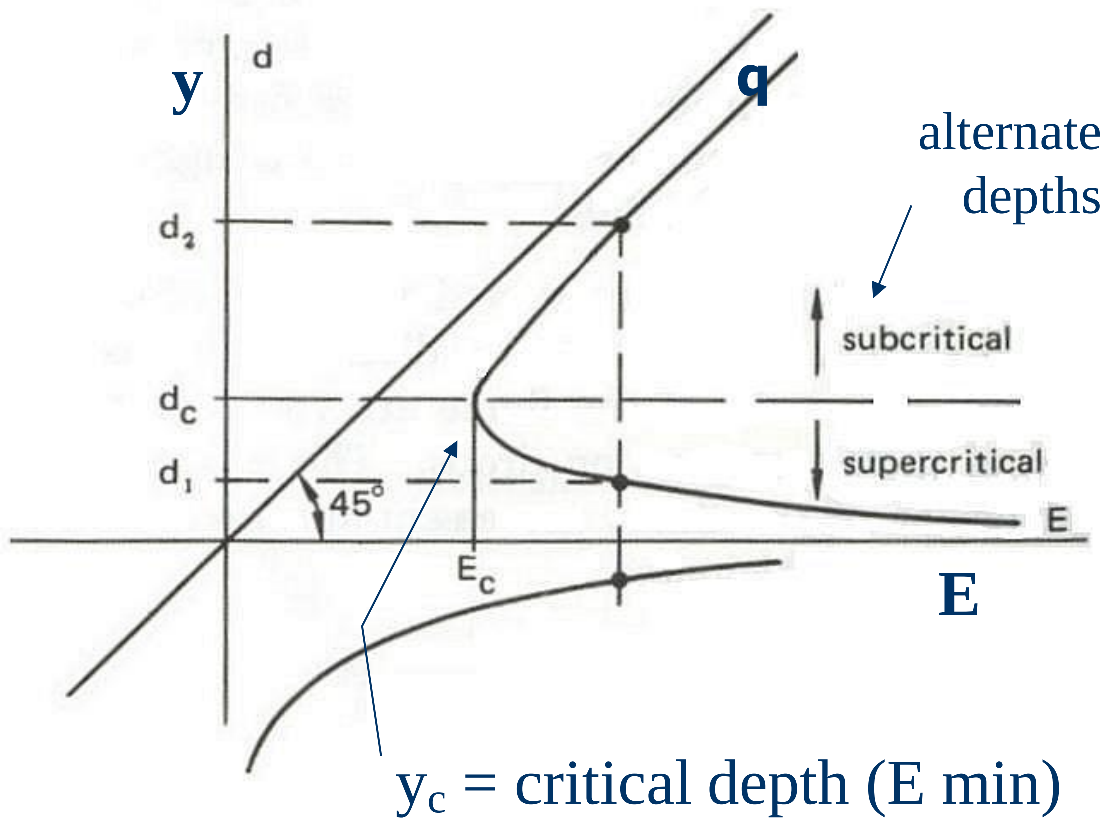
  $$(E - y)y^2 = q^2/2g = \text{constant}$$

  (3 roots – 2 positive, 1 negative)

Specific Energy Diagram

Alternate Depths:

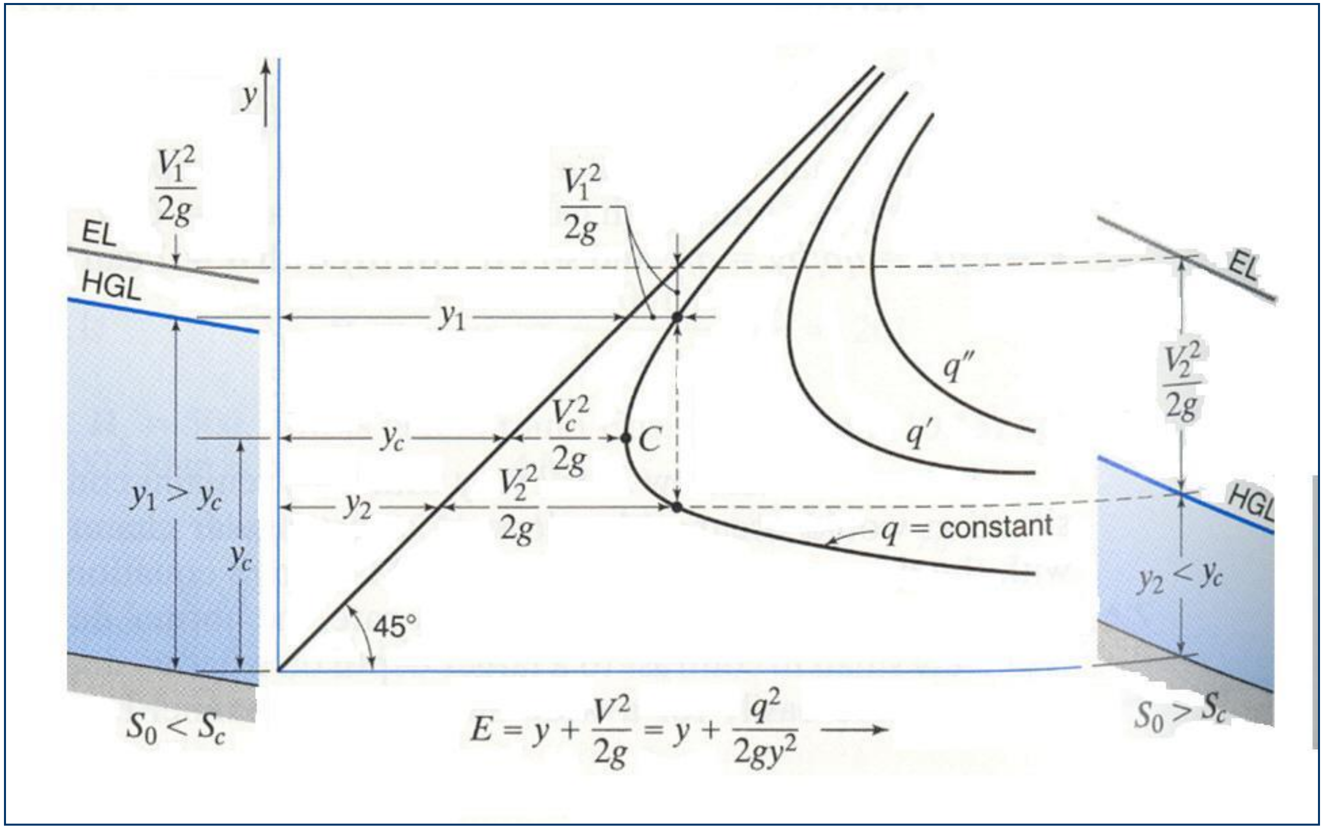
Each value of q will give a different curve, and for a particular q there are two possible values of y for a given value of E (alternate depths)



Specific Energy Diagram

- **Specific-energy diagram** (plot of E vs y):

slow and deep



fast and shallow

Finnemore & Franzini (2002)

Open Channel Hydraulics: Specific Energy

- For the specific energy diagram, the upper limb velocities are less than critical and represent **subcritical flow**.
- The lower limb corresponds to velocities greater than critical representing **supercritical flow**.
- **Critical flow** is a unique point on the curve for a given q.

$$1 = q^2/g \cdot y_c^3 = V_c^2/g \cdot y_c = \mathbf{F}^2$$

(*by definition*)

Open Channel Hydraulics: Specific Energy

- Critical flow state in non-rectangular channels:

$$E = y + \alpha \frac{Q^2}{2gA^2}$$

Differentiating with respect to y, and setting dE/dy = 0 to obtain the minimum specific energy and flow depth

$$\frac{dE}{dy} = 1 - \frac{\alpha Q^2}{gA^3}\frac{dA}{dy} = 0$$

$$\boxed{\frac{\alpha Q^2 B_C}{gA_C^3} = 1}$$

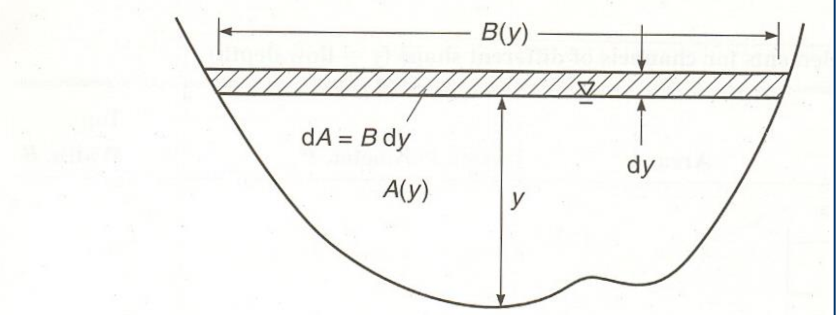


FIGURE 2.12
Geometric properties of general nonrectangular section.

Open Channel Hydraulics: Specific Energy

- For a wide rectangular channel, critical flow occurs when E is minimum: $dE/dy = 0$

- **Critical velocity**: $V_c = (g \cdot y_c)^{1/2} = q/y_c$

- **Critical depth**: $y_c = V_c^2/g$; $y_c = [q^2/g]^{1/3}$

$$y_c = 2/3\ E_c = 2/3\ E_{min}$$

- **Maximum discharge**: $q_{max} = (g y_c^3)^{1/2}$

Open Channel Hydraulics: Specific Energy

Froude Number (Fr): $V/(g \cdot D)^{1/2}$

V = average cross-sectional velocity
g = gravity
D = hydraulic depth (A/B)

A = cross-sectional area
B = cross-sectional width of water surface
D = A/B
D = h ; some textbooks use h rather than D
D = y (flow depth) **for wide and shallow conditions**

Froude number is a dimensionless number, the ratio of inertia forces to gravity forces

Open Channel Hydraulics: Specific Energy

- The **Froude number** (F) is the ratio of inertial force to gravitational force (dimensionless number).
- It can be used to determine if the flow is critical, subcritical, or supercritical:

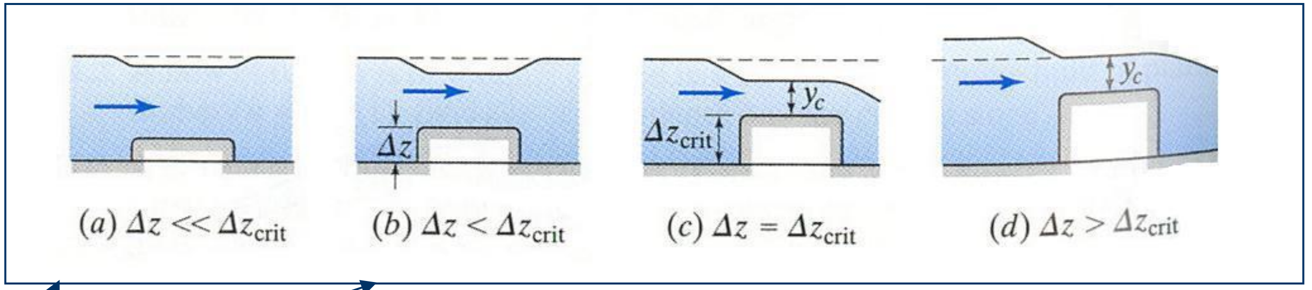
$$\mathbf{F} = V/(g \cdot D)^{1/2}$$

  - F = 1.0 ---> critical
  - F < 1.0 ---> subcritical
  - F > 1.0 ---> supercritical

D = hydraulic depth
D = A/B (xs area/top width)
D = y *for* rectangular channels

Open Channel Hydraulics: Specific Energy

- Critical depth may occur in a channel when the bottom is stepped.
- Steps:



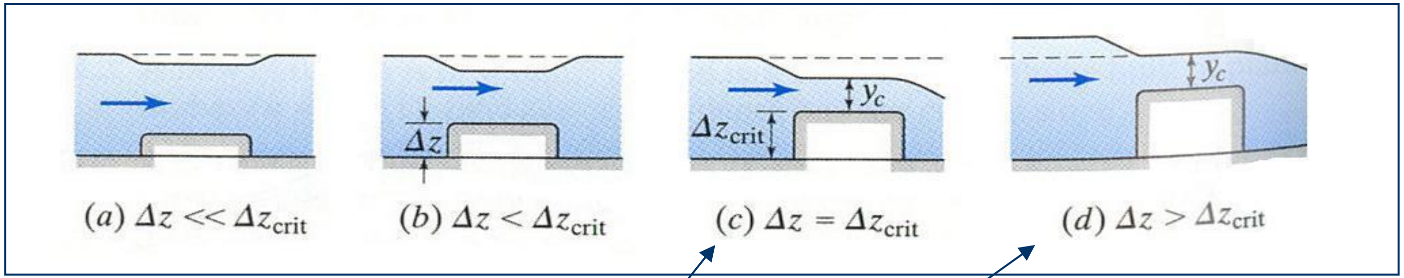
Finnemore & Franzini (2002)

If the flow upstream of the step is **subcritical**, there will be a slight depression in the water surface over the step.

The step causes a *drop* in specific energy E.

A decrease in 'E' with unchanged 'q' results in a decrease in y.

Open Channel Hydraulics: Specific Energy



Finnemore & Franzini (2002)

- **Critical Step Height**: If the height of the step increases, the depression increases, until the depth of the step becomes critical at *minimum* specific energy.

- If the height of the step is further increased, the critical depth remains on the step, and the depth upstream of the step increases, causing a *damming action, choking*.

Subcritical Flow

Open Channel Hydraulics: Specific Energy

Specific Energy Diagram:

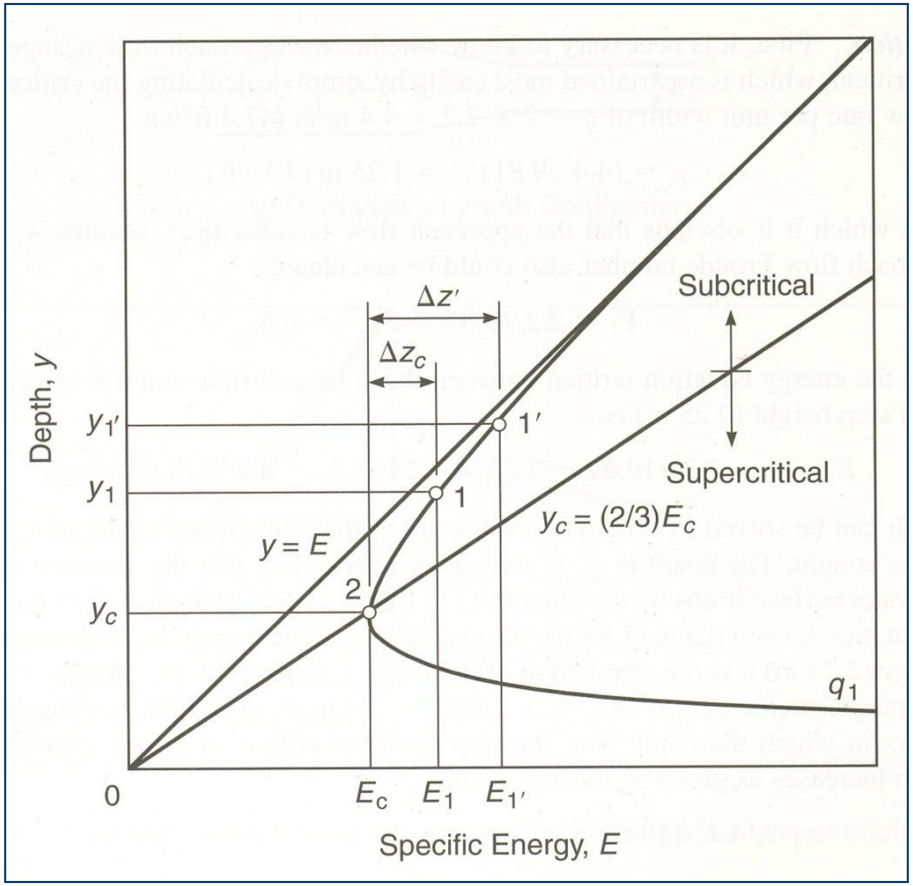
Subcritical Flow

Choking by Step

1 = initial channel

2 = increased bed height to critical, minimal energy.

1′ = increase height more… remains at critical depth but upstream flow depth increases.



Sturm (2001) Figure 2.5

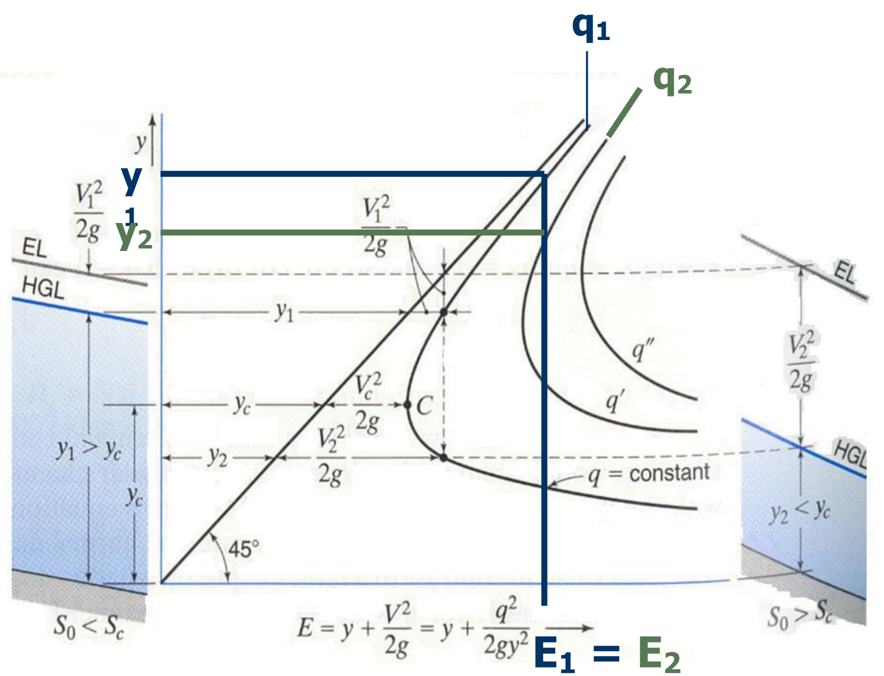
Open Channel Hydraulics: Specific Energy

- **Contractions:** Critical depth may occur in a channel when the channel sidewalls are contracted.
- Contractions change 'q', the flow per unit width, instead of 'E'

In the specific energy diagram, curves for increasing q move to the right, therefore contractions have the same effect of a step on E.

$$\boxed{E_1 = E_2}$$

$$\boxed{q_1 \neq q_2}$$



Open Channel Hydraulics: Specific Energy

- **Channel contractions:**
  - If the approaching flow is **subcritical**, a small contraction will cause a slight depression on the water surface, until critical depth occurs in the contraction.
  - Increasing the contraction further creates a damming action; however, the *depth* in the contracted section continues to *increase* since the critical depth depends upon q as in: $y_c = (q^2/g)^{1/3}$
- **Channel expansions:**
  - If the approaching flow is **subcritical**, an expansion contraction will cause a slight increase on the water surface. Same $E_1 = E_2$ and $q_1 = q_2$

Open Channel Hydraulics: Specific Energy

- **Supercritical Flow vs Subcritical Flow:**

With **supercritical approach flow**, bed steps/depressions and channel contractions/expansions behave differently:

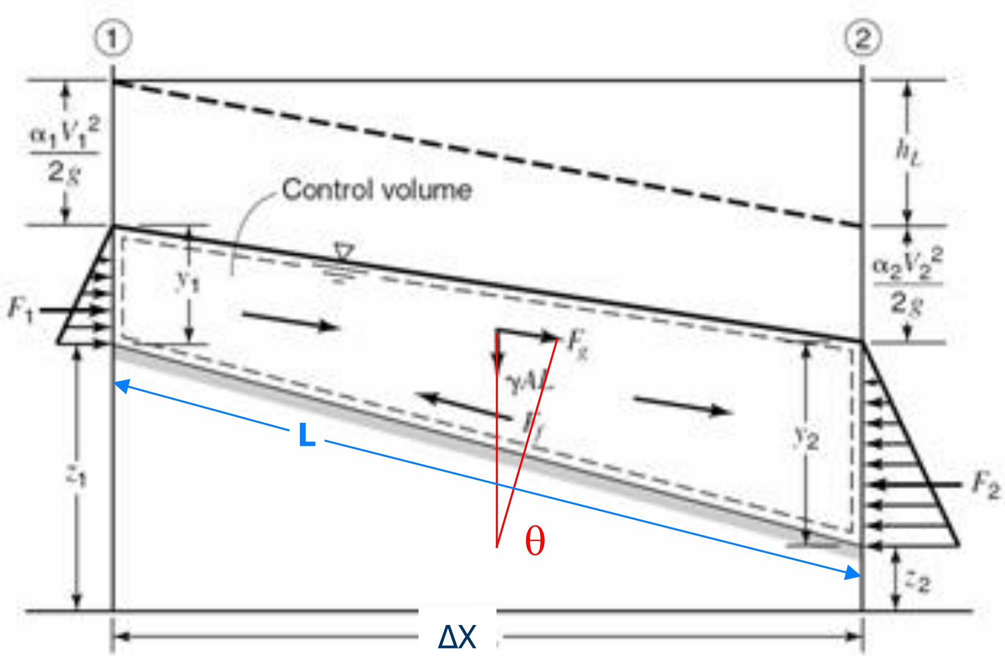
Water depth at the step or contraction increases with increasing step height or contraction until it reaches critical depth; beyond that it causes a damming action or choking.

Supercritical approach flow can be complicated by the occurrence of a hydraulic jump.

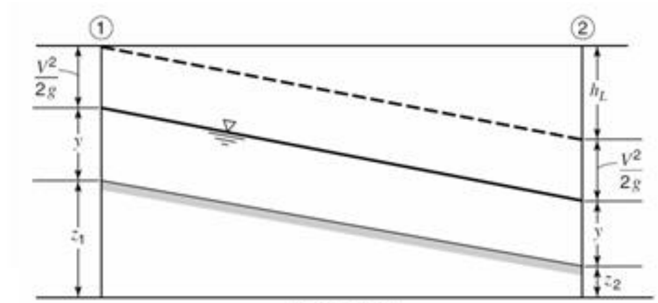
Fluvial Processes: Hydraulics

Open Channel Slopes

- Bed slope: $S_o = -\Delta z/\Delta x$
- Water surface slope: $S_w = -\Delta(z+y)/\Delta x$
- Energy slope: S or $S_e = h_f/L$



Nonuniform flow


Uniform flow

Uniform Flow

$S_o = S_w = S_e$

when θ is small; θ < 5.7°

Gradually-varied Flow

$S_o \neq S_w \neq S_e$

Velocity and depth changing with distance

Sturm (2010)

Fluvial Processes: Hydraulics

Uniform Steady Flow (Q): *in cubic feet per second* (cfs)

Manning Equation: $V = (1.49/n) \cdot R_h^{2/3} \cdot S_o^{1/2}$ // $Q = (1.49/n) \cdot A \cdot R_h^{2/3} \cdot S_o^{1/2}$

V = cross-sectional area velocity
R_h = hydraulic radius (area / wetted perimeter)
n = Manning roughness coefficient
A = cross-sectional area
S_o = bed slope

Design for channel size:
$A \cdot R_h^{2/3} = (Q \cdot n)/(1.49 \cdot S_o^{1/2})$

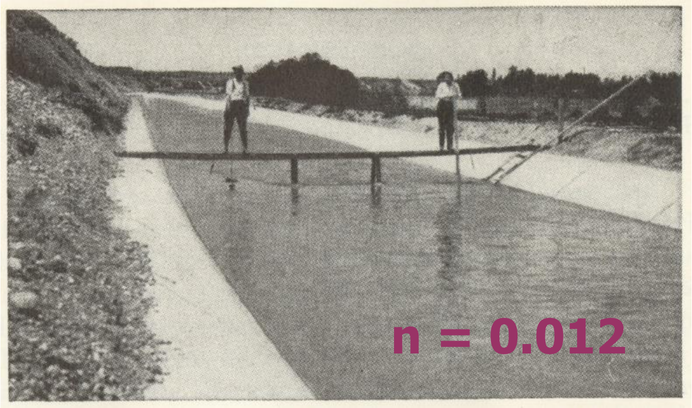
Use Q for design flow through during a stream flood wave Q is unsteady flow.

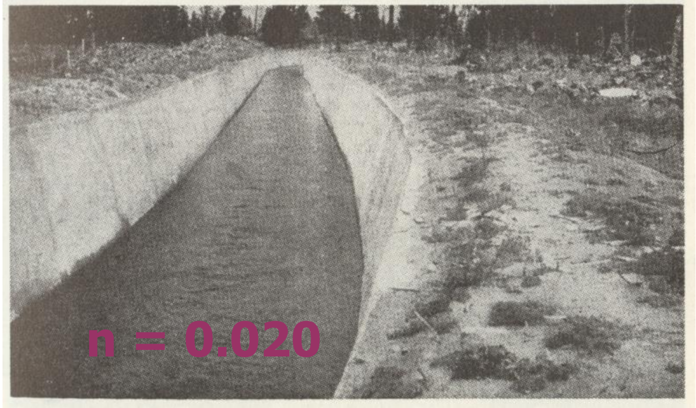
Flow in balance with:

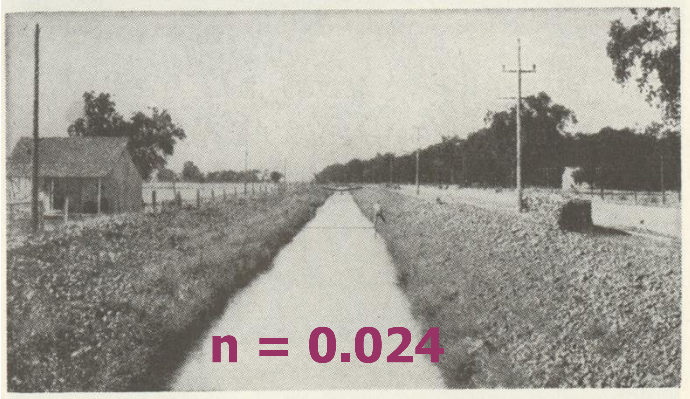
Gravity	⇔	Resistance
Slope	⇔	Conveyance
S_0	⇔	$K = (A \cdot R_h^{2/3}/n)$

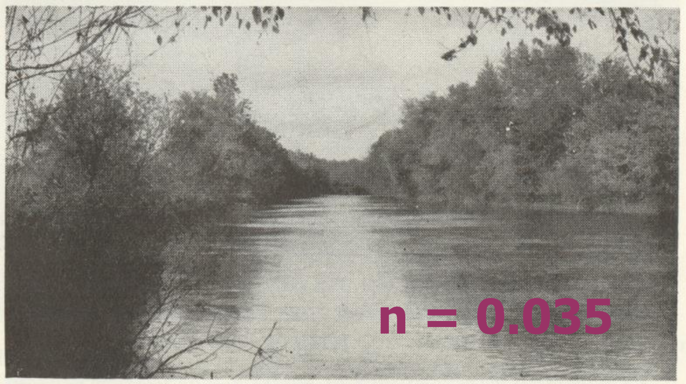
Fluvial Processes: Hydraulics

Manning's n roughness coefficient









Chow (1959)

Fluvial Processes: Hydraulics

Manning's n roughness coefficient:

Manning-Strickler formula:

$n = 0.047\,(D_{50})^{1/6}$ (in meters) $n = 0.0389(D_{50})^{1/6}$ (in feet)

Limerinos Equation:

$$n = \frac{(0.0926)R^{\frac{1}{6}}}{\left(1.16 + 2.0\log\left(\frac{r}{D_{84}}\right)\right)} \quad \text{(eq. 6–30)}$$

$R = r = R_h$

where:

R = hydraulic radius, in ft

D_{84} = particle diameter, in ft, that equals or exceeds that of 84 percent of the particles

Relation between Darcy-Weisbach friction factor *f* and Manning's n

$f = (8 \cdot g \cdot R_h)/V^2$ substitute $V = (1.49/n) \cdot R_h^{2/3} \cdot S_o^{1/2}$

Ch 6 NRCS (2007)

Fluvial Processes: Hydraulics

Roughness coefficient: Darcy-Weisbach *f*

Hey, Thorne, Newson formula:

for gravel bed rivers with a W/D ratio > 15

Notes: $d = d_{max}$; $R = R_h$

$$\frac{1}{\sqrt{f}} = 2.03 \log \frac{aR}{3.5D_{84}} \quad \text{(SI units)} \qquad \text{(eq. 6–27)}$$

or

$$\left(\frac{8}{f}\right)^{0.5} = 5.75 \log \frac{aR}{3.5D_{84}} \quad \text{(English units)} \qquad \text{(eq. 6–28)}$$

where:
R = hydraulic radius
D_{84} = bed-material size for which 84 percent is smaller

The dimensionless a is given by (Thorne, Hey, and Newson 2001):

$$a = 11.1\left(\frac{R}{d_{max}}\right)^{-0.314} \qquad \text{(eq. 6–29)}$$

where:
d_{max} = maximum flow depth

Chang (1988) Equation:

Hydraulically rough flow

$$f = \left(0.248 + 2.36 \log \frac{d}{D_{50}}\right)^{-.5} \qquad \text{(eq. 6–47)}$$

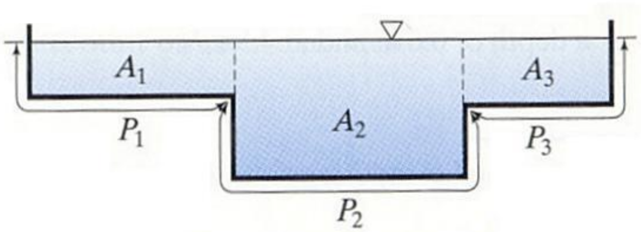
Ch 6 NRCS (2007)

Fluvial Processes: Hydraulics

- Use Manning's equation: Composite roughness channels
  - The selection of an appropriate n is critical to the accuracy of the results.
  - When applying Manning's equation to a more complex channel geometry, break the cross section into several parts:

for English units

$$Q = \frac{1.486}{n_1} A_1 R_{h_1}^{2/3} S_0^{1/2} + \frac{1.486}{n_2} A_2 R_{h_2}^{2/3} S_0^{1/2} + \cdots$$



one equation for a composite n … (n_c)

$$Q = \frac{K_n}{n_c} A_C R_{hc} S_0^{1/2}$$

Fluvial Processes: Hydraulics

Energy Equation for Gradually Varied Flow

Energy equation:

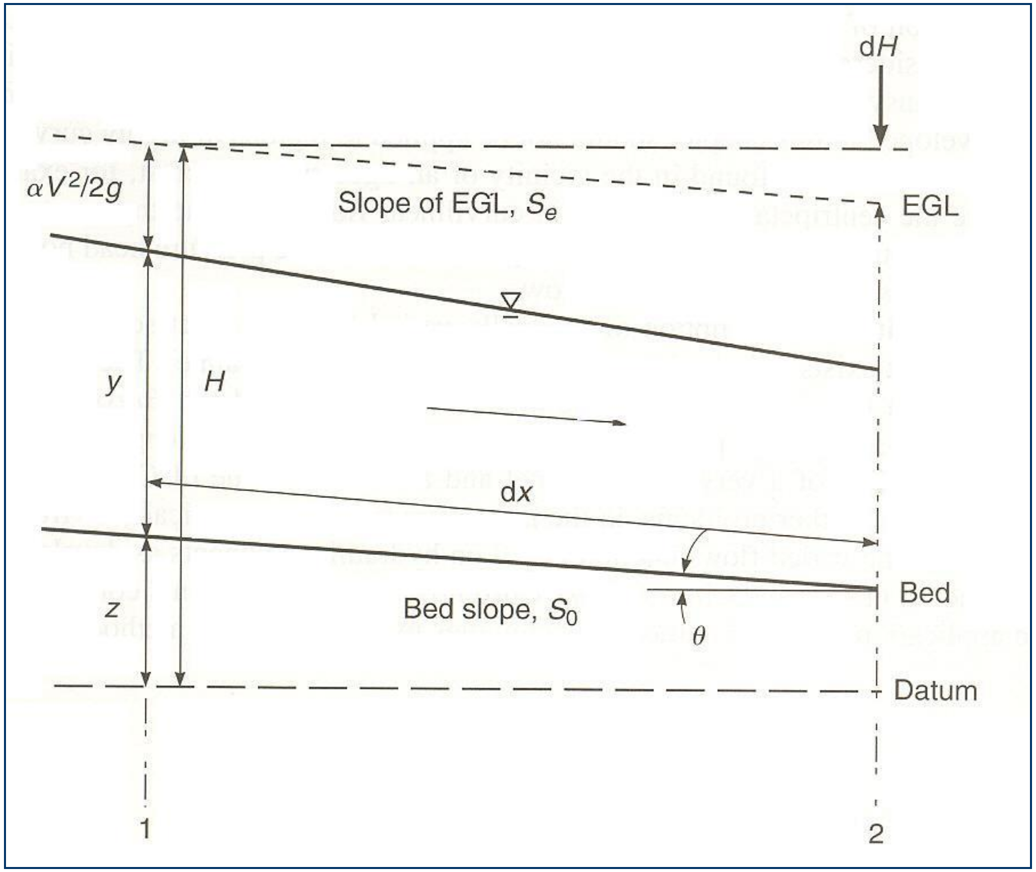
$$H = z + y + \alpha\frac{V^2}{2g}$$

Differentiate H with respect to x:

$$\frac{dH}{dx} = -S_e = -S_0 + \frac{dE}{dx}$$

Solving for dE/dx:

$$\frac{dE}{dx} = S_0 - S_e$$



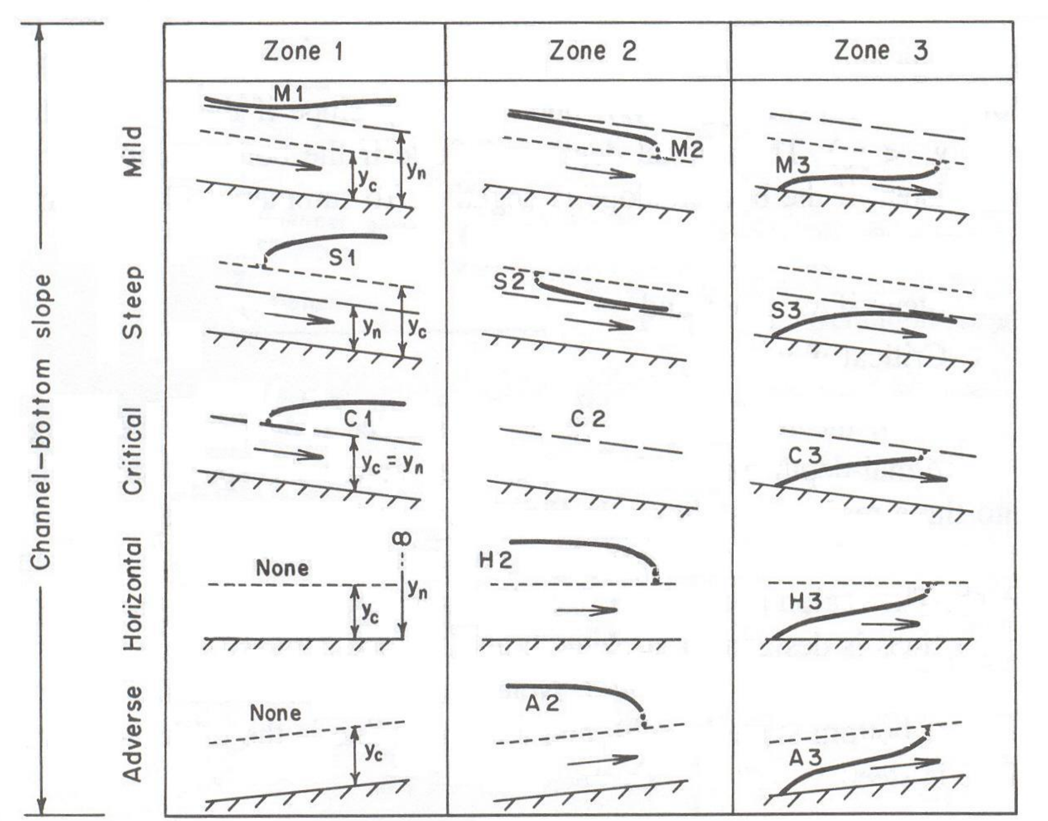
Sturm Figure 5.1

Fluvial Processes: Hydraulics

- **Water Surface Profiles:**
- 12 Fundamental Types of Gradually-Varied Flow

- Slopes: Mild, Steep, Critical, Horizontal, and Adverse

- WS relationships to y_o and y_c: depth increasing/ decreasing



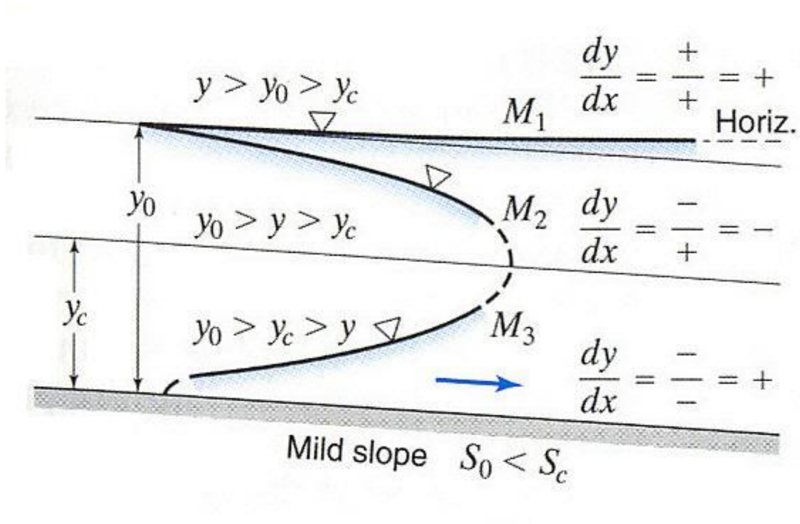
Water surface profiles

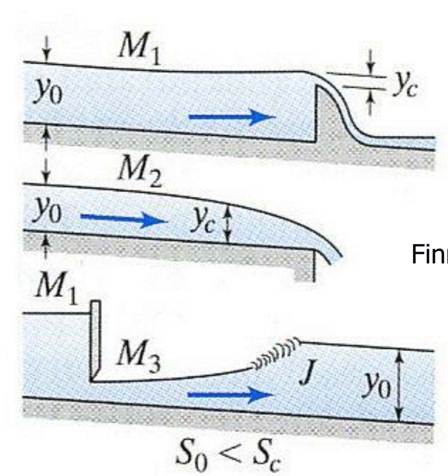
normal depth = $y_0 = y_n$

Chow (1959)

Fluvial Processes: Hydraulics

- Gradually-varied Flow: Mild slope water surface profiles: $y_0 > y_c$





Finnemore & Franzini (2002)

Relevant profiles for stream restoration

- **M_1** is a Backwater Curve (elevated riffle structure, log/rock weir)
- **M_2** is a Drawdown Curve (drop over a log/rock weir)

Fluvial Processes: Hydraulics

Gradually Varied Flow

Modeling Standard:
US Army Corps of Engineers:
Hydrological Engineering Center – River Analysis System (HEC-RAS)

HEC-RAS developed from the HEC-2 water surface profile program based on one-dimensional steady flow, utilizing the gradually-varied flow equation.

HEC-RAS has a graphical user interface (GUI) for pre- and post-processing; and several features the HEC-2 model did not have such as flood encroachment analysis optimization, stable channel design, and accounting ice cover resistance.