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# Initiation of river meandering

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Meanders, or frequent sinuous reversals of channel curvature, are a fundamental characteristic of river planform. Although the cause of meandering has attracted attention since the beginning of civilization, as yet there is no completely satisfactory explanation of this phenomenon. The self-similarity of meander geometry over a wide range of scales and environmental conditions suggests that meandering reflects the influence of some general control (Davy and Davies, 1979). Various arguments have been advanced to explain *why* rivers meander, including dissipation of excess energy (Jefferson, 1902; Inglis, 1947), minimization of energy expenditure (Yang, 1971; Chang, 1988), and minimization of the variance in bed shear stress and boundary friction (Langbein and Leopold, 1966). These theories provide predictions that conform well with observed forms, but they are primarily teleological in nature and thus difficult to evaluate scientifically; minimization of energy or shear stress could just as easily be the result of meandering as it could the cause (Richards, 1982: 28). More important, these arguments do not describe *how* meanders develop.

Over the past 25 years, considerable effort has been devoted to identifying the physical processes that initiate the development of meanders in straight channels. This research has focused on inter-relationships among fluid dynamics, sediment transport, bed morphology, and bank erosion; the goal is to develop a hydrodynamic explanation of meandering. Although much of this work has occurred outside geography, it has relevance to physical geographers, particularly fluvial geomorphologists, for at least two reasons. First, it relates directly to geomorphic theory concerning the basic nature of meandering rivers and their sedimentary deposits. Secondly, geomorphologists are in a position to contribute substantively to theory development because many postulates of hydrodynamic theories have not been rigorously tested in the field. The purposes of this paper are: 1) to review current hydrodynamic concepts related to meander initiation; and 2) to provide direction for future geomorphologic investigations so that they may contribute directly to theory enhancement. The approach taken here is to summarize the salient ideas embodied in various theories. For detailed reviews of mathematical formulations the reader is referred to Diplas *et al.* (1988) and Seminara and Tubino (1989).

## I Flow oscillation and meander initiation

The development of a meandering stream from an initially straight channel requires retreat of the banks at regularly-spaced intervals along alternate sides of the channel. Although bank erosion is a necessary condition for meander initiation (Friedkin, 1945), it is clearly the effect of some original cause, not the cause itself (Ackers and Charlton, 1970). Because bank erosion results primarily from removal of sediment at the base of the bank by hydraulic action (Thorne and Tovey, 1981), meandering is often associated with oscillating flow. Hydrodynamic theories of meander initiation focus on mechanisms that produce flow oscillation in a straight channel. These theories can be separated into two groups: 1) those that view oscillation as an inherent property of turbulent flow; and 2) those that view oscillation as the result of interaction between flow and a mobile channel bed. Both types of explanations attempt to account for the distinctive bedforms associated with the initiation of meandering in straight channels. The characteristics of these bedforms are discussed below.

## II Bedforms in straight erodible channels

Both laboratory experiments and field investigations show that periodic deformation of the channel bed and the development of a sinuous thalweg precedes or is closely associated with the initiation of meandering. Laboratory work has focused on meander development both in straight channels (Quraishy, 1944; Friedkin, 1945; Einstein and Shen, 1964; Ackers and Charlton, 1970; Karcz, 1971; Schumm and Khan, 1972) and in straight channels with an entrance bend (Tiffany and Nelson, 1939; Friedkin, 1945; Schumm and Khan, 1972). An entrance bend induces meandering at a lower gradient than that required to initiate meandering in a channel with a straight entrance (Schumm and Khan, 1972). Erosion at the upstream bend propagates rapidly along the channel producing a series of bends that decrease in amplitude downstream (Tiffany and Nelson, 1939; Schumm and Khan, 1972). Bank erosion precedes bed deformation at the entrance bend, but transmission of the erosion downstream occurs in association with the development of alternating deeps and shoals (Friedkin, 1945; Schumm and Khan, 1972).

In straight, erodible, plane-bed channels without an entrance bend, initial motion of bed material occurs parallel to the channel sides, but scour holes develop rapidly at regular intervals along alternate sides of the channel (Quraishy, 1944). Material scooped out of the scour holes is deposited downstream in fan-shaped wedges that overlap in a 'fish-scale pattern' (see Figure 1). The scour hole (pool) and depositional wedge together form what has been referred to as a bar unit (Dietrich, 1987) or single-row alternate bar (Ikeda, 1989). Each bar terminates in an oblique lobe front, the lowest portion of which is equivalent to a riffle. The highest portion of the bar is located slightly upstream of the bar front toward the centre of the channel (see Figure 1).

Alternate bars significantly affect patterns of flow and sediment movement in straight channels. Shoaling and divergence of flow over the bars leads to convergence of flow in the scour holes (Ikeda, 1984a; Fukuoka, 1989). This movement of flow around alternate bars produces an abrupt shift in the position of the thalweg (zone of maximum velocity) from one side of the channel to the other (see Figure 1). Under straight entrance conditions, the amplitude of this meandering thalweg increases downstream (Schumm and Khan,

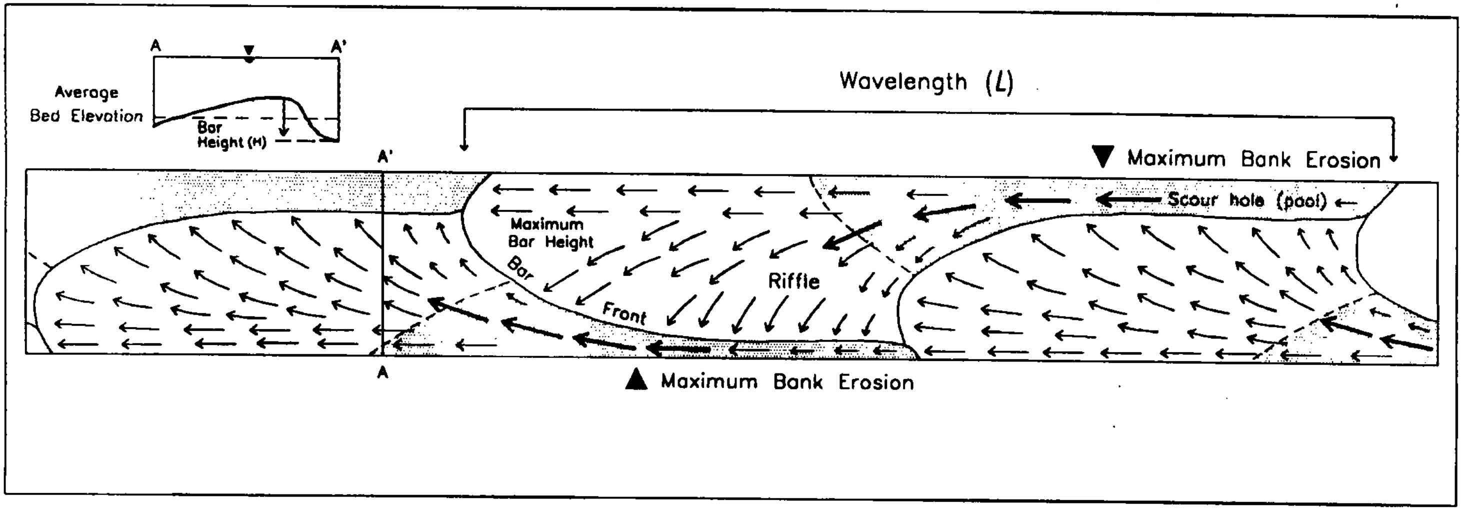


**Figure 1** The morphology of single-row alternate bars in straight channels. Shaded areas are below reach-averaged bed elevation. Arrows indicate pattern of flow; heavy arrows mark position of the thalweg (after Quraishy, 1944; Dietrich, 1987; Fukuoka, 1989).

1972). The orientations of sediment transport and velocity vectors generally coincide except in the vicinity of the bar front where sediment transport vectors deviate towards the scour hole due to the gravitational effect of the steep transverse bed gradient (Fukuoka, 1989). As sediment moves across the lobe front the bars migrate downstream; increases in sediment load enhance bar growth and rates of migration (Einstein and Shen, 1964; Ackers and Charlton, 1970; Sukegawa, 1973). Stabilization of migrating bars in straight channels involves deposition of suspended load on bar surfaces by shallow flows. The veneer of fines prevents erosion, but scour along the thalweg decreases water levels, leading to bar emergence and cessation of bed material transport across the bar surface (Schumm and Khan, 1972).

If the equilibrium size of a bar is large enough to deflect flow sufficiently to overcome the strength of the channel banks, erosion will occur opposite the bars, thus initiating channel curvature (Quraishy, 1944; Ackers and Charlton, 1970). The locus of bank erosion is dependent on the transverse slope of the bed, flow velocity, water level, and streamline deviation, but is generally located near the longitudinal midpoint of the scour hole (see Figure 1) (Fukuoka, 1989). Bank erosion provides additional sediment for bar growth downstream, which propagates deflection of flow and the initiation of curvature along the channel (Ackers, 1964; Ackers and Charlton, 1970; Schumm and Khan, 1972).

Recent experimental work has defined more precisely the hydraulic conditions under which alternate bars form and the morphologic characteristics of these features. Fujita and Muramoto (1985) demonstrated that alternate bars do not develop uniformly along laboratory flumes but instead exhibit sequential development. The formation of a prominent bar upstream triggers a chain reaction of bar propagation in the downstream direction. During the initial phase of bar development, bar edges become conspicuous and bar length increases more rapidly than bar height. This phase continues until bar length stabilizes. In the second phase, bar length remains fairly constant, but bar height continues to increase until it reaches a maximum value. The increase in bar height is associated primarily with an increase in the depth of scour rather than enhanced deposition of sediment on bar tops. This second phase is usually two to three times longer than the initial phase. A third phase is an equilibrium state wherein bar height and length fluctuate slightly about constant values.

Alternate bars form in moderately wide, shallow, and steep channels where excess shear stress on the bed is rather small (Sukegawa, 1973; Ikeda, 1973) (see Table 1). In narrow, deep, low-gradient channels only mesoforms (ripples, dunes, plane bed, antidunes) are present, whereas multiple row bars characteristic of braiding occur in exceptionally wide, shallow, and steep channels (Ikeda, 1989). The formation of alternate bars at low values of excess shear stress and high width/depth ratios suggests that they are a low-flow phenomenon, particularly in streams transporting sand and fine gravels or large suspended loads. Cursory field observations of bar formation in sand-bed rivers generally support this premise (Matthes, 1941; Fahnestock and Maddock, 1964). The notion that alterate bars form during low flow is at odds with the geomorphic viewpoint that these features develop in response to bankfull events (Knighton, 1982). The geomorphic viewpoint is based largely on Lewin's (1976) observation of bar formation during high flow in an artificially straightened reach of the gravel-bedded River Ystwyth in Wales. This observation probably reflects the influence of bed material caliber on alternate bar formation. Coarse heterogeneous bed material normally promotes bar formation; however, if armouring occurs the lower limit of alternate bar formation increases in proportion to the increase in mean grain size of the armour layer (Jaeggi, 1984) (see Table 1). Thus, armouring tends to inhibit bar

**Table 1** Empirical equations to predict formative conditions, wavelength, and height of alternate bars.

| | |
|---|---|
| *Formative conditions* | |
| **Chang *et al.*** (1971) | |
| $W/D > 12$ | |
| **Ikeda** (1973) | |
| $U_*/U_{*c} \leqslant 1.4(SW/D)^{0.33}$ | multiple linguoid bars |
| $3.5(SW/D)^{0.33} \geqslant U_*/U_{*c} > 1.4(SW/D)^{0.33}$ | alternate bars with well-developed bar fronts |
| $10(SW/D)^{0.33} \geqslant U_*/U_{*c} > 3.5(SW/D)^{0.33}$ | alternate bars with poorly defined bar fronts |
| $U_*/U_{*c} > 10(SW/D)^{0.33}$ | ripples and dunes |
| **Sukegawa** (1973) | |
| $U_*/U_{*c} \leqslant 3.35(SW/R)^{0.25}$ | |
| **Jaeggi** (1984) | |
| lower limit $\theta/\theta_c \geqslant 1$ (unarmoured beds) | |
| $\theta/\theta_c \geqslant (d_{ma}/d_m)^{0.67}$ (armoured beds) | |
| upper limit $\theta/\theta_c \leqslant 2.93\ln(\theta_b/\theta_c) - 3.13(W/d_m)^{0.15}$ | |
| *Wavelength* | |
| **Ikeda** (1984a: 1984b) | |
| $L = 5(WD/C_f)^{0.5}$ or $L/W = 5(D/WS)^{0.5}F$ | $F < 0.8$ |
| $L/W = 5.3(W/D)^{0.55}(D/d_{50})^{-0.45}$ | $F \geqslant 0.8$ |
| *Bar height* | |
| **Ikeda** (1984b) | |
| $H/D = 0.0442(W/D)^{1.45}(D/d_{50})^{-0.45}$ | $6 < W/D < 40$ |
| **Jaeggi** (1984) | |
| $H = .22W^{0.85}{d_m}^{0.15}$ | |

*Notes:*
$U_*$ = shear velocity; $U_{*c}$ = critical shear velocity at incipient motion; $W$ = flow width; $D$ = flow depth; $S$ = channel gradient; $R$ = hydraulic radius of flow; $d_m$ = mean grain size of bed material; $d_{ma}$ = mean grain size of armour layer; $d_{50}$ = median grain size, $\theta$ = dimensionless Shield's parameter $[DS/(s-1)d_m$, where $s$ is the relative density of the bed material]; $\theta_c$ = dimensionless critical Shield's parameter; $\theta_b = [WS/(s-1)d_m]$; $L$ = bar wavelength; $C_f$ = resistance coefficient ($gDS/U^2$; where $g$ = gravitational acceleration and $U$ = mean velocity); $F$ = Froude number; $H$ = bar height.

formation. This factor may explain the discrepancy between Lewin's (1976) observation and those in sand-bed streams.

Well-developed alternate bars form in subcritical flow (mean Froude number <1). A similar bedform, the diagonal bar, develops at mean Froude numbers near one (Einstein and Shen, 1964). This feature, which has also been called a point dune (Hickin, 1972), produces 'pseudomeandering' in straight erodible laboratory channels (Wolman and Brush, 1961). Jaeggi (1984) noted that some confusion exists concerning the distinction between alternate and diagonal bars and that in some threshold studies diagonal bars may be classified as alternate bars. Einstein and Shen (1964) proposed that diagonal bars form along the path of a transverse wave with a period equal to the time required for the main flow to travel one length of a bar. The scour holes associated with diagonal bars are shallower and rates of migration are higher than those for alternate bars (Jaeggi, 1984).

Alternate bars have an average wavelength of approximately nine times the channel width (Ikeda, 1984a; 1984b). This value is slightly less than the average wavelength of meanders in curved channels, which is approximately 10–14 channel widths (Leopold and Wolman, 1960). However, variation about the mean is considerable; the proportional constant between bar wavelength and width ranges from 4 to 17. The wavelength of individual bars depends on flow geometry and the caliber of the bed material load (see Table 1). Chang *et al.* (1971) showed that dimensionless bar wavelength ($LS/D$) is positively related to the Froude number. This work was furthered by Ikeda (1984a; 1984b), who developed separate relationships to predict bar spacing for Froude numbers less than and greater than 0.8 (see Table 1). The need for separate equations may reflect the transition from true alternate bars ($F < 0.8$) to diagonal bars ($F > 0.8$).

Bar height, defined as the vertical distance between the top of the bedform and the deepest part of the scour hole (Figure 1), depends on channel form and is independent of hydraulic factors such as discharge, slope, shear stress, and Reynolds number (Ikeda, 1984b; Jaeggi, 1984). Ikeda's (1984b) equation (Table 1) indicates that the height of alternate bars is greatest in wide channels with coarse bed material. This formula was developed from data for streams with Froude numbers greater than 0.7; thus, it may be based on a heterogeneous mix of alternate and diagonal bars. Jaeggi (1984) noted that alternate bars are much higher than diagonal bars.

Field investigations are few, but limited observations suggest that alternate bars in straight reaches of natural channels are similar to those observed in the laboratory (Leopold and Wolman, 1957; Fahnestock and Maddock, 1964; Leopold, 1982; Dietrich and Whiting, 1989). An exception is that some natural bars appear to be stable and do not migrate (Leopold and Wolman, 1957). Cursory observations of meander initiation in channelized streams confirm that the development of alternate bars precedes the initiation of channel curvature via bank erosion (Kinoshita, 1961; Keller, 1972; Lewin, 1976; Simon, 1989); however, detailed investigations of the interactions among bed morphology, bank erosion, and fluvial processes as meanders develop are lacking.

## III Periodically reversing helical flow and meander initiation

It has long been recognized that periodically reversing helical motion is a fundamental characteristic of flow in meandering rivers (Thomson, 1876; Rozovskii, 1957). Helical flow consists of spiral motion superimposed on the primary flow (Richards, 1982); whereas secondary currents represent circulation of fluid around the axis of primary flow (Ciray, 1967; Bathurst *et al.*, 1979). If periodically reversing helical motion is an inherent property of open channel flow, it represents an obvious mechanism for initiating periodic deformation of the channel bed and banks along alternate sides of the channel. In fact, this mechanism is commonly invoked in geomorphologic conceptual models of meander initiation (Hey, 1976; Brotherton, 1979; Knighton, 1982; Thompson, 1986).

Helical flow occurs in a meandering channel because the curvature-induced centrifugal force acting in the cross-channel direction carries near-surface flow outward, whereas the counteracting pressure-gradient force generated by superelevation of the water surface moves water inward along the bed (Dietrich, 1987). Asymmetry of the channel bed also plays an important role in that the point bar deflects flow outward, thus restricting helical motion to the outer part of the channel (Dietrich and Smith, 1983). According to this theory, helical flow is a product of curvature and therefore should not occur in straight, plane-bed channels. However, at least two factors may lead to the production of weak helical vortices in straight open channels: 1) Coriolis acceleration; and 2) anisotropic turbulence. The following discussion reviews current theories related to the generation of helical flow in straight channels and also explores mechanisms that may produce asymmetry and periodic reversal of this helical motion.

## 1 Coriolis-induced secondary circulation

All fluids that move on the earth are affected by the Coriolis force associated with the earth's rotation. This force deflects moving objects to the right in the northern hemisphere and to the left in the southern hemisphere. The Coriolis-induced force per unit mass of water ($F_c$) is defined as (Kabelac, 1957):

$$F_c = 2\Omega \sin\phi U \tag{1}$$

where $\Omega$ is the angular velocity of the earth ($7.294 \times 10^{-5}$ radians $s^{-1}$), $\phi$ is the angle of latitude, and $U$ is mean velocity of flow. Around the turn of the century it was proposed that deflection of streamflow by the Coriolis force produces preferential erosion of right-hand channel banks in the northern hemisphere and left-hand channel banks in the southern hemisphere (Von Baer, 1860; Gilbert, 1884; Eakin, 1910). Eakin (1910) and Einstein (1926) argued that erosion associated with the Coriolis force is related to the production of secondary currents. These currents are generated in much the same manner as those produced by centrifugal forces in curved channels: a local imbalance exists between the Coriolis-induced force and the resulting transverse pressure gradient force. Flow near the surface thus moves outward in the direction of the Coriolis force, whereas flow near the bed moves in the opposite direction under the influence of the pressure gradient force.

The driving force for Coriolis-induced secondary currents can be obtained by substituting deviations from the depth-averaged velocity for $U$ in equation (1) (Booij, 1988). Neu (1967) used this approach to derive the following expressions for deviation angles $\alpha$ from a straight flow path based on consideration of vertical variations in gravitational, friction, and Coriolis forces in a straight channel:

$$\alpha_s = 0.036\ (D/U) \tag{2}$$

$$\alpha_b = 0.060\ (D/U) \tag{3}$$

where $D$ is flow depth, $U$ is mean velocity, and the subscripts $s$ and $b$ refer to the surface and near-bed layers, respectively. Most sediment-bearing rivers have $D/U$ ratios less than 20 and thus deviation angles of secondary currents are less than 1 to 2°. Larsson (1986) related the magnitude of Coriolis-induced surficial secondary currents ($W_s$) to the Rossby number ($R$) as:

$$W_s/U = 4R^{-1} \tag{4}$$

where $R = U/\Omega\sin\phi D$. This equation indicates that the Rossby number must be below 400 before secondary velocities become greater than 1% of the downstream velocity. Values of $R$ this small occur only in deep slow-moving water bodies such as large rivers, reservoirs, and estuaries, where secondary velocities may exceed 5% of the main flow velocity (Booij, 1988). Thus, although Coriolis-induced secondary flow has been mentioned as a possible cause of meander initiation (Einstein, 1926), it is unlikely that the magnitude of the Coriolis force in most rivers is great enough to cause significant deformation of the channel perimeter. Further work is required to determine whether Coriolos-induced currents are capable of producing preferential channel erosion or meandering in deep, slow-moving waterways.

## 2 Turbulence-induced helical flow in straight channels

Whereas field measurements of primary and secondary flow components in meandering streams have shown that twin asymmetric surface-convergent helical cells occur in pools (Hey and Thorne, 1975; Bathurst *et al.*, 1977; 1979) and twin surface-divergent (Hey and Thorne, 1975) or vertically-stacked (Thorne and Hey, 1979) helical cells occur at riffles, information on secondary currents in straight channels is comparatively meager. Indirect evidence, such as: 1) depression of the maximum velocity core beneath the water surface (Francis, 1878; Wood, 1879; Stearns, 1883); 2) float studies (Cunningham, 1883; Gibson, 1909); 3) computation of secondary flow from patterns of primary flow (Chui *et al.*, 1978; Bhowmik, 1982); or 4) patterns of dye movement, sediment transport, and water surface topography (Leopold, 1982); suggests that flow in straight channels is characterized by twin symmetrical surface-convergent helical cells. Float studies indicate that the velocity of secondary currents associated with these cells is about 5% of the primary flow (Gibson, 1909).

The occurrence of helical motion in straight streams has been attributed to anisotopic turbulence in three-dimensional open channel flow (Prandtl, 1927). These turbulence-induced currents, which are referred to as secondary flows of the second kind to distinguish them from secondary currents associated with mean flow skewing, i.e., flow in curved channels (Prandtl, 1952: 148), are described by the mean streamwise vorticity equation for turbulent uniform flow:

$$V\frac{\partial E}{\partial y} + W\frac{\partial E}{\partial z} = \frac{\partial^2}{\partial y\,\partial z}(\overline{v^2} - \overline{w^2}) + \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial y^2}\right)\overline{vw} \tag{5}$$

where $y$ and $z$ are co-ordinates in the vertical and cross-channel directions, respectively, $V$ and $W$ are the $y$ and $z$ components of mean velocity, $v$ and $w$ are the velocity fluctuations corresponding to $V$ and $W$, $E$ is streamwise vorticity ($\partial W/\partial y - \partial V/\partial z$), and the overbars indicate time-averaged values. Secondary flow results from gradients in the difference between the turbulent normal stresses, i.e., nonzero values of the vorticity production term (first term on the right-hand side of equation (5)) (Townsend, 1956; Einstein and Li, 1958a; Tracy, 1965).

The relationship between turbulence-induced vorticity and twin symmetrical surface-convergent helical cells in straight open channels is unclear. Experimental studies have shown that corner regions are zones of high vorticity production in channelized flow (Brundrett and Baines, 1964; Gessner and Jones, 1965; Ligget *et al.*, 1965; Tracy, 1965;

Perkins, 1970; Melling and Whitelaw, 1976; Gerard, 1978; Knight, 1989). In square ducts, the pattern of secondary flow consists of a pair of helical cells with opposite circulations separated by the corner bisector. Flow moves towards the corner along the bisector and away from the corner along the walls. The velocity of these secondary currents is less than 3% of the mean primary flow velocity (Tracy, 1965). In open channels the symmetry of corner vortices is distorted by the reduction in the length scale of vertical velocity fluctuations near the free surface (Naot and Rodi, 1982; Nezu *et al.*, 1989; Tominaga *et al.*, 1989). As a result, the upper vortices become stronger and larger than the lower ones. The two surface-convergent upper cells occupy most of the total flow area in relatively narrow channels; however, as the width/depth ratio of the flow exceeds four, the width of each upper cell stabilizes at two times the depth of flow (Naot and Rodi, 1989; Nezu *et al.*, 1989). Thus, it seems unlikely that this mechanism could produce twin flow-filling surface-convergent cells in natural channels, most of which have width/depth ratios in excess of four. Alternatively, corner-induced secondary currents may spawn small multiple roll cells in wide channels with erodible beds (Nezu and Nakagawa, 1984; Kotsovinos, 1988; Nezu *et al.*, 1988). These roll cells are capable of deforming the bed at regularly spaced intervals (Casey, 1935; Wolman and Brush, 1961; Culbertson, 1967; Karcz, 1966; Ikeda, 1981), but they have little effect on the overall geometry of the channel (Ikeda, 1981).

Another factor that can produce helical flow in straight channels is gradual or abrupt variations in boundary roughness (Hinze, 1967; 1973; Muller and Studerus, 1979; Naot, 1984). The direction of roughness-induced secondary currents is dependent on the sign of the vorticity production term in equation (5), which is related to the shear velocity ($U_*$) as (Perkins, 1970):

$$\frac{\partial^2}{\partial y\,\partial z}(\overline{v^2} - \overline{w^2}) = \frac{U_*^2}{l} \quad \frac{2}{U_*}\frac{\partial U_*}{\partial z} - \frac{1}{l}\frac{\partial l}{\partial z} \tag{6}$$

where $l$ is the vertical length at which $\overline{v^2} = \overline{w^2}$ (roughly equal to the local thickness of the boundary layer) and $z$ is the distance along the transverse axis. Thus, mean streamwise vorticity is produced by anisotropy of turbulence whenever a transverse gradient in $U_*$ occurs. Where $\partial U_*/\partial z$ is increasing (transition from smooth to rough boundary) the vorticity production term is greater than zero and secondary circulation is clockwise; conversely where $\partial U_*/\partial z$ is decreasing (transition from rough to smooth boundary) the production term is less than zero and secondary circulation is counterclockwise. The transverse variation of $U_*$ in wide straight channels is usually symmetrical about the channel centreline (Ghosh and Roy, 1970; Kartha and Leutheusser, 1970; Odgaard, 1984); as one moves across the channel, shear velocity steadily increases until it reaches a maximum near the centre of the channel then steadily declines towards the opposite bank. Boundary roughness in natural channels often follows a similar pattern with the coarsest material located in the centre of the stream and finer material occurring near either bank. According to equation (6), these patterns of boundary shear and roughness should produce two contrarotating surface-convergent circulation cells on opposite sides of the channel centreline.

## 3 Boundary-layer theory and helical motion

An alternative theory of helical motion in straight channels draws upon concepts of turbulent eddy generation proposed by Einstein and Li (1958b) and Theodorsen (1952).

Einstein and Shen (1964) maintained that low-velocity, high-vorticity eddies are shed continuously from the wall region into the outer region of the flow. These small, horseshoe-shaped eddies are attached either to the boundary sublayer or, along rough boundaries, to wakes between roughness elements that protrude through the sublayer. Vortices generated along the banks have a tendency to be swept downstream, thereby creating a streamwise vortex that moves water up along the bank and toward the centre of the channel at the surface. If vortices develop along both banks, the result is twin surface-convergent helical cells.

Detailed investigations of boundary-layer flows support some aspects of Einstein and Shen's (1964) theory and contradict others (see Cantwell, 1981; Allen, 1985). Momentum exchange between inner and outer regions of the boundary layer occurs via a process known as bursting (Kline *et al.*, 1967). The relationship between bursting and macroscale turbulence is not completely understood, but recent work suggests there is a connection between the two. Experimental studies indicate that the bursting cycle involves the formation and decay of spanwise vortices that are lifted and stretched into horseshoe-shaped vortices similar to those described by Theodorsen (1952) (Kline *et al.*, 1967; Head and Bandyopadhyay, 1978; 1981). These horseshoe vortices are transient, but they produce coherent turbulent 'bulges' in the outer region flow (Thomas and Bull, 1983). The periodicity of bursts also scales closely with the periodicity of 'boils' or 'kolks' (Matthes, 1947) observed in natural rivers, suggesting that a connection exists between bursting and macroturbulence (Jackson, 1976). Despite these relationships, there is no direct evidence that the process of bursting produces large, persistent helical vortices along the margins of open channels as proposed by Einstein and Shen (1964).

## 4 Oscillation of helical flow

If the twin-surface convergent helical cells that reputedly occur in straight open channels develop an oscillatory motion, they may be capable of moving sediment laterally across the channel at periodic intervals, thereby producing alternate bars. For this process to occur, a mechanism that induces asymmetry of the cells and periodic reversal of the dominant cell must be identified. Einstein and Shen (1964) argued qualitatively that inequalities in bank roughness may initiate cell asymmetry and periodic reversal. Their theory of near-bank vortex distortion described above predicts that clockwise circulation occurs along the left bank of a straight channel and counterclockwise circulation exists along the right bank. If at a given section, the clockwise circulation cell is dominant, water will accumulate on the right side of the channel, resulting in an increase in shear stress along this bank. This process will favour the development of counterclockwise circulation along the right bank, which after a given time, will become the dominant circulation pattern. The counterclockwise current will move sediment from the left bank towards the right bank. The process repeats itself along the channel resulting in the formation of a meandering thalweg and alternating bars (see Figure 2A).

A weakness of this theory is that Einstein and Shen (1964) did not clearly indicate how the initial dominance of one of the cells occurs. The diagram they used to illustrate the concept (Einstein and Shen, 1964; Figure 3) suggests that a pre-existing bar may in some instances be the initiating mechanism. Thus, cause and effect are somewhat ambiguous in this model. Also, subsequent references to this work imply that Einstein and Shen (1964) actually observed periodic reversal of asymmetric helical cells (e.g., Richards, 1982: 184; Thompson, 1986), but their paper clearly indicates that this pattern was inferred from the

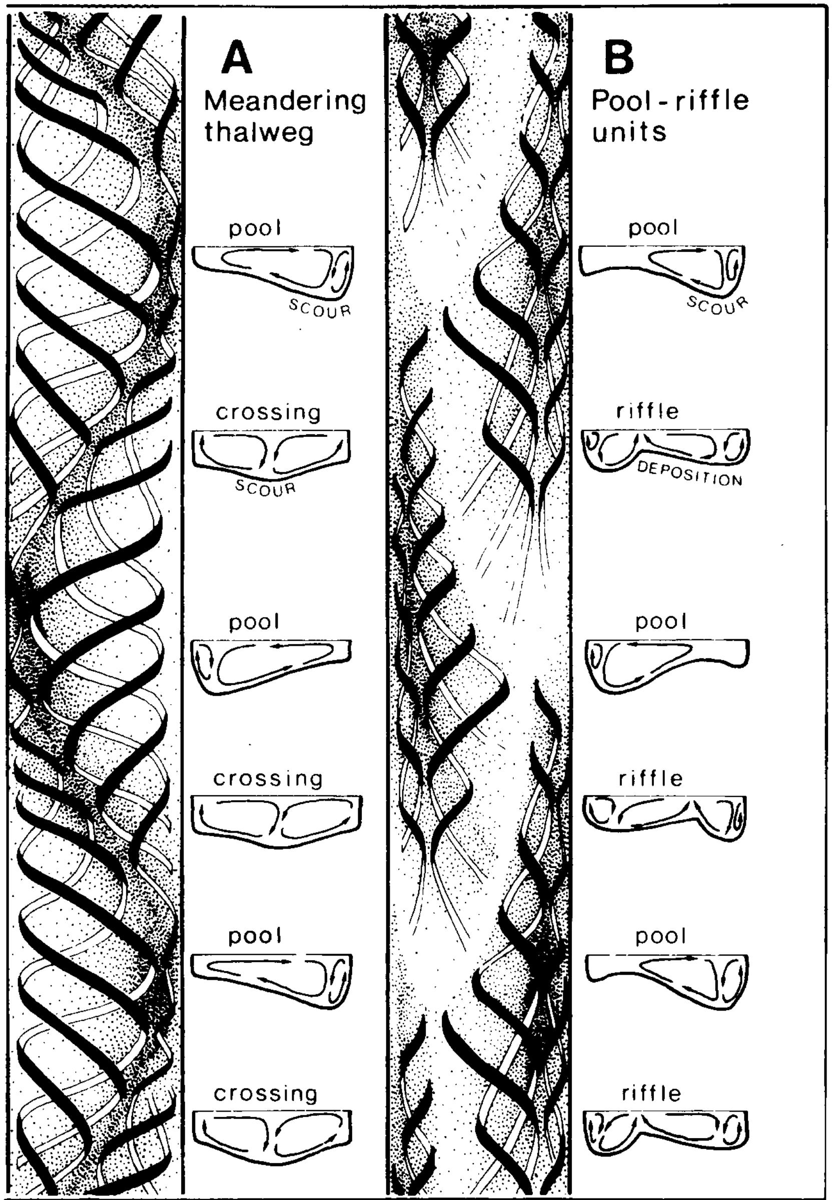


**Figure 2** **A** Einstein and Shen's (1964) model of twin periodically reversing, asymmetric, surface-convergent helical cells. **B** Thompson's (1986) model of modification of the surface-convergent flow pattern by interactions between the flow and a mobile bed. Black lines indicate surface currents, white lines indicate near-bed currents. (Reprinted from Thompson, A. 1986: Secondary flows and the pool-riffle unit: a case study of the process of meander development, by permission of John Wiley & Sons Ltd.)

pattern of bed deformation and not observed directly. The theory also does not directly explain the development of surface divergent cells at thalweg inflection points. These divergent cells may develop as emerging bars and scour holes modify the original surface-convergent pattern (Thompson, 1986) (see Figure 2B).

Shen and Komura (1968) advanced Einstein and Shen's (1964) theory by using equation (5) and an expression for the velocity of secondary currents to demonstrate that the strength of helical motion should be enhanced by wall roughness and flow acceleration. Their experiments indicated that alternate bars form only when flow is accelerating and both banks are rough. This finding suggests that cell instability (asymmetry and periodicity) and alternate bars develop during the rising stage of floods in channels with resistant banks. However, other experimental and theoretical studies have demonstrated that bars can form under steady flow conditions (e.g., Chang *et al.*, 1971; Fredsoe, 1978).

Quick (1974) used Shen and Komura (1968) and Einstein and Li's (1958a) ideas of vorticity production as the basis for a simple lumped-parameter model of meander initiation in which the only variables are angles. This model also incorporates three assumptions about the mechanical processes that produce periodic reversal of helical flow. Operation of the model suggests that stable meandering is associated with moderately erodible channel perimeters over the entire range of values for the precession coefficient, which defines the ratio of vorticity decay and production. Conversely, highly erodible channels with small precession coefficients exhibit pattern instability (braiding), whereas highly inerodible channels with large precession coefficients are generally straight. Although these results are consistent with geomorphic evidence (Ferguson, 1987), the model does not provide an explanation based on the basic equations of fluid motion and its validity hinges on the correctness of its underlying assumptions, which are largely untested.

The notion that flow oscillation is precipitated by local deflection of the flow by an obstacle or by a change in channel shape or alignment is perhaps the most widely cited mechanism for meander initiation (Prus-Chacinski, 1954; Leliavasky, 1955: 120–28). According to this qualitative argument, local erosion or deposition along a channel bank initiates incipient curvature and weak centrifugally induced secondary currents. Erosion associated with the development of these secondary currents enhances the initial disturbance, which strengthens the curvature of flow and associated secondary currents. The deflection of flow into the opposite bank initiates additional erosion downstream. Thus, the disturbance grows and propagates along the channel in much the same manner as meanders propagate downstream from an initial bend in an otherwise straight laboratory channel (e.g., Friedkin, 1945).

The response of a stream to a local disturbance has been modelled as an oblique gravity wave (Werner, 1951) and via a pendulum analogy (Anderson, 1967). Werner (1951) derived the following equation for dimensionless oscillation wavelength:

$$L/W = 2U/(\partial^2 gD - U^2)^{0.5} \tag{7}$$

where $\partial$ is an empirical constant ($\leq 1$) whose value depends on the bedload of the stream. Anderson (1967) derived a similar formula:

$$L/A^{0.5} = (2\pi^3/\beta)^{0.5} F^{0.5} \tag{8}$$

where $A$ is the cross-sectional area of flow and $\beta$ is an empirical constant. Although Werner (1951) did not quantitatively evaluate the accuracy of his formula, Parker (1976) concluded that it underpredicts meander wavelengths. Anderson (1967) showed that experimental data from several studies conform well with equation (8) if $(2\pi^3/\beta) = 72$. However, Chang *et al.* (1971) found this function did not describe the wavelengths of the bars that formed in their experiments. Neither Werner (1951) nor Anderson (1967) considered secondary currents in their analyses. They also assumed that the flow oscillation initiated by an obstacle is independent of damping forces.

One of the most popular explanations for meander initiation in geomorphology is Yalin's (1971) theory of the stochastic structure of turbulent eddies. This theory is based on the assumption that the largest macroturbulent eddies, whose vertical and horizontal dimensions correspond to those of the channel cross section, are ultimately responsible for periodic deformation of the channel bed and planform. Yalin (1971) proposed that the turbulent fluctuations associated with these eddies are confined to a relatively narrow band of frequencies defined by the dimensions of the eddies, which are assumed to be elliptical. Consideration of the autocorrelation function of this band process indicates that velocity fluctuations at cross sections separated from each other by even multiples of the distance $\pi L$, where $L$ is the length scale, are positively correlated, whereas fluctuations at sections separated by distances that are odd multiples of $\pi L$ are negatively correlated. A disruption of the statistical structure of the largest eddies by a permanent local discontinuity will induce a change in the average velocity profile of the flow at that location. And, because the turbulent structure is autocorrelated, similar deviations of the velocity profile will occur at distances of $2\pi L$, $4\pi L$, etc, whereas opposite deviations will occur at distances of $\pi L$, $3\pi L$, etc. Thus, this theory provides a potential mechanism for generating oscillation of twin helical cells along straight channels (Hey, 1976). If the channel perimeter is erodible, sediment transport rates and corresponding erosion and deposition will vary in the same periodic manner. Yalin (1971) claimed that the appropriate length scale for meandering is the channel width. Thus, similar perturbations are located at distances of roughly $2\pi W$ along the channel. Although this distance conforms well with the average spacing of pool-pool or riffle-riffle sequences in meandering streams, which is generally between five and seven channel widths (Richards, 1982: 188), it is somewhat longer than the average spacing of successive alternate bars in straight channels ($4.5W$) (Ikeda, 1984b).

The validity of Yalin's theory hinges on the underlying assumption regarding the structure of macroturbulent eddies. Recent work suggests that large eddies have diameters of 0.5 to 1.5 times the depth of flow and thus do not occupy the entire flow width in most natural rivers (Jackson, 1976; Cantwell, 1981; Allen, 1985). Geomorphologists have also given less prominence to Yalin's (1971) caveat that the correlation structure of velocity fluctuations will die out with increasing distance from the disturbance; meandering on the other hand, displays remarkable longitudinal persistence. Moreover, a complex assemblage of competing correlation structures will exist along natural rivers, which have an infinite variety of local disturbances.

## 5 Evidence of periodic reversal of helical flow

Direct evidence of periodic reversal of helical flow derives primarily from streamplate experiments. Several studies have shown that a stream of water flowing down a smooth inclined surface develops meandering tendencies (Tanner, 1960; 1962; Gorycki, 1973a; Davies and Tinker, 1984). Tanner (1960) attributed these surface-tension meanders to periodically reversing helical cells that form prior to stream curvature, but he did not directly observe these cells. Gorycki (1973a), on the other hand, used dye to observe the structure of flow within surface-tension streams. He found that as the rate of flow is increased, an initially straight stream develops a series of symmetrical, low-sinuosity curves simultaneously along its length. With increased flow, one or more of these curves develop into meanders that migrate downstream and trigger the development of other meanders. At still higher rates of flow, the stream pattern becomes highly unstable and exhibits characteristics of braiding (Gorycki, 1973b). Flow within the straight stream is characterized by curved filaments that become more slow-moving and sinuous with depth. Undulations of the water surface occur in conjunction with the sinuosity, with relatively deep flow occurring at curves. The spacing of the undulations is approximately two to three times the stream width, a distance that conforms well with some field observations of alternate bar spacing (Keller, 1972). As incipient sinuosity develops, upper layers of the flow become more sinuous and the wavelength of the sinuous filaments increases slightly. The amplitude of the sinuous flow also increases and upper filaments begin to overroll as they approach the outside of the curves, resulting in periodically reversing helical flow.

Gorycki (1973a) proposed that sinuous flow in the straight stream is caused by distortion of laminar flow near the channel boundary as it is squeezed between the boundary and the fast upper levels of the flow. He also suggested that the sinuous flow represents the initial manifestation of periodically reversing helical flow and that this sinuous flow may be present in straight reaches of natural streams. This argument was rejected by Davies and Tinker (1984), who claimed that no intrinsic hydraulic phenomenon, such as periodically reversing helical flow, is required to explain the development of surface tension meanders. They showed that at low rates of flow a train of meanders propagates downstream from a large bend introduced at the upstream section of an initially straight stream. This finding suggests that meandering results from disturbances that produce lateral displacement of flow, such as a local irregularity of the bankline. Davies and Tinker (1984) also demonstrated that a long straight segment induced artificially at the downstream end of a meander curve showed no tendency to reverse its direction despite the active development of meanders downstream of the straight reach. Thus, either periodically reversing helical flow is not present in the straight reach or it is ineffective at producing meanders. Davies and Tinker (1984) did not directly examine flow structure within the surface-tension streams; their conclusions were based on observations of planform only. Moreover, their experiments indicate that at high flow rates meanders can form spontaneously in the manner described by Gorycki (1973a).

Although the sequential development of planform in streamplate experiments is similar to that which occurs in erodible laboratory streams (Schumm and Khan, 1972), the analogy between surface-tension streams and natural rivers must be seriously questioned because of the extreme differences in scale. Direct evidence of oscillatory helical flow in straight channels is lacking and detailed field measurements of flow structure are required to rigorously evaluate the assumptions associated with flow initiation theories.

## IV Instability and meander initiation

A fundamentally different approach to the problem of meander initiation involves the analysis of the stability of an erodible channel to periodic perturbations of flow, bed topography, and channel planform. This paradigm adopts the view that meandering is the result of inherent instability between flow and a mobile channel perimeter. It can be divided into two themes: bar theory and bend theory. These themes and the emerging link between them are discussed below.

## 1 Bar theory

Bar theory proposes that periodic deformation of the channel bed is the fundamental cause of meandering (Callander, 1978). This view emerged from laboratory studies (e.g., Quraishy, 1944) and field observations (e.g., Kinoshita, 1961) that showed that alternate bars develop prior to the initiation of channel curvature. The underlying premise of bar theory is that a straight, erodible channel will lose stability to migrating infinitesimal doubly harmonic perturbations of the bed (see Figure 3). In other words, these infinitesimal disturbances will grow in amplitude. The wavelength of the fastest-growing perturbation is viewed as the dominant disturbance, i.e., the one that determines the wavelength of alternate bars. Because it is assumed that the amplified bars initiate channel curvature by deflecting flow into the banks, the wavelength of the dominant perturbation is also viewed as the wavelength of incipient meandering.

Most stability analyses are based on two-dimensional models of flow and sediment transport (e.g., Callander, 1969; Hayashi, 1973; Parker, 1976; Fredsoe, 1978); an exception is the work by Engelund and Skovgaard (1973) in which a three-dimensional model was developed to more fully account for helical motion associated with a nonuniform vertical velocity distribution. The basic equations in the two-dimensional approach include, either in depth-averaged or unit volume form: 1) the momentum-force balance equations in the longitudinal and transverse directions; 2) the continuity equation for water; and 3) the continuity equation for sediment. Linearization of these functions yields corresponding equations for slight perturbations about the initial undisturbed flow, which is assumed to be steady, uniform, and unidirectional. The postulated form of the solution (doubly harmonic migrating perturbation) is substituted into the perturbed form of the

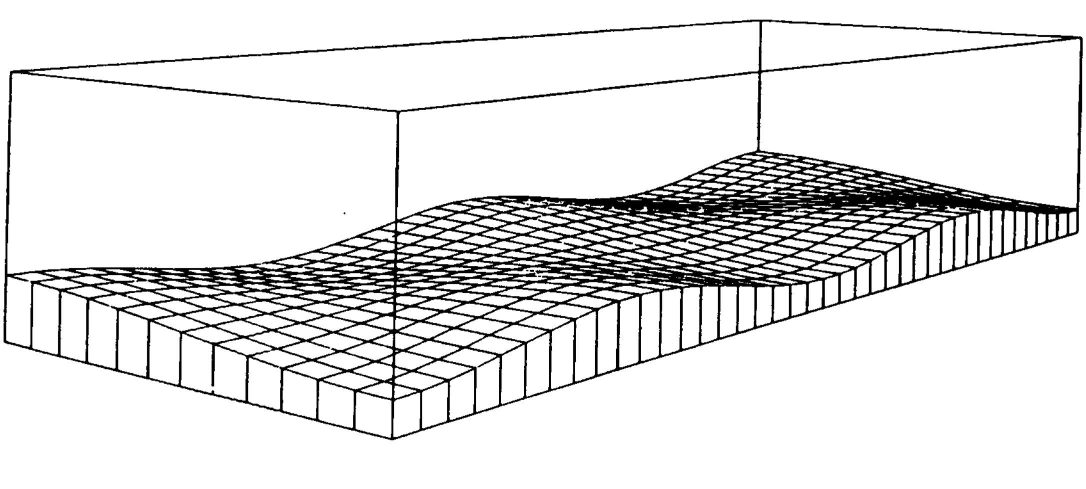

**Figure 3** Three-dimensional view of the doubly harmonic bed perturbation employed in linear stability analyses (from Nelson and Smith, 1989).

basic equations and solved for the complex migration velocity. This parameter consists of a real part and an imaginary part. If the imaginary part is positive, instability occurs and the perturbation grows exponentially. The real part of the complex celerity is proportional to the rate of migration of the perturbation.

Linear stability analyses have focused primarily on three issues: 1) determining the necessary conditions for instability; 2) distinquishing among straight, meandering and braided regimes; and 3) predicting the wavelengths of alternate bars or meanders. Parker (1976) demonstrated through an analysis of the roots of the characteristic equation for complex celerity that sediment transport is a necessary condition for instability. This result is at odds with the occurrence of meandering in sediment-free streams such as those on stream plates, glacial meltwater streams, and the Gulf Stream. Parker (1976) identified other factors that assume the role of sediment transport in these systems including differential heating and melting for glacial meltwater streams, surface tension for

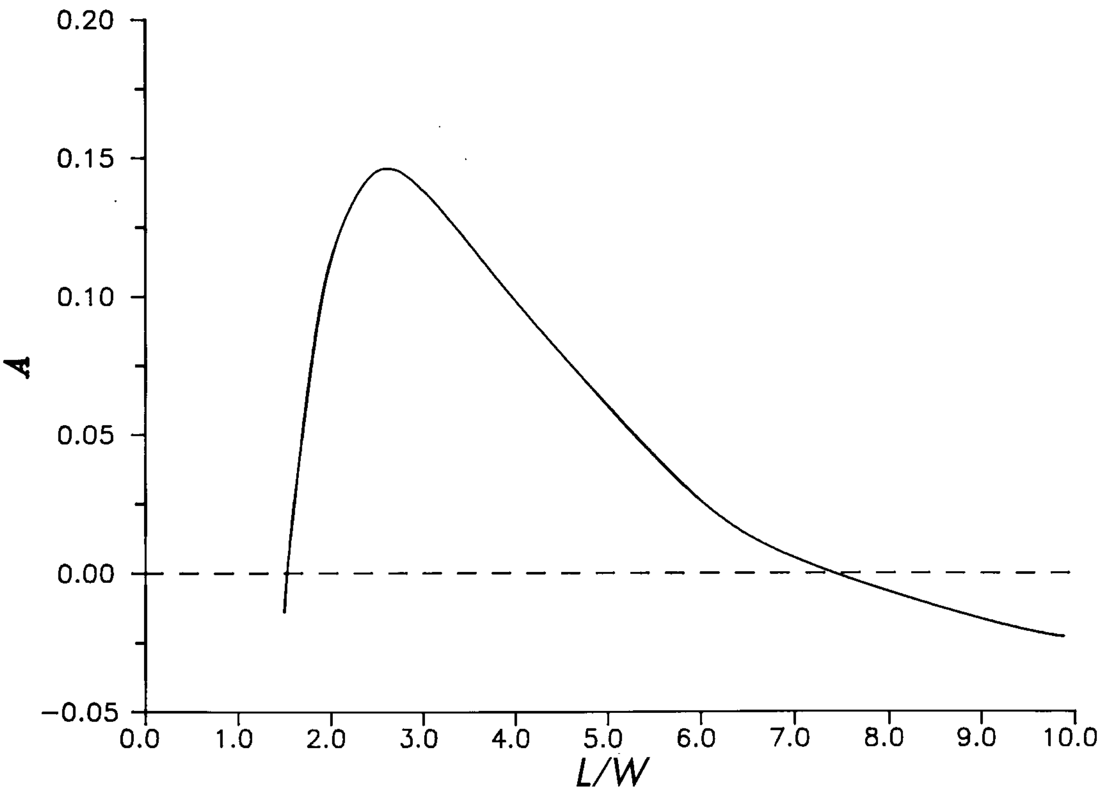


**Figure 4A** Bar amplification factor ***A*** versus wavelength-width ratio for $W/D = 20$, $\theta = 0.2$, $s = 2.65$, and $C_D$ (drag coefficient) $= 7$.

miniature stream-plate rivers, and the Coriolis effect for the Gulf Stream. A stability analysis of the glacial meltwater hypothesis indicates that such streams have an inherent tendency to meander, but that meanders form only during supercritical flow and do not migrate downstream (Parker, 1975). The latter prediction is contradicted by field evidence (Marston, 1983).

The results of stability analyses are commonly presented as a plot of amplification factor versus dimensionless wavelength (see Figure 4A). The peak of this curve for a given set of hydraulic conditions specifies the wavelength at which amplification is a maximum; this wavelength is assumed to correspond to that of the developing meanders. The wavelength with peak amplification rate is strongly dependent on width/depth ratio (see Figure 4B) and the characteristic 'humped' shape of the plot of maximum amplification factor versus width/depth ratio can be used to distinquish between straight, meandering, and braided regimes (Engelund and Skovgaard, 1973). On such a plot there will be a certain value of

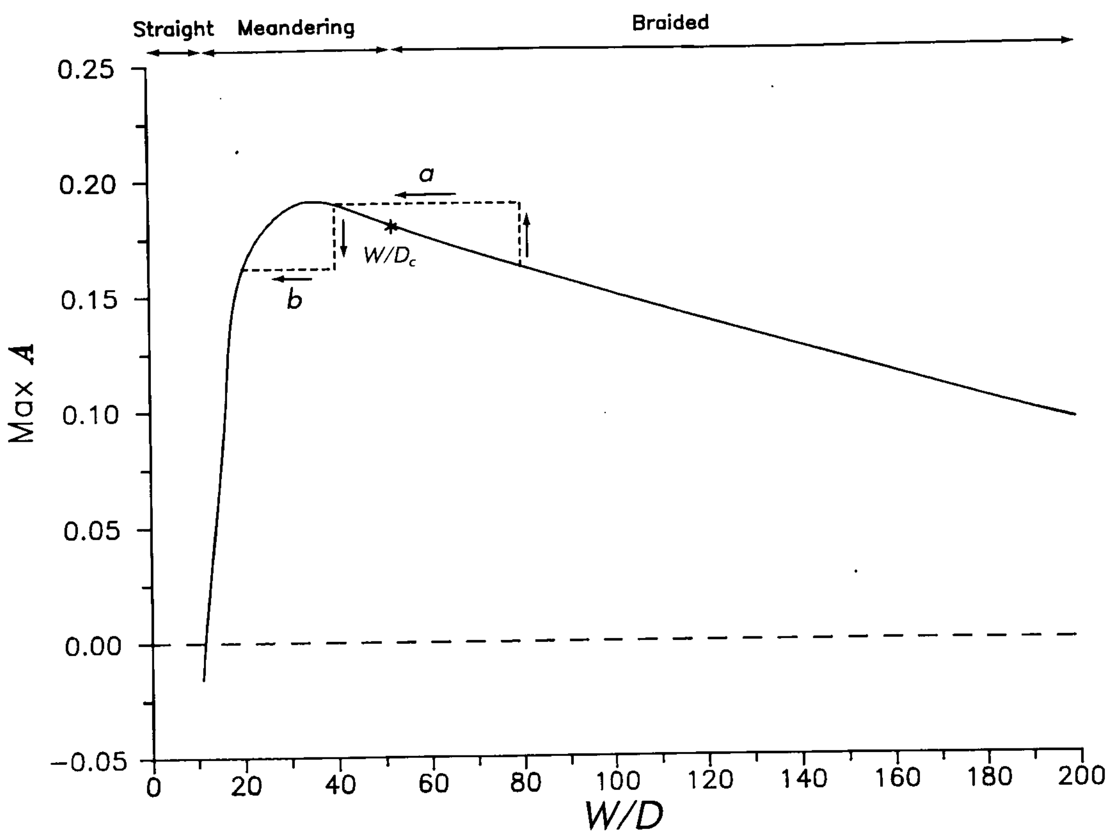


**Figure 4B** Maximum amplification factor versus width/depth ratio for $\theta$, $s$, and $C_D$ as in Figure 4A. If $W/D > W/D_c$ (a) Max $\boldsymbol{A}$ for $0.5(W/D)$ exceeds that for $(W/D)$ and two adjacent meandering patterns develop on the bed, resulting in braiding. If $W/D < W/D_c$ (b) Max $\boldsymbol{A}$ for $0.5(W/D)$ is less than that for $(W/D)$ and only a single oscillatory pattern develops, resulting in meandering (after Fredsoe, 1978).

$W/D$ ($W/D_c$) for which the amplification rate will be the same for $0.5W/D$. For values of $W/D$ less than this critical value the stream will always meander. However, for values of $W/D$ slightly greater than this critical value the amplification rate will be greater for a channel with $0.5W/D$ and two adjacent wave patterns will form on the bed, resulting in braiding. For still larger values of $W/D$ the amplification rate will be higher for channels of $0.33W/D$, $0.25W/D$, and so on, resulting in more intense braiding as more and more adjacent wave patterns develop across the stream. The 'humped' shape of the maximum amplification-width/depth ratio plot depends on the inclusion of the effect of transverse bed slope on sediment transport in the linear stability analysis (Fredsoe, 1978). If this factor is not included, the relationship between maximum amplification and width/depth ratio decreases monotonically, suggesting that infinite braiding will occur (Engelund and Skovgaard, 1973).

Through an analysis of the roots of the equation describing maximum amplification, Parker (1976) found that differentiation between meandering and braided regimes depends on the parameter $\varepsilon^* = SW/\pi FD \approx SW/UD^{0.5}$. Meandering corresponds to $\varepsilon^* << 1$, braiding is associated with $\varepsilon^* >> 1$, and the transition between these regimes occurs at $\varepsilon^* \approx 1$. This result conforms well with geomorphic evidence which suggests that relatively narrow, deep, low-gradient, high velocity streams tend to meander, whereas wide, shallow, high-gradient streams tend to braid (Leopold and Wolman, 1957). Parker (1976) found no evidence for a domain of stability, but concluded that meandering does not occur for $W/D < 10$ based on experimental evidence. His linear stability analysis yielded a predictive equation for meander wavelength that is similar in form to Anderson's (1967) relation (equation (8)). Unfortunately, this equation is difficult to test for natural rivers since it requires detailed information on hydraulic conditions of the initial nonmeandering flow. Hayashi (1973) derived the exact form of Anderson's (1967) equation from linear stability analysis, but his model is limited by the assumption of potential flow and by the constraint that perturbations to the bed and flow are either in phase or 180° out of phase.

Fredsoe (1978) improved the basic two-dimensional model by partitioning the total sediment load into suspended load and bedload components and by solving the linearized equations numerically rather than analytically. These refinements produce a plot of maximum amplification rate versus width/depth ratio that becomes negative for small width/depth ratios, indicating that narrow streams remain straight (Figure 4B). The specific values of $W/D$ at which a stream will meander, braid, or remain straight depend on the dimensionless Shields parameter $\theta$ (see Figure 5). Fredsoe's (1978) analysis suggests that the dimensionless meander wavelengths ($L/W$) for bankfull flow should be between 4 and 15, values that conform well with the range of $L/W$ (5–15) for most natural rivers (Leopold and Wolman, 1957). More recent work has shown that the critical $W/D$ for bar instability is dependent not only on $\theta$, but also on relative roughness ($D/d$), with the critical $W/D$ increasing as $D/d$ increases (Colombini *et al.*, 1987).

Recently, Nelson and Smith (1989) and Nelson (1990) illuminated the roles that various hydraulic components play in producing bar instability. The factors they considered include streamwise convection of streamwise momentum (SCSM), streamwise convection of cross-stream momentum (SCCM), and Froude number ($F$). The full analysis (all terms included) indicates that the migration rates of perturbations decrease exponentially from large positive values (downstream migration) for short bars to small negative values (upstream migration) for long bars. Moreover, sensitivity analyses reveal that each factor alone is sufficient to produce instability and that the selected wavelength in the full analysis (all terms included) is dependent on all three factors. The analysis also provides a deeper

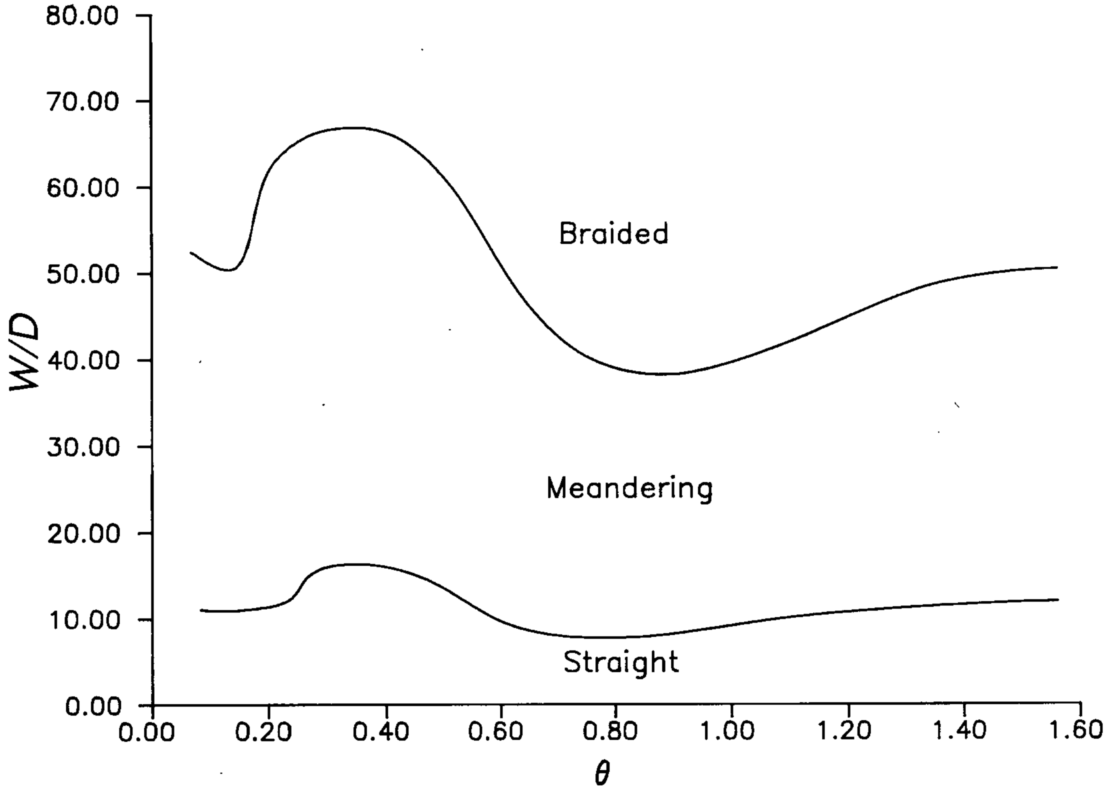


**Figure 5** Stability threshold diagram for a dune-covered river bed ($s$ = 2.65, $D/d$ = 1000, and $C_D$ = 7). (after Fredsoe, 1978).

understanding into the physical mechanisms that produce bar instability. As flow approaches a symmetrical bump in the channel bottom, water surface elevation increases upstream of the obstacle, producing deceleration of streamwise flow and a cross-stream pressure gradient force which generates cross-stream flow. Deceleration of streamwise flow promotes a converging sediment flux (i.e., deposition), whereas the acceleration of cross-stream flow promotes a diverging sediment flux (i.e., erosion). This effect of the bump on the flow and sediment flux influences bar migration. If the bump is long, SCSM dominates; deceleration of streamwise flow on the upstream side of the bump results in deposition, whereas acceleration of streamwise flow downstream of the bump produces erosion. The net effect is that the bar migrates upstream. Conversely, for short bumps, SCCM dominates, leading to sediment divergence (erosion) upstream, sediment convergence (deposition) downstream, and downstream migration of the bar. The length of the bump also influences its rate of growth. For very short bumps, the inertial lag between the bed topography and the flow results in erosion at the crest and inhibition of bar growth. For very long bumps, deceleration of streamwise flow and associated sediment convergence are weak; thus, bar growth is minor. At intermediate wavelengths the transition from erosion to deposition will occur near the apex of the bump, promoting bar growth. At some optimal intermediate wavelength, the impact of the bump on SCSM, SCCM, and the sediment flux will be such that bar growth is maximized.

Nelson and Smith (1989) also found that the optimal wavelength and the peaked nature of the amplification curve are dependent on boundary roughness, Froude number ($F$), and width/depth ratio. Increases in wavelength are associated with decreases in roughness and increases in $F$, whereas increases in roughness produce more peaked amplification curves. Width-depth ratio has a minor influence on optimal wavelength and peakedness with narrow, deep streams having longer wavelengths and less peaked amplification curves than wide, shallow streams. These relationships suggest that alternate bars develop best in relatively, wide, shallow streams that transport coarse bed material.

Major limitations of linear bar theory are that:

1) It ignores nonlinear effects of the growing perturbations on flow and sediment transport.
2) It generally underpredicts alternate bar wavelengths.
3) It cannot explain the longitudinal asymmetry and diagonal fronts of alternate bars (see Figure 1).
4) It implies that bars develop uniformly along a reach of stream.
5) It must be reconciled with the fact that bank erosion associated with lateral migration of the channel occurs at a much slower rate than the migration velocities of the fastest growing bar perturbations.
6) It assumes that channel banks are inerodible and thus does not incorporate a mechanism for the initiation of channel curvature.

Over the past ten years, refinements to bar theory have focused on the first five shortcomings. The last issue is addressed by bend theory, which will be discussed in the next section.

Linear theory predicts unrestricted exponential growth of alternate bars; it cannot account for the elongation, subsequent growth, and eventual stabilization of these bars. Recently, 'weakly' nonlinear bar theories (i.e., those restricted to the neighbourhood of neutral stability) have been developed in an attempt to evaluate finite amplitude effects on bar development (Colombini *et al.*, 1987; Fukuoka, 1989). These nonlinear effects inhibit bar growth, leading to stabilization of bar beight. Bar amplification is suppressed by two factors: 1) enhanced transverse movement of sediment from the bar tops to the scour hole as the transverse gradient of the bed increases; and 2) a decrease in the lateral variation of boundary shear stress and sediment discharge caused by strong nonlinear interactions between the flow and bed profile (Fukuoka, 1989). The equilibrium height of the bars is a complex function of width-depth ratio, Shields parameter ($\theta$), and relative roughness (Colombini *et al.*, 1987).

Although linear bar theory predicts the wavelength of incipient alternate bars quite well, it generally underpredicts the equilibrium wavelengths by 30–40% (Nelson and Smith, 1989). Equilibrium wavelengths predicted by nonlinear stability analysis are somewhat longer than the wavelengths of maximum amplification in the linear approach, but are still shorter than measured wavelengths (Fukuoka, 1989). However, weakly nonlinear theory successfully predicts the development of diagonal bar fronts and longitudinal bar asymmetry (Colombini *et al.*, 1987). It also provides reasonable predictions of the temporal development of bar height under steady and unsteady flow conditions (Seminara and Tubino, 1989; Tubino, 1991). The response of bars to unsteady flow depends on $U$, the ratio between the rate of unsteadiness and the rate of bar growth (Tubino, 1991). If $U$ $>>1$ bar growth occurs much more slowly than the rate of change in flow conditions and thus the flow has little impact on bar development. Conversely, if $U <<1$ bars develop much more rapidly than the time scale of flow unsteadiness and the bars assume equilibrium characteristics conforming with instantaneous flow conditions. If $U \approx 1$ flow unsteadiness has a cumulative impact on bar development. During the rising stage of a flood, the width-depth ratio of the flow decreases, whereas the critical width-depth ratio for bar formation increases. Bar growth is therefore damped during this stage. Near the peak of the flood, the width-depth ratio of the flow may be less than the critical width-depth ratio for bar formation, leading to gradual decay of the bar. During the long descending stage of the flood, substantial bar growth occurs as the width-depth ratio of flow exceeds the critical value for bar formation.

The harmonic perturbations employed in linear and weakly nonlinear bar theories imply that bars develop uniformly along a channel, a notion that is contradicted by experimental evidence (Fujita and Muramoto, 1985). Moreover, weakly nonlinear bar theories do not adequately predict the elongation of bar wavelength that occurs during the initial phase of bar evolution (Fujita and Muramoto, 1985). To address these issues, Nelson and Smith (1989) and Nelson (1990) developed a fully nonlinear flow-sediment transport model to simulate the evolution of bed topography in a channel subjected to a localized, nonharmonic perturbation. This model, which is a generalized version of the meander flow model of Smith and McLean (1984), fully incorporates: 1) gravitational effects of transverse bed slope on bedload transport; 2) the production of helical flow associated with streamline curvature induced by 'topographic steering' of the flow around the bars; and 3) the nonlinearity of flow and sediment transport. Their simulations accurately reproduce the sequence of bar evolution observed by Fujita and Muramoto (1985). In particular, the model predicts bar elongation and the formation of diagonal bar fronts. Elongation is caused by an increase in the inertial lag between the bed topography and the flow as the bar grows, leading to an increase in the wavelength with maximum amplification rate. Diagonal fronts and longitudinal asymmetry occur due to an increase in the migration rate of the bar top relative to the rest of the bar as it increases in height. The model also indicates that twin-surface convergent helical cells should exist immediately downstream from the bar tops. This prediction is consistent with field observations (Leopold, 1982) and implies that helical cells in straight reaches of natural rivers may be generated by deformation of the bed.

Linear and nonlinear bar theories predict that the fastest growing perturbations have rather large propagation velocities. Because it is difficult to explain how these rapidly migrating bars can produce slow bank erosion at discrete locations along a channel, linear steady-state bar theories have been proposed as an alternative explanation of meandering (Oleson, 1984; Struiksma and Klassen, 1988; Struiksma and Crosato, 1989). Stationary perturbations have positive (but not maximum) growth rates and the wavelengths of these perturbations are two to three times longer than those for unsteady perturbations. The major weakness of these models is that they do not identify the mechanism which promotes the development of steady perturbations over those with maximum amplification rates.

## 2 Bend theory and bar-bend interactions

The greatest limitation of bar theory is that it fails to provide a mechanistic explanation of the initiation of channel curvature. The theory considers only the formation of alternate bars within a straight channel with inerodible banks; bank erosion is not allowed. Moreover, it implicitly assumes that the bars control the initial wavelength of meandering by deflecting flow laterally into the banks. Although this process has been observed experimentally (e.g., Ackers and Charlton, 1970), it is not described explicitly by bar theory. In response to these shortcomings, an alternative approach to meander initiation has emerged that considers the stability of a straight channel to an infinitesimal perturbation of the channel centreline. This approach is known as bend theory. As in bar theory, a two-dimensional or three-dimensional physical model of the problem is developed and subjected to stability analysis. The wavelength associated with maximum instability is viewed as the wavelength of meandering.

Ikeda *et al.* (1981) developed a simple two-dimensional model based on the continuity and momentum equations of fluid motion, a relationship describing lateral variation in bed topography as a function of channel curvature, and a simplified bank erosion function that relates the erosion rate to near-bank velocities. This formulation considers only point bars determined by curvature; a mechanism for migrating alternate bars is not included. Ikeda *et al.* (1981) found that bar and bend theories predict meander wavelengths that are approximately the same order of magnitude, providing justification for the assumption that bank erosion associated with alternate bars triggers a second type of instability associated with perturbations of channel planform. This bend instability operates at a similar scale as the bar instability, implying that each incipient bend contains a single nonmigratory alternate bar. In a companion paper, Parker *et al.* (1982) expanded this model to include nonlinear effects, which inhibit bend growth and produce fattening and skewing of the bends.

Kitandis and Kennedy (1984) modified the basic two-dimensional model by explicitly examining the influence of secondary currents induced by channel curvature on bend initiation. To isolate the effect of secondary currents on the instability process, they assumed a flat channel bed (i.e., no point bars). Stability analysis yielded a dominant meander wavelength of the same order of magnitude as that derived from Ikeda *et al.*'s (1981) depth-averaged model with point bars. Thus, it is unclear whether secondary currents or changes in average flow properties are responsible for bend instability.

Over the past several years considerable attention has been given to developing a unified bar-bend theory that fully accounts for: 1) the formation of alternate bars in an initially straight channel; 2) the subsequent development of planform curvature; and 3) the interactions between alternate bars, point bars, and curvature. Advanced linear bend models include full coupling between sediment motion and flow, allowing analysis of the total response of the system to perturbations (Blondeaux and Seminara, 1984; 1985; Johannesson and Parker, 1989; Parker and Johannesson, 1989; Seminara and Tubino, 1989). Such models reveal that a resonance phenomenon exists wherein curvature forces a natural solution that corresponds to quasisteady bar perturbations. In other words, in straight channels where no curvature-induced forcing occurs, the selected wavelength is the one with the fastest rate of amplification, which corresponds closely to measured wavelengths of alternate bars. These perturbations also migrate downstream. However, as incipient sinuosity develops, the alternate flow pattern associated with curvature reinforces the wavelengths of perturbations with small migration and amplification rates (see Figure 6). The wavelengths of these forced, quasisteady perturbations, which are longer than those of the fastest-growing perturbations, conform well with the measured wavelengths of meanders (Blondeaux and Seminara, 1985). Thus, the resonance mechanism explains how steady perturbations are selected over those with maximum amplification rates and provides a first step towards reconciling the difference between the wavelengths of alternate bars and the wavelengths of meanders. The unified theory also implies that as curvature develops, migrating alternate bars are suppressed and forced bars (steady point bars) emerge (Seminara and Tubino, 1989) and that for low values of sinuosity migrating alternate (free) bars and point (forced) bars may coexist (Tubino and Seminara, 1990).

Unequivocal evidence of resonance has yet to emerge, but experimental findings provide some support for this phenomenon. Kinoshita and Miwa (1974) found that as the sinuosity of an experimental channel increases the migration rate of alternate bars steadily decreases. At a critical angle of planform deflection migration ceases and the bars become fixed at the bends. The critical angle varies inversely with dimensionless meander wavelength, but is usually about 20° for natural streams. The behaviour of alternate bars depends not only on sinuosity, but also on the channel-forming index ($SW/D$) (Ikeda, 1989). When the channel-forming index is low, no migrating bars form in straight channels (see Table 1), but forced bars develop in mildly sinuous channels. For high values of the channel-forming index, migrating alternate bars develop in straight channels with fixed banks, whereas free and forced bars coexist in mildly sinuous channels. In the latter situation, bed relief is a maximum when free and forced bars are in phase and the bed is flat when the bars are out of phase (Ikeda, 1989). Alternate bar migration ceases in highly sinuous channels that exceed the critical deflection angle and have high channel-forming indexes. In this case, the migrating alternate bars meld with the forced bars to form fixed

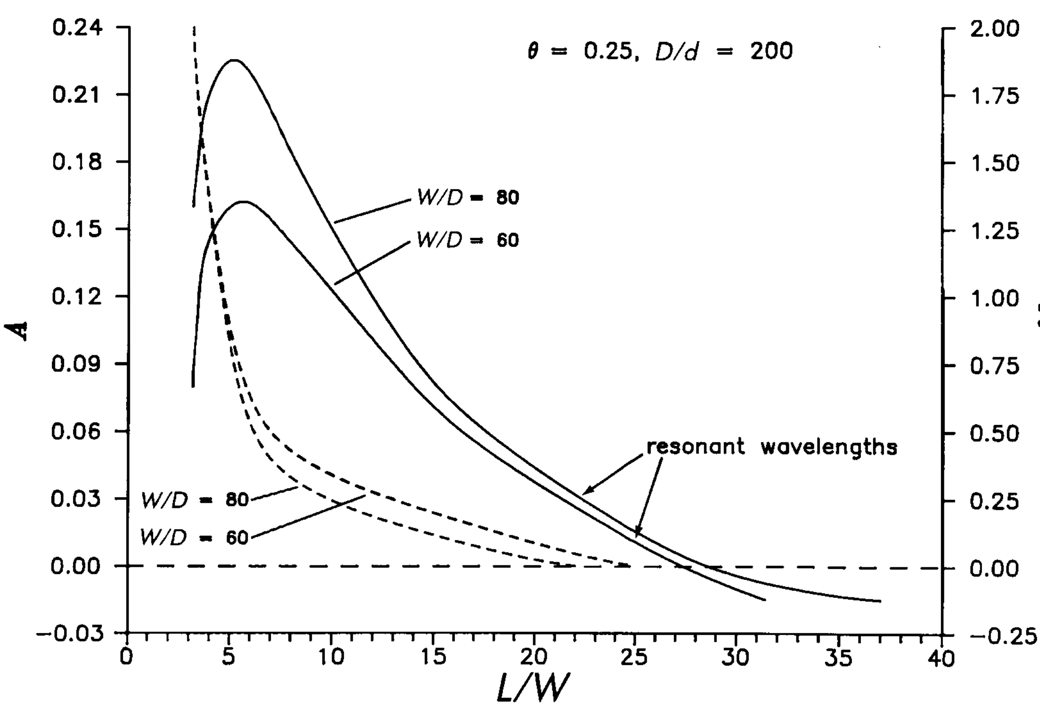


**Figure 6** Nondimensional bar amplification factor $\boldsymbol{A}$ (solid lines) and migration rate $\boldsymbol{M}$ (dashed lines) versus wavelength-width ratio for given values of $W/D$, $\theta$, and $D/d$. Resonant wavelengths are also shown (after Blondeaux and Seminara, 1985).

bars. Recent experimental findings by Colombini *et al.* (1990) indicate that the wavelengths of fixed bars in curved experimental channels are shorter than those associated with the resonant wavelengths predicted by linear bar-bend theory. Preliminary theoretical analysis suggests that a nonlinear theory of resonance may provide more accurate predictions of forced bar wavelengths than does linear theory.

Resonance may also play a role in late-stage meander evolution. In elongated meander bends, the abrupt increase in curvature at the entrance to the bend triggers the development of forced bars with wavelengths corresponding to quasi-steady free bars (Johannesson and Parker, 1989; Parker and Johannesson, 1989). This process leads to overdeepening of the pool at the entrance to the bend and a damped oscillatory pattern of the bed along the margins of the channel through the bend (Struiksma *et al.*, 1985; Colombini *et al.*, 1990). In other words, curvature induces an oscillatory pattern of bed topography that is superimposed on the pattern of planform curvature. This mechanism may explain the occurrence of multiple pool riffle sequences in enlarged meander loops (e.g., Hooke and Harvey, 1983; Thompson, 1986). Nonlinear analysis of free-forced bar interactions in curved channels suggests that free bars, which are initially suppressed as sinuosity exceeds a critical threshold, may re-emerge and coexist with forced (point) bars as bends become elongated during later stages of meander evolution (Tubino and Seminara, 1990). This prediction is supported by recent experimental evidence (Colombini *et al.*, 1990).

## V Conclusion

A universal theory of meander initiation has yet to emerge, but considerable progress towards this goal has occurred over the last few decades. In particular, bar-bend theory appears to hold great promise for the development of a general theory. Thus far, this theory has successfully predicted the formative conditions and morphologic characteristics of alternate bars, point bars, and meander bends. Because it is derived from basic principles of fluid and sediment motion in straight and meandering channels, this theory provides a physically based explanation of the initiation process.

Bar-bend theory is not without its shortcomings. The notion of a migrating doubly harmonic infinitesimal perturbation is less intuitive than that of a local random perturbation and the theory provides no explanation for the origin of these perturbations. The notion of an infinitesimal perturbation is also rather difficult to test empirically; preliminary experimental evidence contradicts the hypothesis that bars develop and grow uniformly in the form of a doubly harmonic wave (Fujita and Muramoto, 1985). Moreover, laboratory studies and field data suggest that the development of bars does not always produce meandering (e.g., Ackers and Charlton, 1970; Leopold and Wolman, 1957) and more work is required to establish a mechanical link between bar development and bank erosion (e.g., Pizzuto and Meckelnburg, 1989).

Despite these limitations, bar-bend theory rests on a stronger foundation than theories that invoke oscillation of helical flow as a meander initiation mechanism. Flow initiation theories are based largely on untested assumptions regarding mechanisms that trigger flow oscillation. Even if such oscillation does occur it seems unlikely that velocities of the secondary currents associated with weak surface-convergent helical cells are strong enough to produce widespread deformation of the bed and banks. In the light of these problems, it is somewhat surprising that most conceptual models of meander development proposed by geomorphologists view helical flow as the primary meander initiation mechanism (e.g., Brotherton, 1979; Knighton, 1982; Thompson, 1986).

Geomorphologists can contribute substantially to the refinement of meander initiation theories. Their expertise in field methods is well suited for designing experiments to test the applicability of these theories to real-world systems. Careful documentation of the meander initiation process in the field will identify inadequacies of theoretical models, allowing for further refinement. The success of such efforts requires a firm understanding of theory to ensure that appropriate variables are measured. Field data currently consist of measurements in streams with well-developed meanders, whereas most theories of meander initiation require information on initial conditions prior to meandering. Data needs include accurate measurements of primary and secondary flow components to test theories concerning the production and oscillation of helical flow in straight streams. If this type of flow does occur, it may be capable of generating the small perturbations required to trigger bar instability. The need also exists to document in detail the interactions that occur among flow structure, sediment transport, bed morphology, and bank erosion as an initially straight stream evolves into a sinuous form. Particular attention should be given to the influences of unsteady flow, heterogenous bed and bank materials, deposition of fines, and riparian vegetation on channel response since these factors are generally neglected in theoretical models. Recently channelized streams provide an obvious setting for this type of field experiment. Lewin's (1976) work serves as an example of this approach, but more ambitious studies are required to rigorously test meander initiation theories. This line of research should be inherently attractive to geomorphologists since it promises to contribute not only to theory development, but also to the solution of practical problems associated with stream channelization (e.g., Brookes, 1988).

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