Soil Science and Soil Physics

Lecture 8

Water flow in soil, hydraulic conductivity

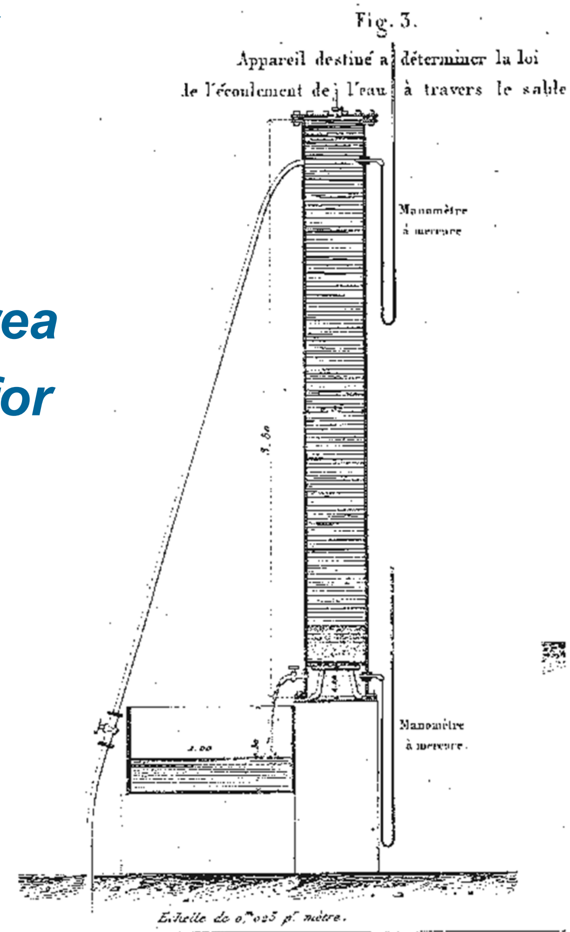
Saturated flow

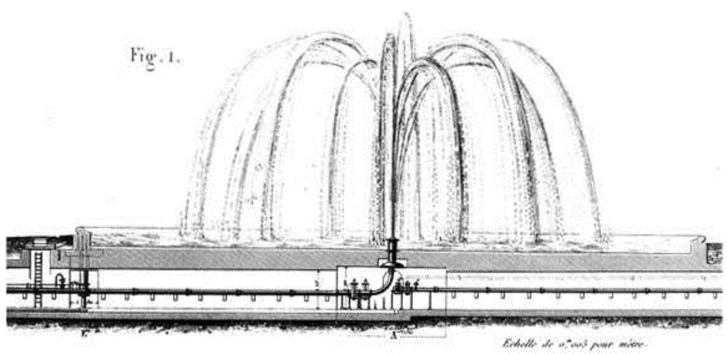
Henry Darcy (1856) *filtration of water for fountains in Dijon*

After many experiments he found that ***water flow*** *through the soil column depends on:*

- ***directly proportional to pressure drop***
- ***inversely proportional to the length***
- ***directly proportional to the crossectional area***
- ***dependent on coefficient which is specific for each media***

Henry Darcy

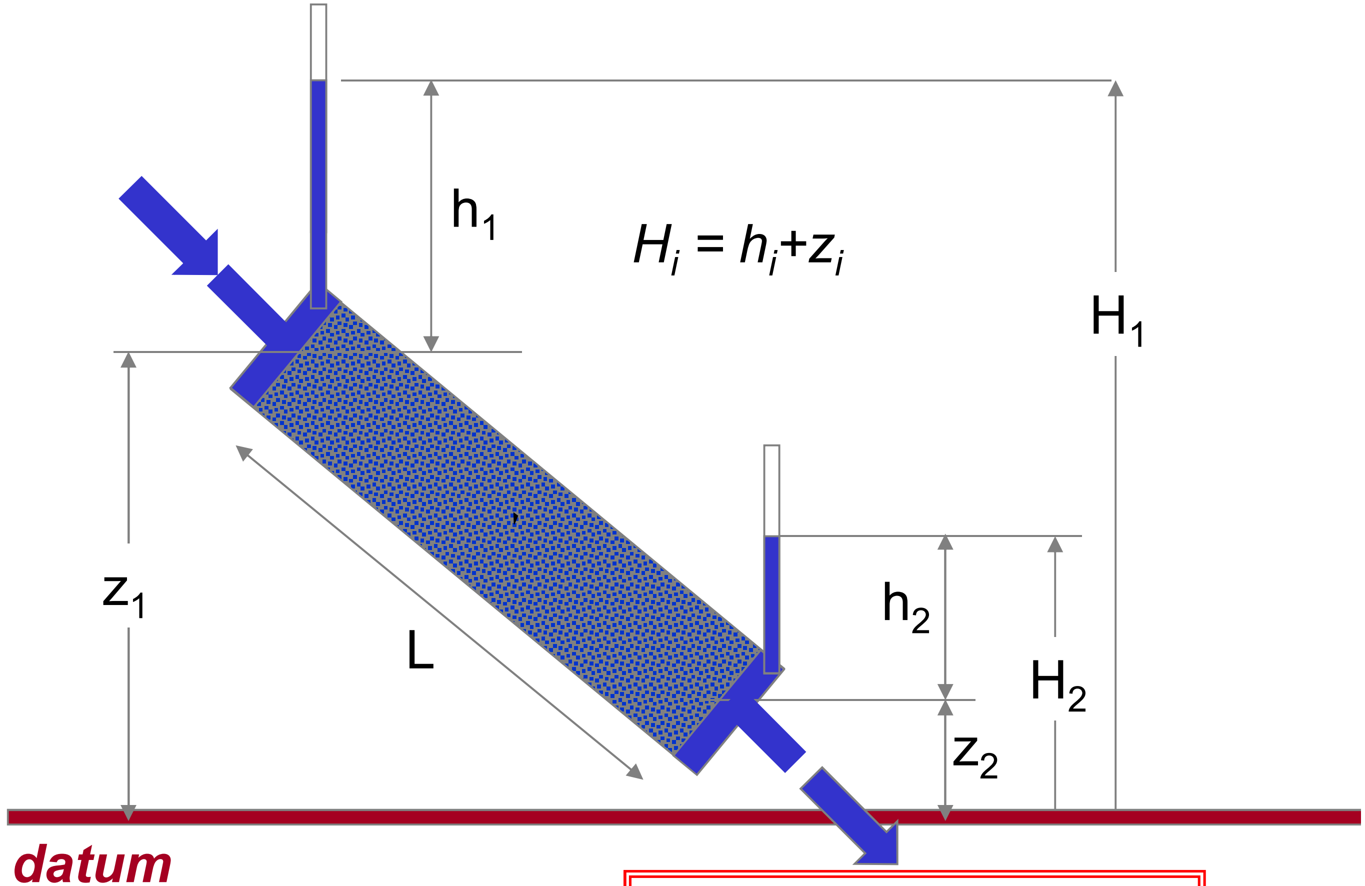




Darcy, H., 1856. *Les Fountaines de la Ville de Dijon*

Darcy law

$$Q = \frac{K_S A \Delta H}{L}$$



Q = flow [$L^3.T^{-1}$]
A = crossectional area [L^2]
K_s = saturated hydraulic conductivity [$L.T^{-1}$]
$\triangle H = H_1 – H_2$ (hydraulic head drop) [L]
L= sample lenght [L]

valid in fully saturated porous media
For example: under the ground water level

for:

$$q = \frac{Q}{A}$$

kde:
q ... Volume flux [$L.T^{-1}$]
Q ... Flow rate [$L^3.T^{-1}$]
A ... Crossectional area [L^2]

Transforms to the:

$$q = K_s \frac{\Delta H}{L}$$

More gereneral form:

$$q = K_s \frac{dH}{dl}$$

Pro 1D vertical flow

$$q = -K_s \frac{dH}{dl} = -K_S \nabla H$$

*poznámka: **negative sign** due to the fact **grad H** aims **against flow direction***

Coefficient or the saturated hydraulic conductivity K_s

Also called (sometimes) ***filtration coefficient**, **darcy's coefficient*** *or* ***permeability (incorrect)***

commonly used units K_s ($m.s^{-1}$), ($cm.d^{-1}$), ($cm.s^{-1}$)

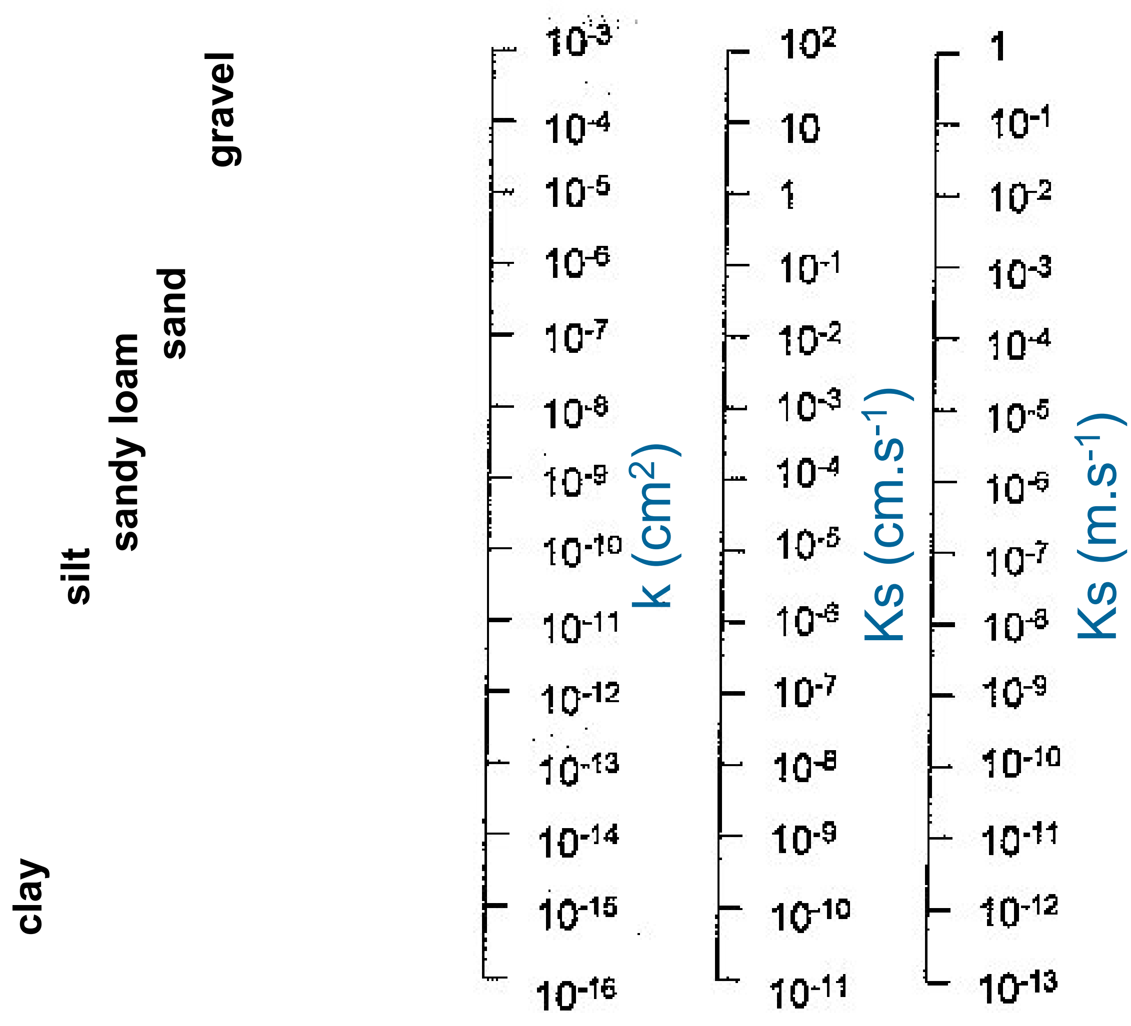
K_s is property of water-solid interaction. Parameter which is related only to the porous media (independently on flowing liquid) is:

Permeability k

$k = K_s \nu / g$ [L^2]

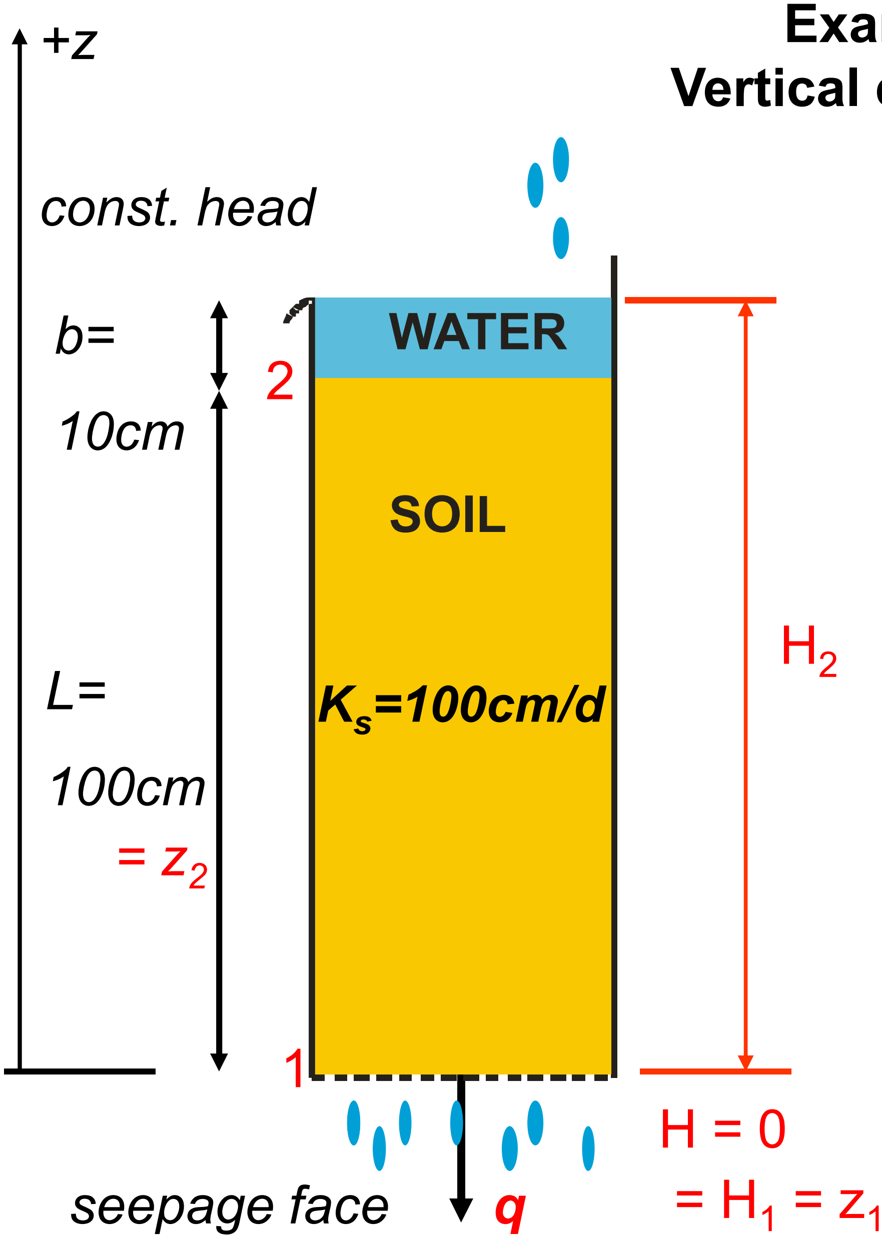
kde ν kinematic viscosity

Ks for different media



Zdroj: Císlerová a Vogel, 1998

Example 1 :
Vertical column: q=?



1) datum definition

2) points 1 and 2 with known hydraulic heads

3) Darcy's law

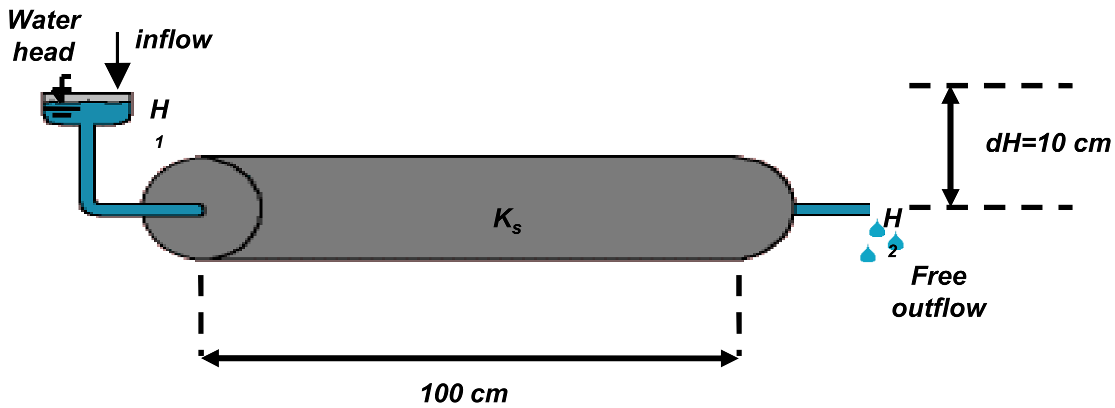
$$q = -K_s \frac{\Delta H}{L} = -K_s \frac{(H_2 - H_1)}{L} =$$

$$= -100 \frac{(110 - 0)}{100} = -110\ cm.d^{-1}$$

Example 2

horizontal column: q = ?

1) Step 1, definition of datum and coordination system



2) Definition of points 1 and 2). Then $x_1 = 0$ and $h_1 = 10$ cm, x_2=100cm, $h_2 = 0$, $z_1 = z_2 = 0$, $L = x_2 - x_1 = 100$ cm

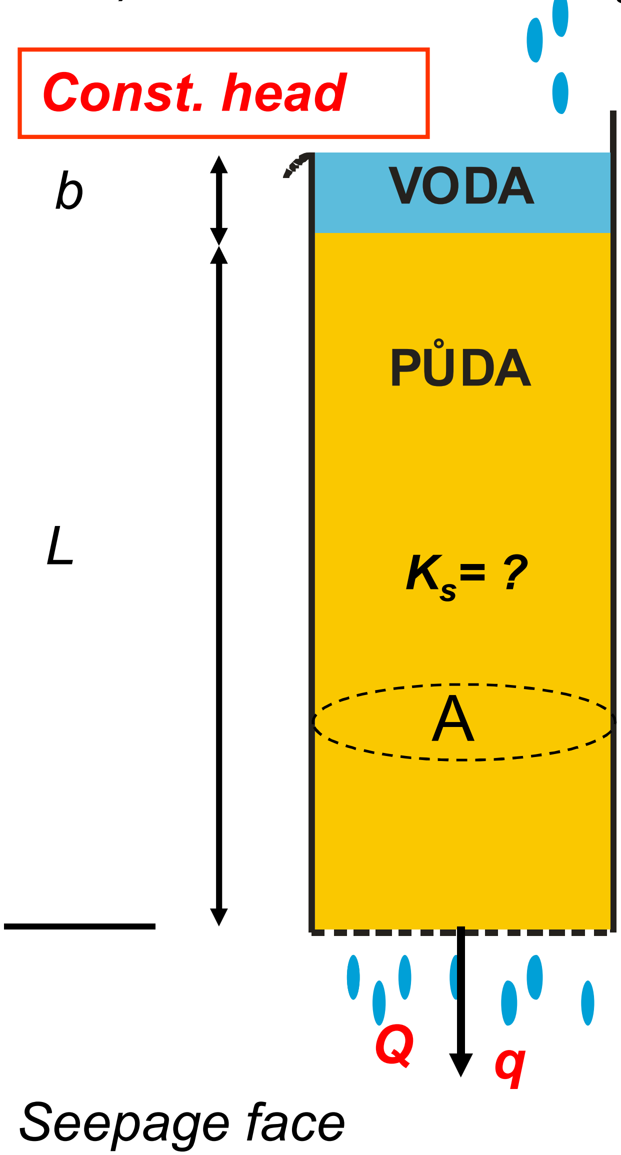
3) Hydraulics heads are then $H_1 = h_1 + z_1 = 10$ cm, $H_2 = h_2 + z_2 = 0$ cm

5) Darcy's law

$$q = -K_s \frac{\Delta H}{L} = -K_s \frac{(H_2 - H_1)}{L} = -100 \frac{(0-10)}{100} = 10\ cm.d^{-1}$$

K_s measurements

1) Measurements of K_s using constant head permeameter



Soil column

$H_1 = 0 + 0$ *(lower end)*

$H_2 = b + L$ *(upper end)*

$\Delta H = (b+L) - 0$

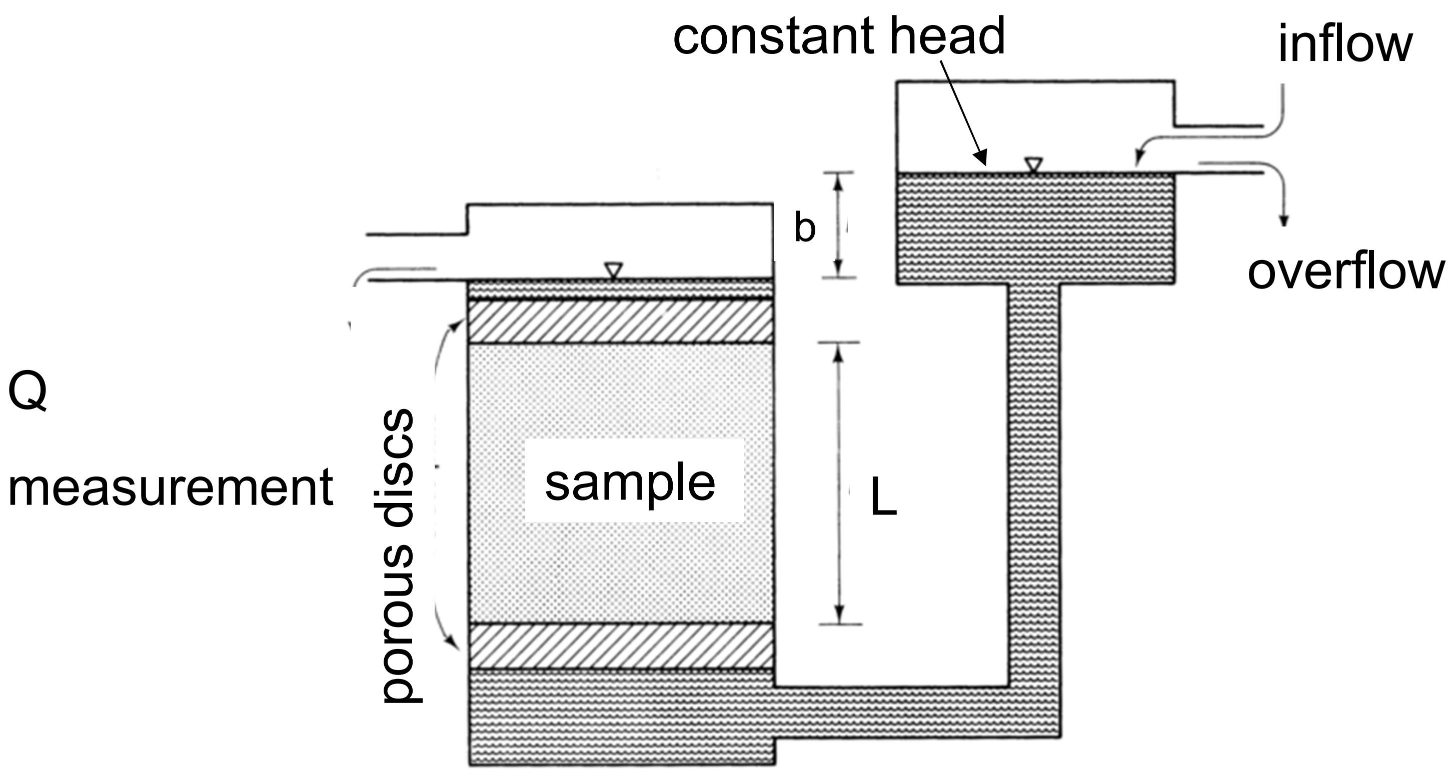
then:

$$K_s = -\frac{qL}{\Delta H} = -\frac{qL}{(b+L)}$$

In practice we measure Q, resp. V/t, then:

$$K_s = \frac{VL}{At\Delta H} = \frac{VL}{At(b+L)}$$

Constant head permeameter

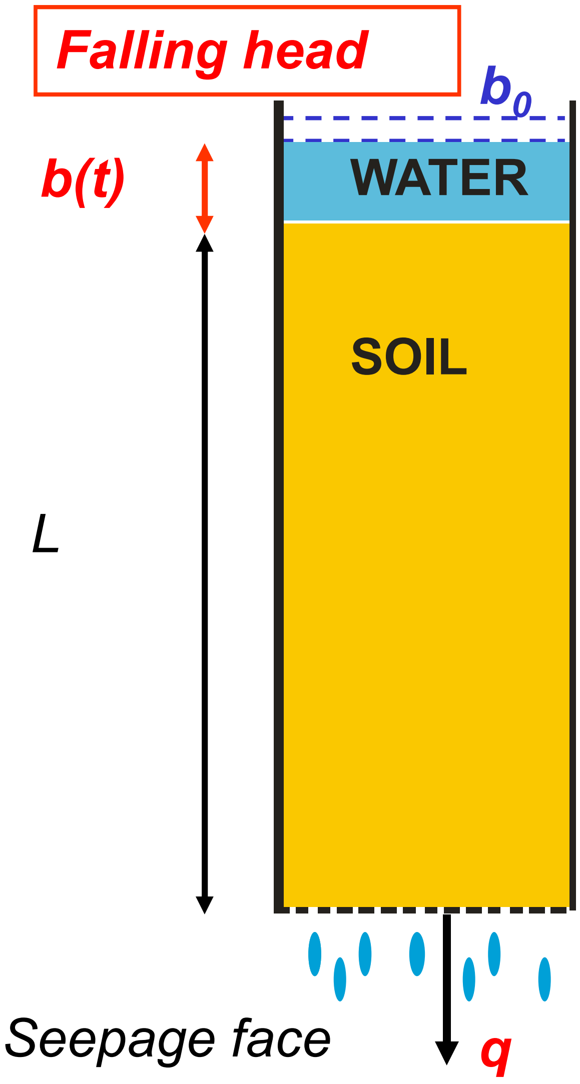


soil sample must be carefully saturated with water to measure "real" ***Ks****.*

Fig. : http://www.utdallas.edu/~brikowi/Teaching/Geohydrology/LectureNotes/Darcy_Law/Permeameters.html

Constant head permeameter

2) Measurement of K_s falling-head permeameter



Experiment is done on soil sample in the laboratory. Initial water level is equal to b_0

$H_1 = 0$, $H_2(t) = L+b(t)$, $\Delta H(t) = [b(t) + L] - 0$

$$q = \frac{db}{dt} = -K_s \frac{(b+L)}{L}$$

becomes:

$$\frac{db}{b+L} = -\frac{K_s}{L} dt$$

Integration of the left side

$$\int_{b_0}^{b_1} \frac{db}{b+L} = \ln(b+L)\Big|_{b_0}^{b_1} = \ln\frac{b_1+L}{b_0+L}$$

Right side integration

$$-\int_0^{t_1} \frac{K_s}{L} dt = -\frac{K_s}{L} \int_0^{t_1} dt = -\frac{K_s t_1}{L}$$

$$\frac{db}{b+L} = -\frac{K_s}{L} dt$$

then:

$$\ln \frac{b_1 + L}{b_0 + L} = -\frac{K_s t_1}{L}$$

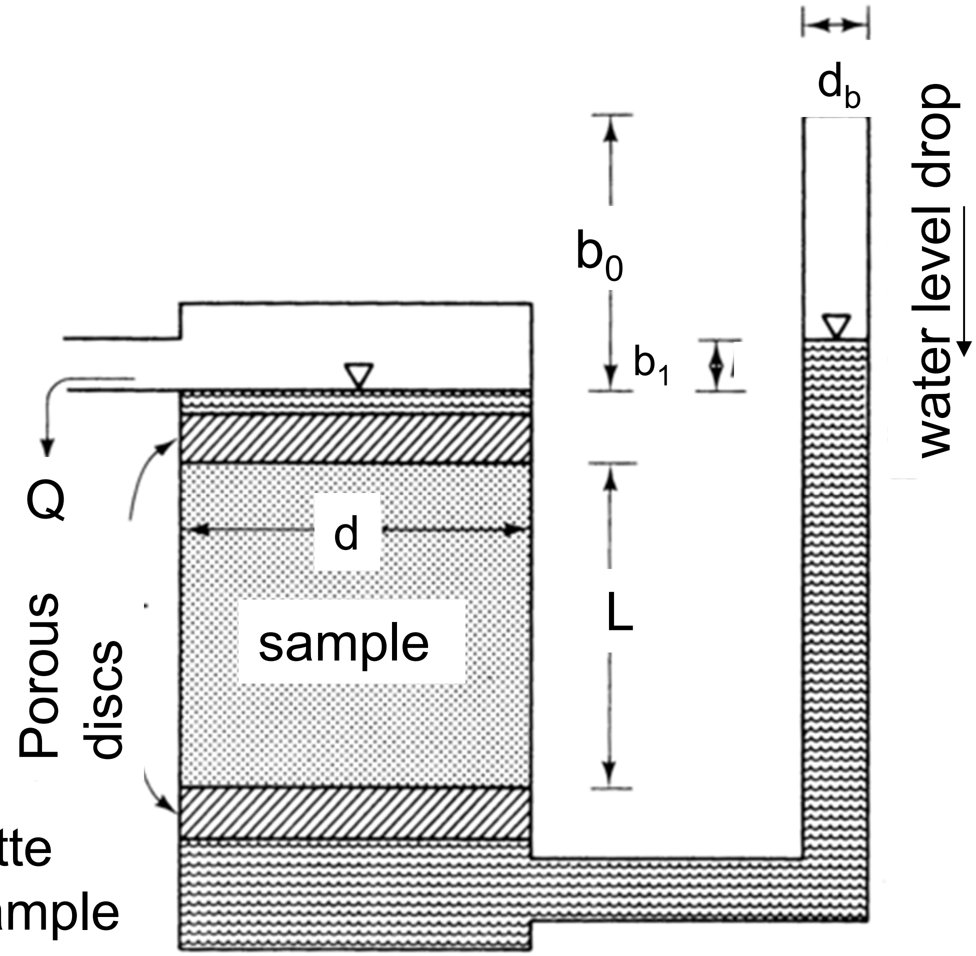
$$K_s = \frac{L}{t_1} \ln \frac{b_0 + L}{b_1 + L}$$

Falling-head permeameter

Consider different crossectional areas of burette and column:

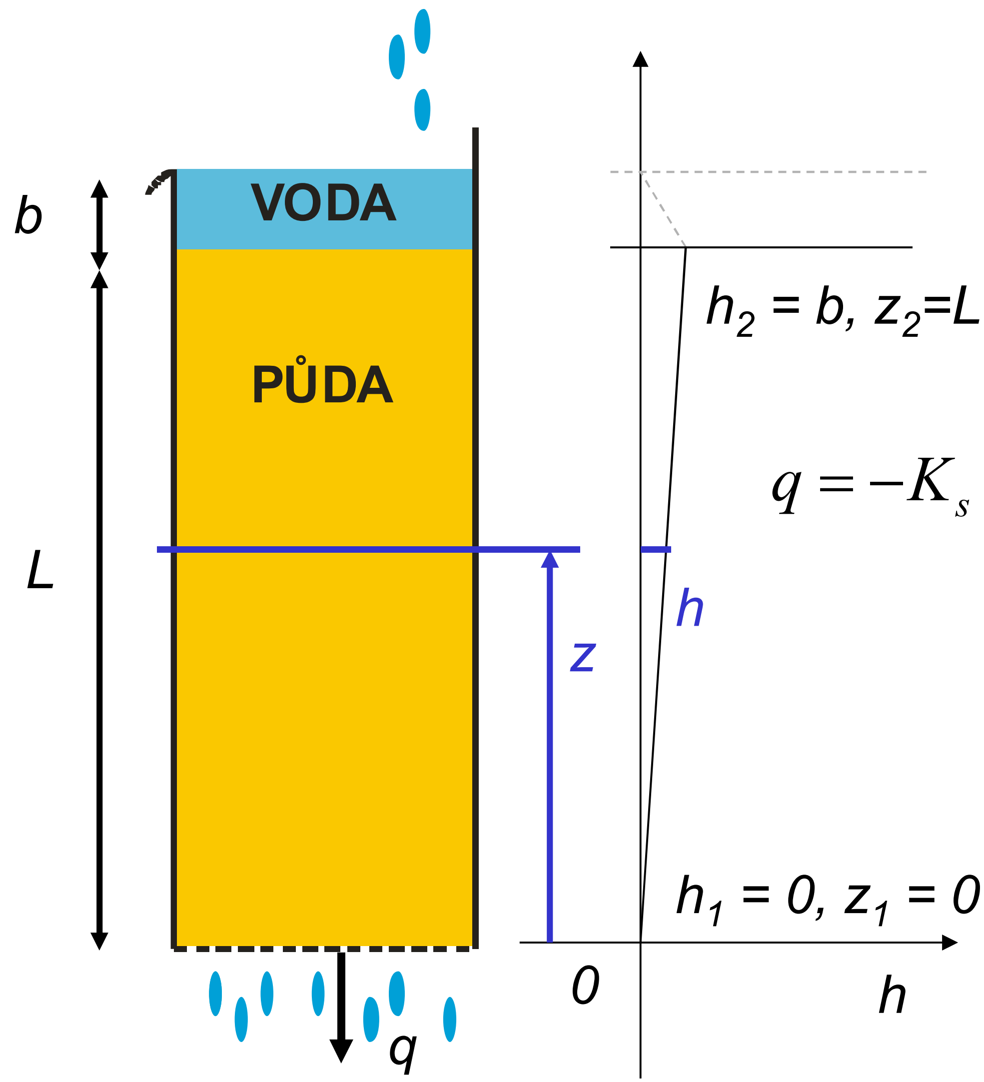
$$K_s = \frac{A_b}{A}\frac{L}{t_1}\ln\frac{b_0 + L}{b_1 + L}$$

where:
t_1.... time
A_b … cross. area of burette
A ... cross. area of the sample



Zdroj: http://www.utdallas.edu/~brikowi/Teaching/Geohydrology/LectureNotes/Darcy_Law/Permeameters.html

Example 3 :
Pressure head along the soil column h(z)=?



K_s is constant, h(z) = ?

Darcy's law:

$$q = -K_s \frac{\Delta H}{L} = \underbrace{-K_s \frac{b+L}{L}}_{q_{12}} = \underbrace{-K_s \frac{h+z}{z}}_{q_{1z}}$$

$$h = \frac{h_2 - h_1}{L} z = \frac{b}{L} z$$

In this case, pressure head profile is linear .

Measurements of K_s in field – infiltration experiments

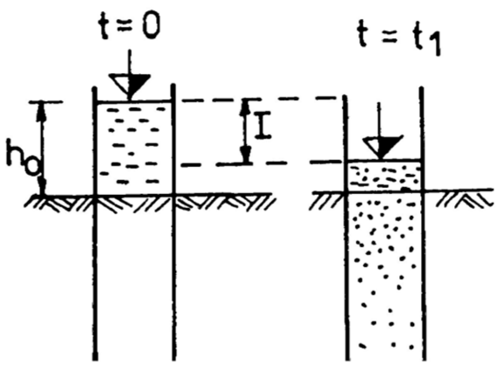
Flux across the surface is called infiltration rate i.

Total amount of water infiltrated is I [L]

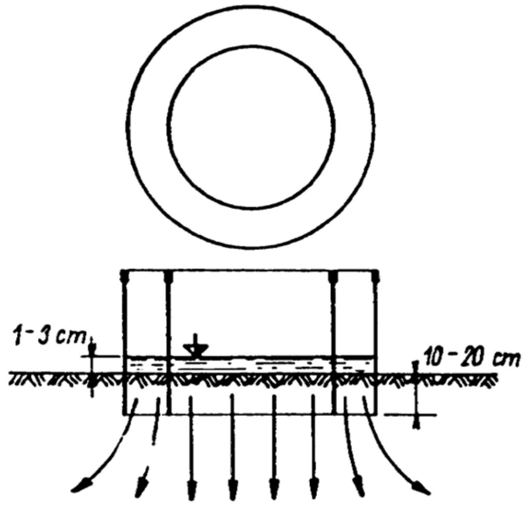
Infiltration rate can be steady or unsteady

Measurements of Ks in field

Ideal case



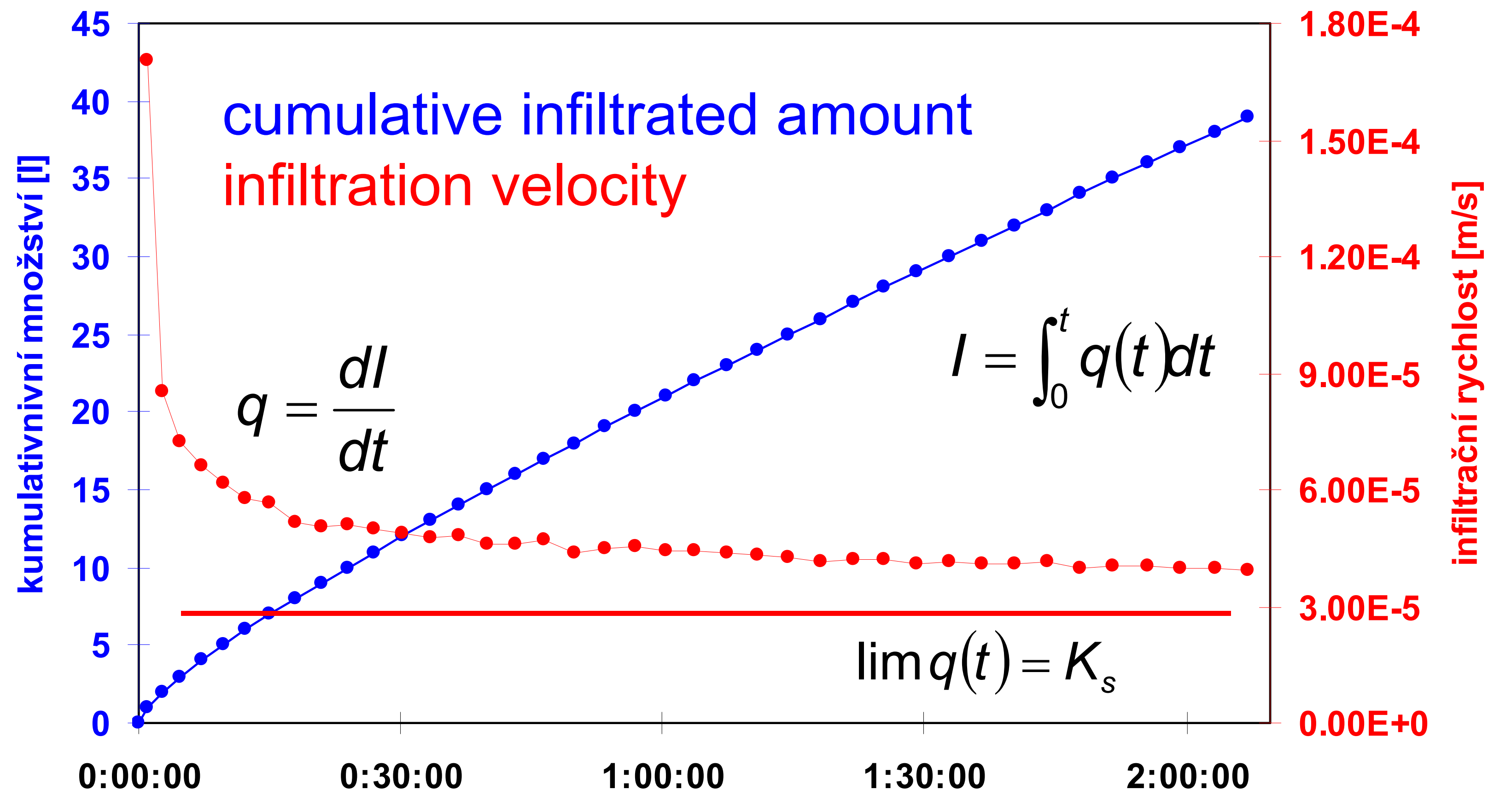
Common practice



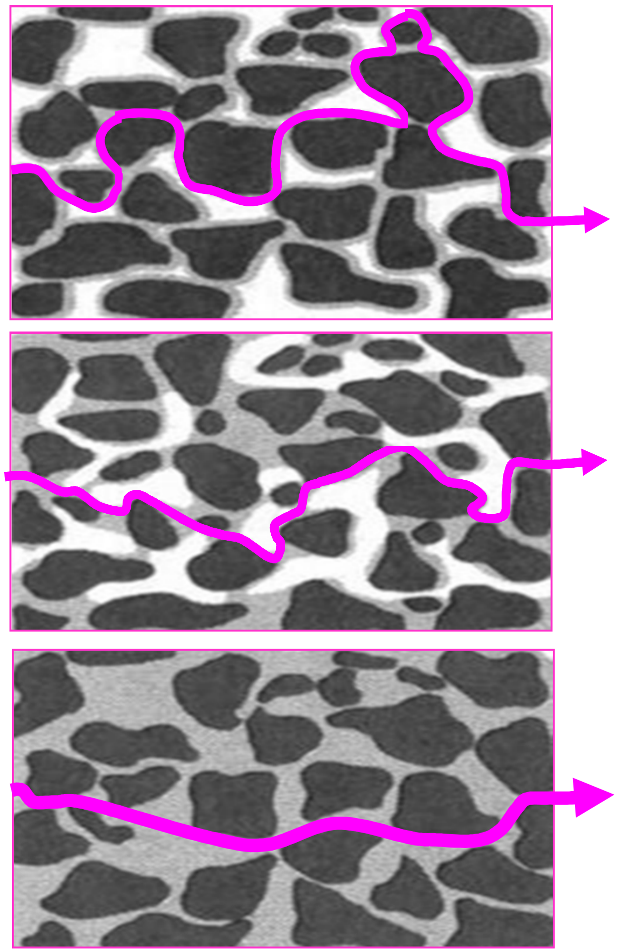
Double-ring infiltration:

- Two rings
- Spike inserted inside of the inner ring
- water batches are being added to the inner ring, every time the water level reaches the spike
- The time between spike appearance is measured for each batch and flux is then calculated for each time interval

Evaluation of double ring infiltration



Unsaturated hydraulic conductivity



Zdroj: E. Sulzman

- *θ changes in time and space*
- *relationship θ(h) e.g. retention curve exist*
- *hydraulic conductivity **depends on θ resp. on h for h < 0***
- *K(θ), resp. K(h) is called hydraulic conductivity function*

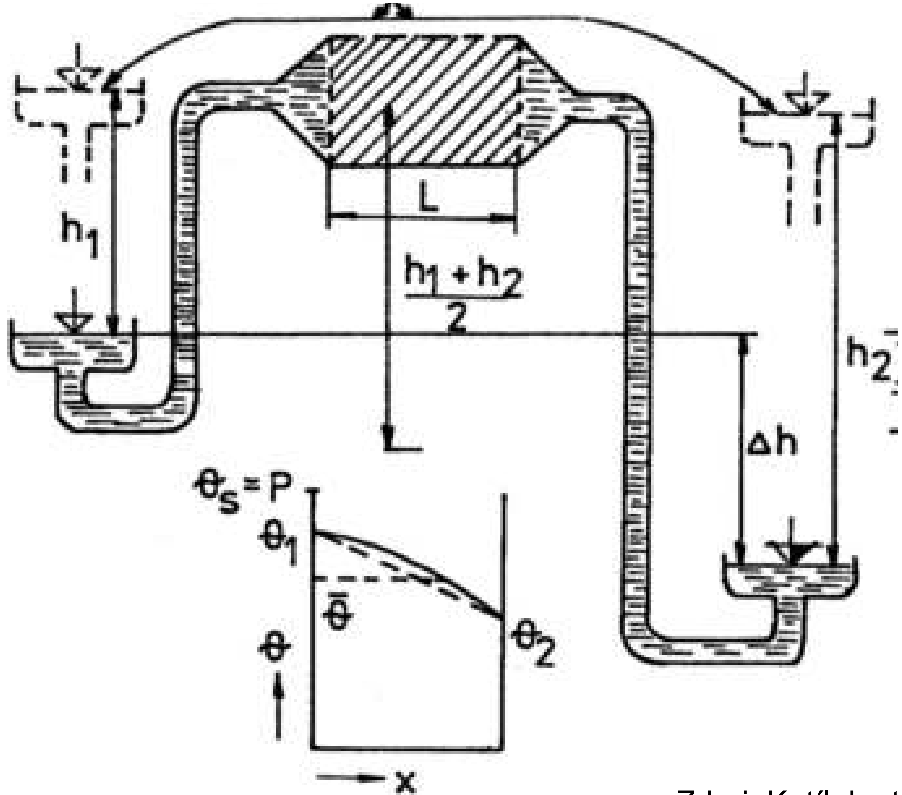
Darcy-Buckingham law

Edgar Buckingham (1907)

$$q = -K(\theta)\nabla H$$

where:
q flux
H hydraulic head

flow direction at dh
original level



Zdroj: Kutílek et al. 1994

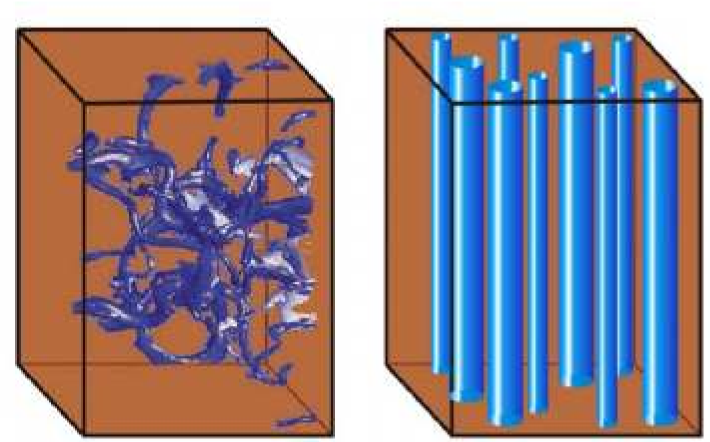
Capillary models

Capillary models (Childs and Collis-George, 1950)

Based on assumption that from known of pore size distribution (from retention curve) and equation and Laplace equation, and Equation describing the flow in individual capillary the unsaturated Hydraulic conductivity can be predicted

Capillary models predict relative hydraulic conductivity

$$K(\theta) = K_r(\theta) K_s$$



where K_s is measured saturated hydraulic conductivity

Relative hydraulic conductivity

We will obtain the hydrulic conductivity of the capillary model as follows:

$$K(\theta) = C_1^2 C_2 \int_0^{\theta} \frac{1}{h^2} d\theta$$

Then for relative hydraulic conductivity K_r

$$K_r(\theta) = \frac{K(\theta)}{K(\theta_S)} = \frac{\int_0^{\theta} \frac{d\theta}{h^2}}{\int_0^{\theta_S} \frac{d\theta}{h^2}}$$

constants C_1^2 a C_2 are canceled

where: $K(\theta_S) = K_S$ Is saturated hydraulic conductivity

Most commonly used models of Kr prediction

$(\theta/\theta_s)^b$ ..... Relative tortuosity is included

Burdine (1953):

$$K_r(\theta) = \left(\frac{\theta}{\theta_s}\right)^2 \frac{\int_0^{\theta} \frac{d\theta}{h^2}}{\int_0^{\theta_s} \frac{d\theta}{h^2}}$$

$$\theta_e(h) = \begin{cases} \left(\frac{H_b}{h}\right)^{\lambda} & h < H_b \\ 1 & h \geq H_b \end{cases}$$

Brooks a Corey

Retention curves functions are substituted……….

Mualem (1976):

$$K_r(\theta) = \left(\frac{\theta}{\theta_s}\right)^{1/2} \left(\frac{\int_0^{\theta} \frac{d\theta}{h}}{\int_0^{\theta_s} \frac{d\theta}{h}}\right)^2$$

$$\theta_e(h) = \begin{cases} \frac{1}{\left(1 + (-\alpha h)^n\right)^m} & h < 0 \\ 1 & h \geq 0 \end{cases}$$

van Genuchten

...... and the final expressions for Kr predictions are then

From Brooks and Corey $K_r(\theta_e) = \theta_e^{b+a\lambda}$

$$K_r(h) = \begin{cases} \left(\dfrac{H_b}{h}\right)^{a+b/\lambda} & h < H_b \\ 1 & h \geq H_b \end{cases}$$

Where parameters a and b are for Burdine model Burdine a=2 a b=3
Mualem a=2 a b=2.5

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$ Efective water content

......

from van Genuchten vztahu

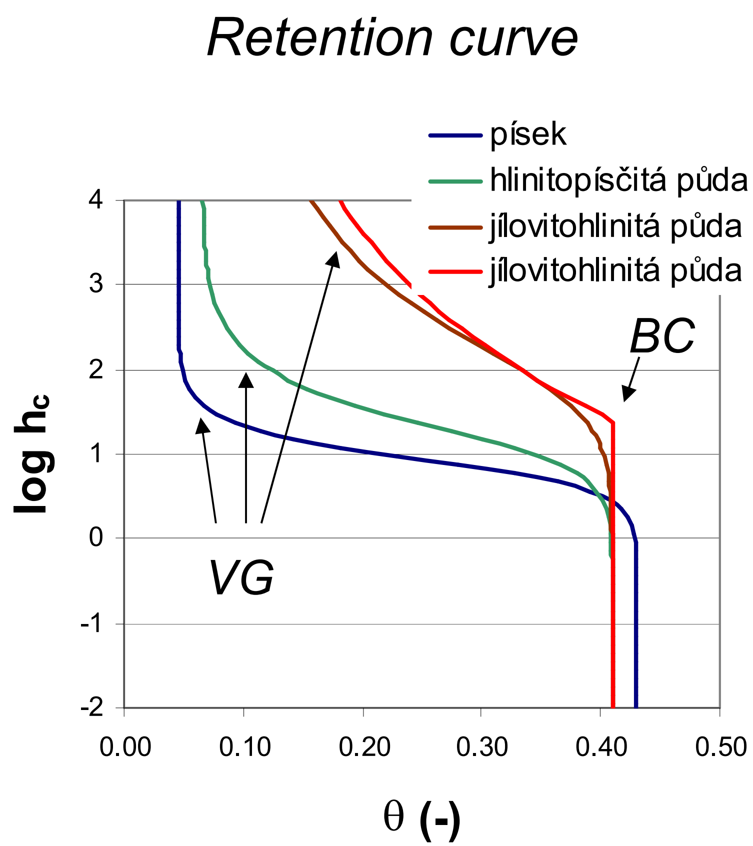
$$K_r(\theta_e) = \theta_e^{0.5}[1-(1-\theta_e^{\frac{1}{m}})^m]^2$$

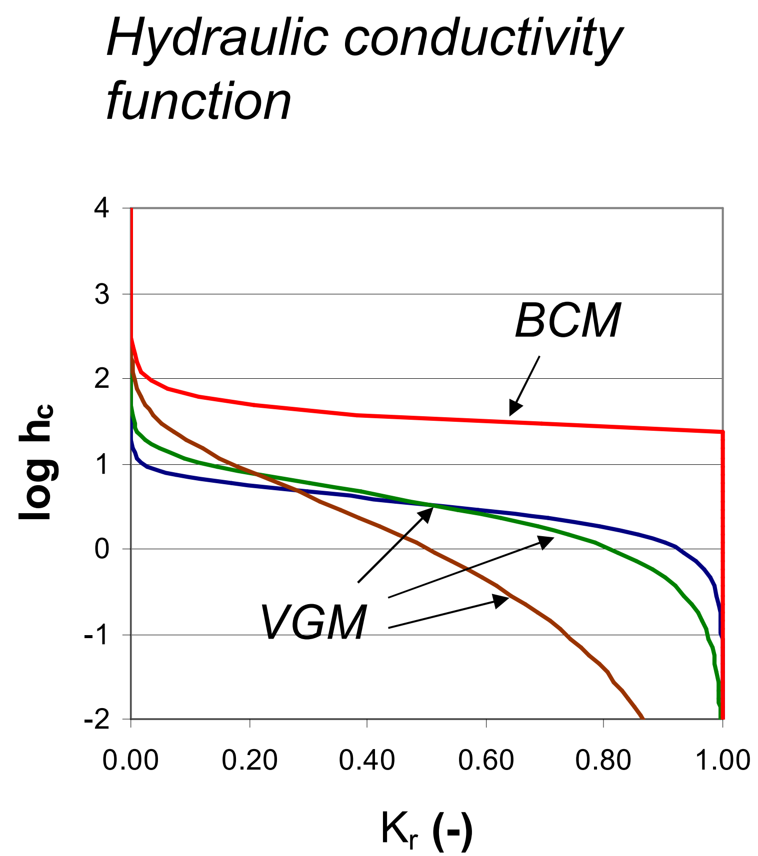
$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

$$K_r(h) = \begin{cases} \dfrac{\left\{1-(-\alpha h)^{mn}\left[1+(-\alpha h)^n\right]^{-m}\right\}^2}{\left[1+(-\alpha h)^n\right]^{m/2}} & h < 0 \\ 1 & h \geq 0 \end{cases}$$

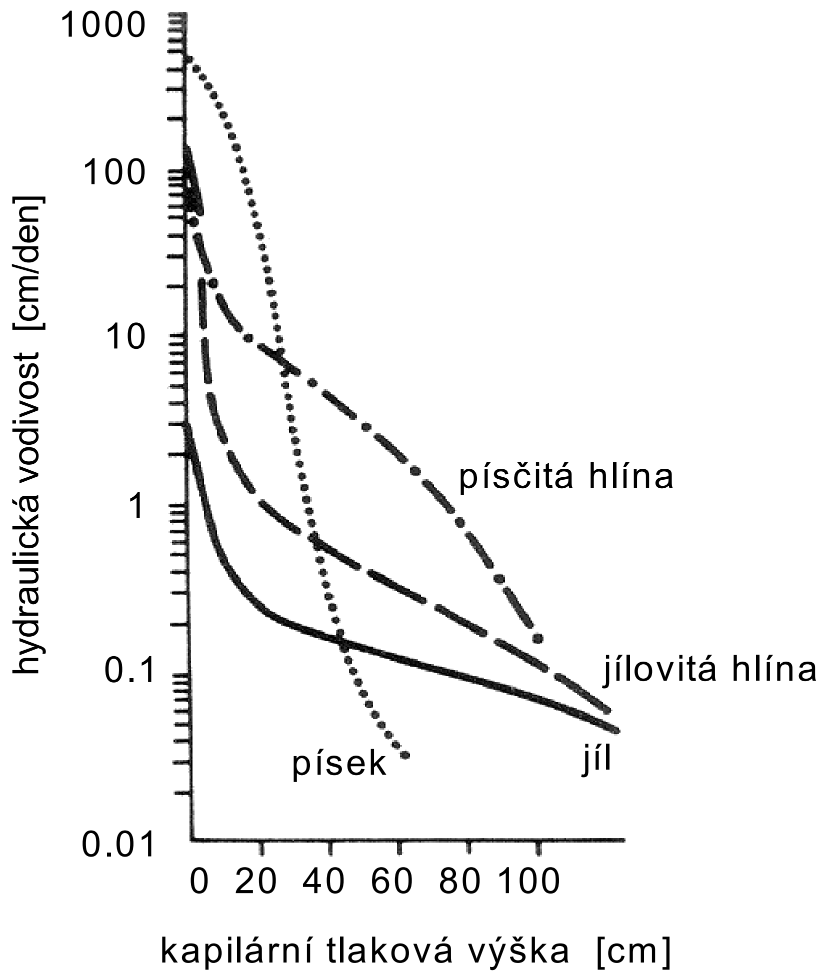
Mualem

Retention curve and hydraulic conductivity function for different soils





Unsaturated hydraulic conductivity

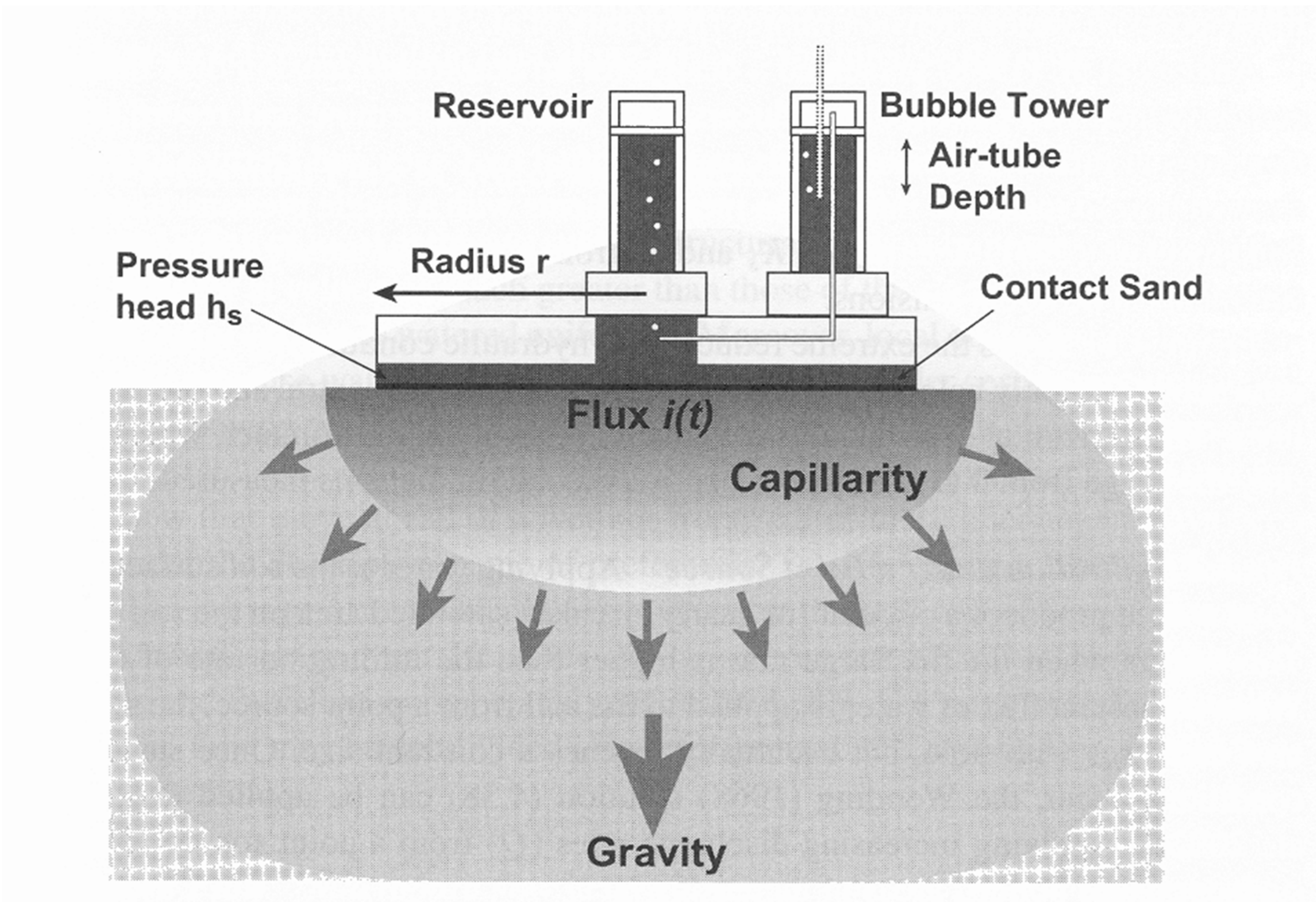


Measurements of hydraulic conductivity in field

Tension infitlrometer

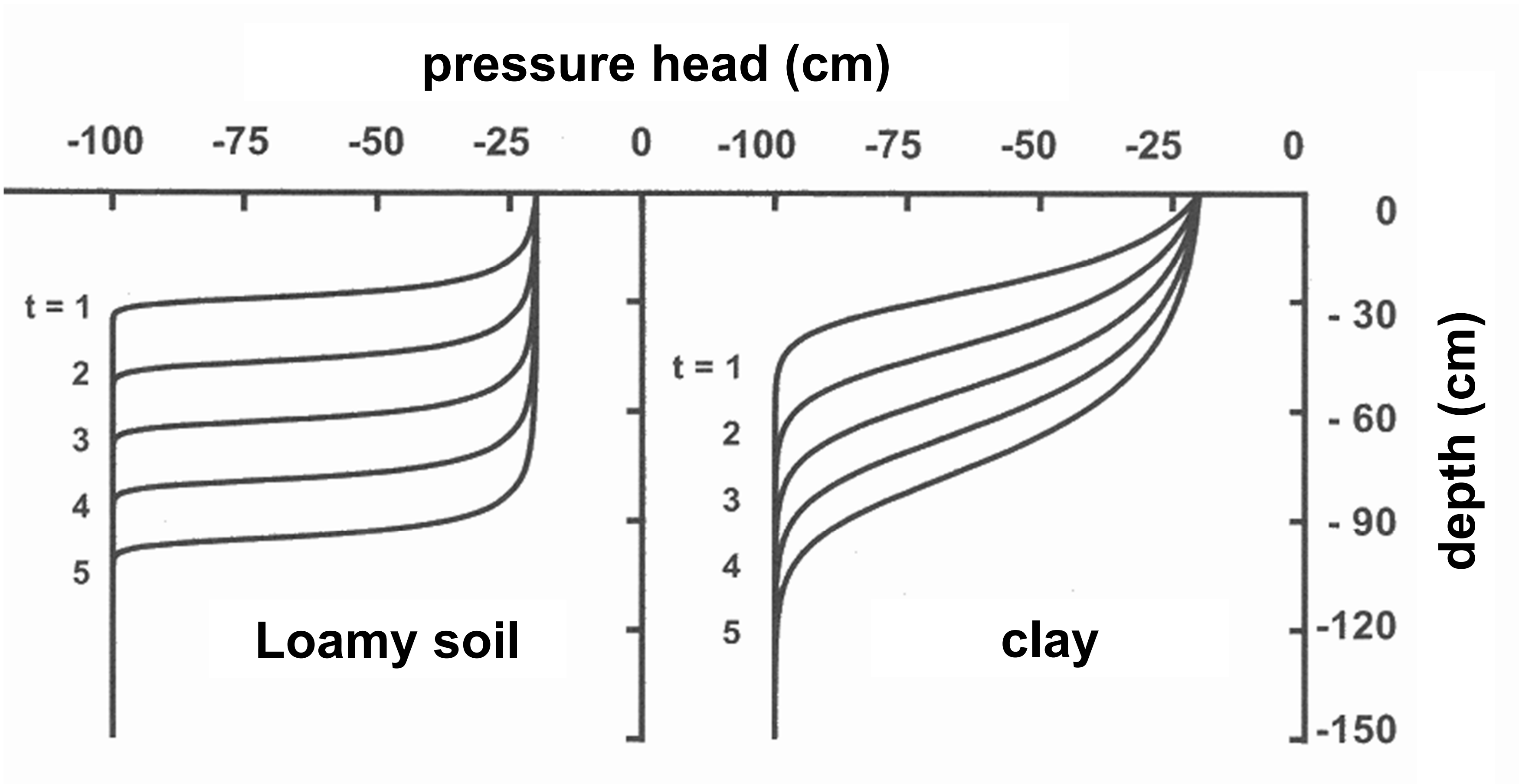
Measurements of I under infitrometer with fixed tension head

Tension infiltrometer



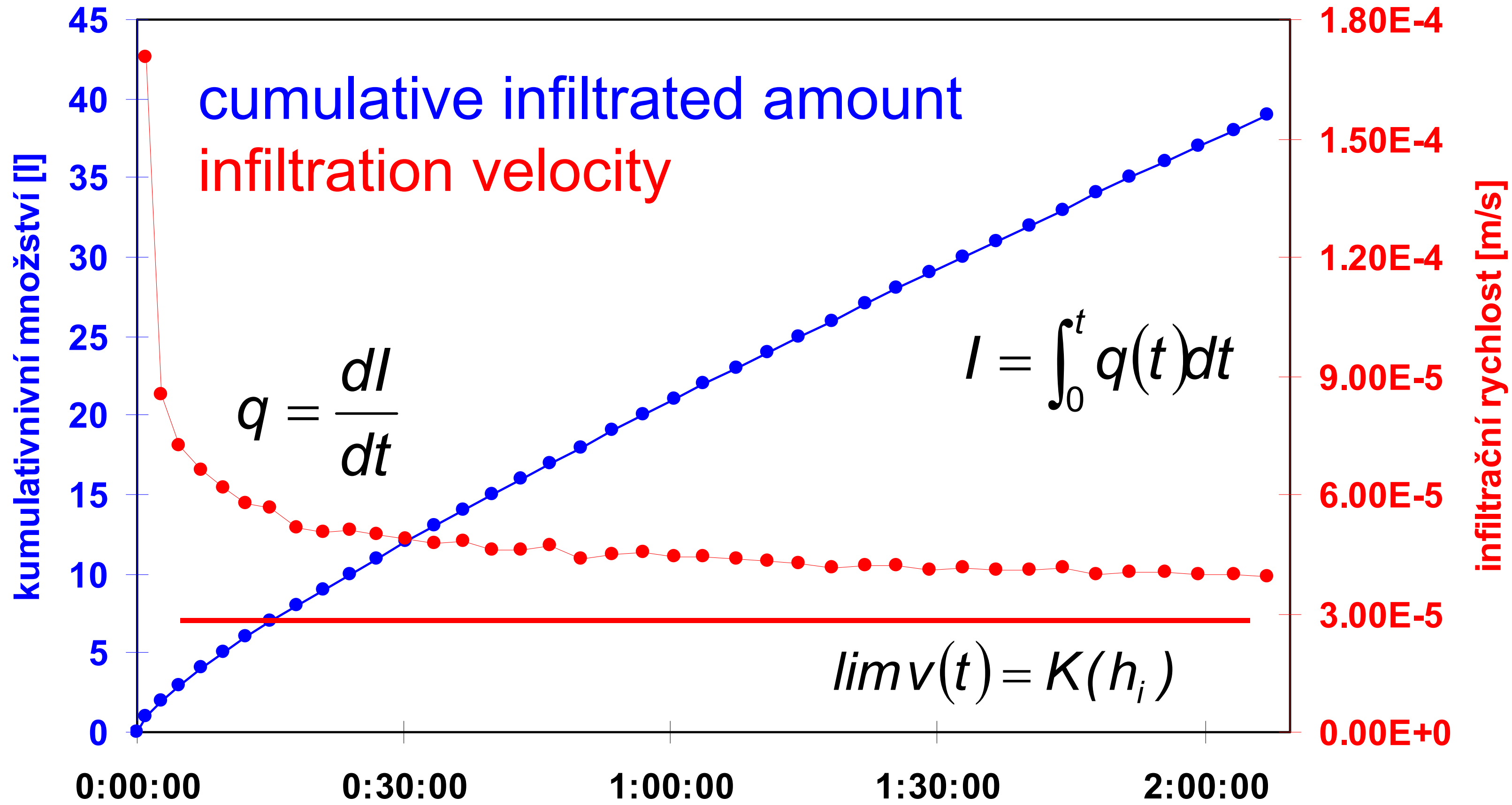
Jury a Horton 2004

Infiltration experiment – the wetting front



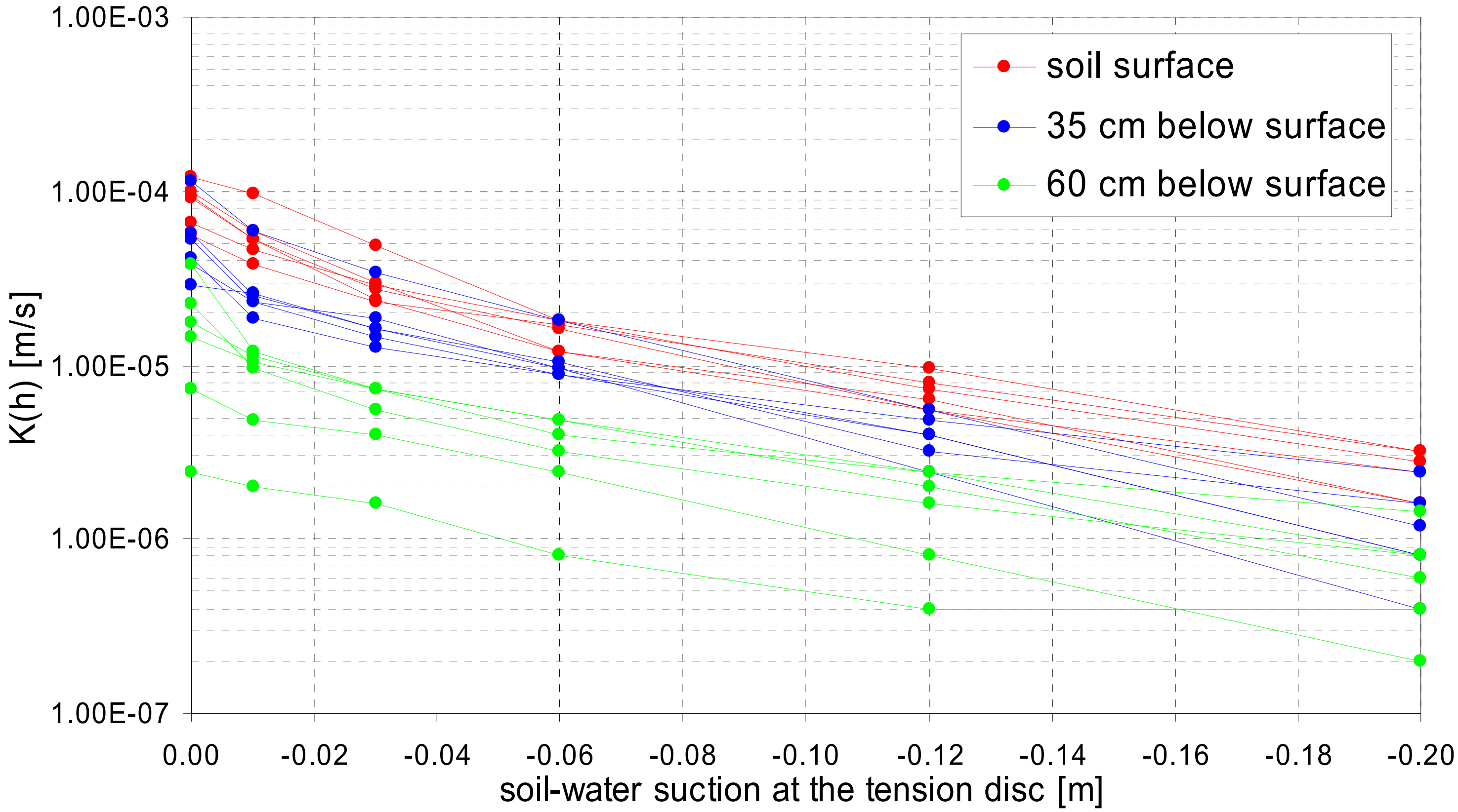
Result of one infiltration experiment

- analogous to K_s
- One $K(h_i)$ value for each infiltration done with the pressure head h_i



Measurement of K(h) in field

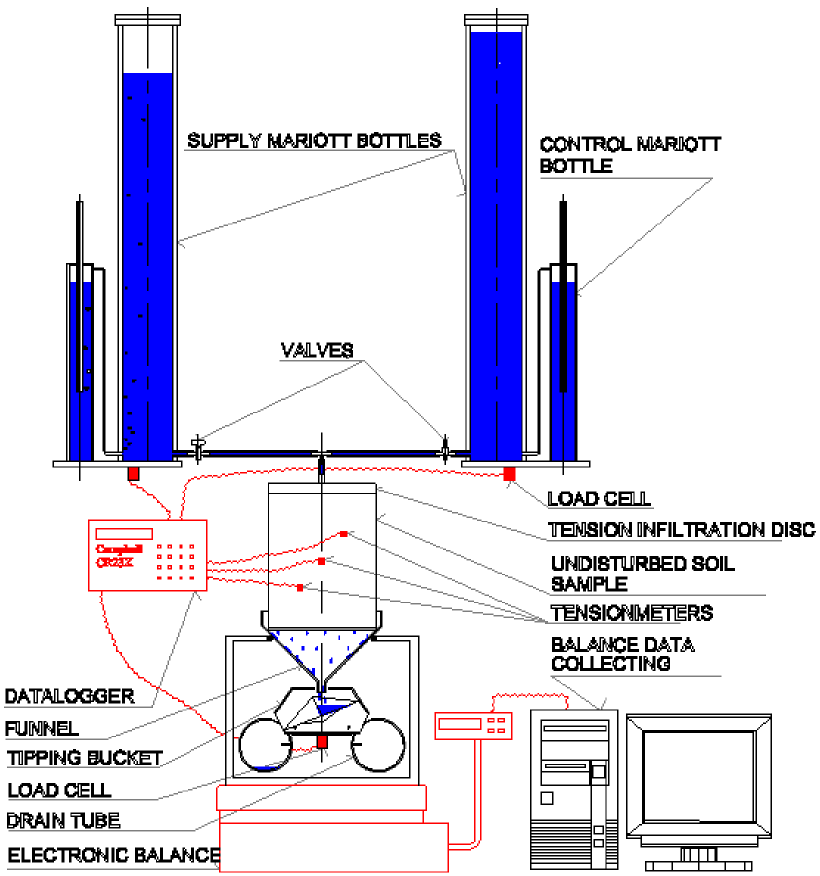
- Clay loam soil



Measurements of K(h) in laboratory

- evaluation using inverse modeling

……



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