Transport processes

Hydraulic conductivity

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(Un)saturated hydraulic conductivity (K)

Measure how a given liquid flows through a given material.

- saturated hydraulic conductivity K_s
  - Scalar or tensor variable
- Unsaturated hydraulic conductivity K
  - function of suction pressure or vol. water content

Saturated hydraulic conductivity K_s

- permeability k is a property of the material alone
- $K = k\frac{\rho g}{\sigma}$
- $\mathrm{k} \sim \mathrm{K}/5$

How to determine it?

(Un)saturated hydraulic conductivity can by obtained

1. by measurement in the lab,
2. by measurement in the field, or
3. from RC with a given mathematical model

In the lab

- K_s is calculated with the Darcy formula
- K is calculated with the Darcy-Buckingham formula
- water flow measurement +
  - stepwise changes of hydraulic gradient
  - gradual changes of hydraulic head gradient (falling head experiment)
- evaporation method

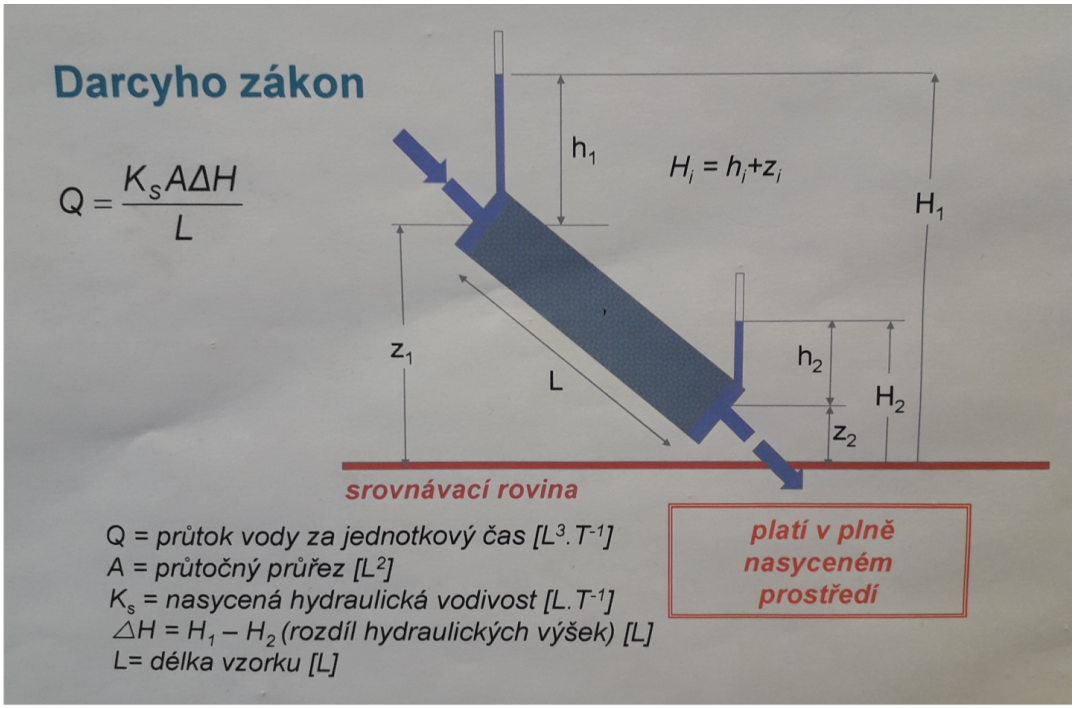
Darcy-Buckingham formula

$$q = -K(h)\nabla H$$

- H = h + z
- for x direction: $q = -K(h)\frac{\mathrm{d}H}{\mathrm{d}x}$

How to determine it?

In the lab



Constant and falling head exepriment to determine K_s

How to determine it?

In the lab

Evaporation experiment to determine K

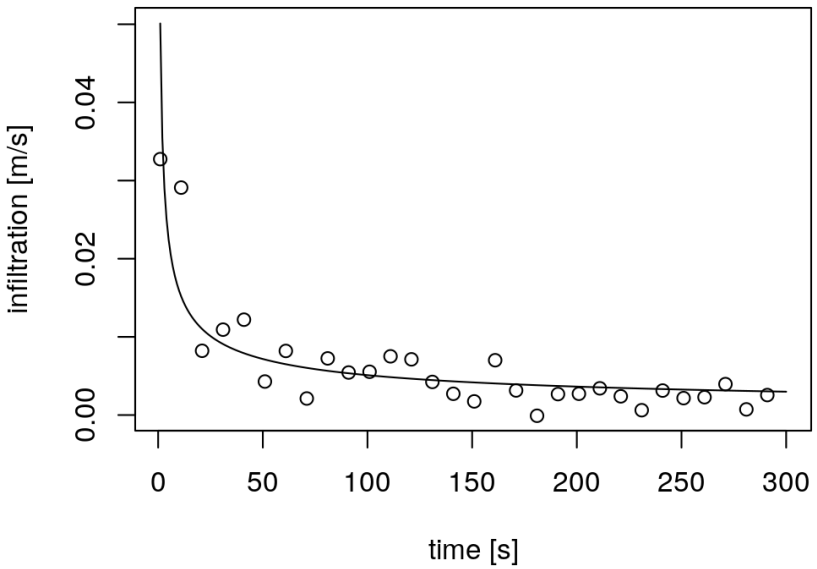
In the field

- Ponding experiment
- Mini-dist infiltration experiment

Philips infiltration

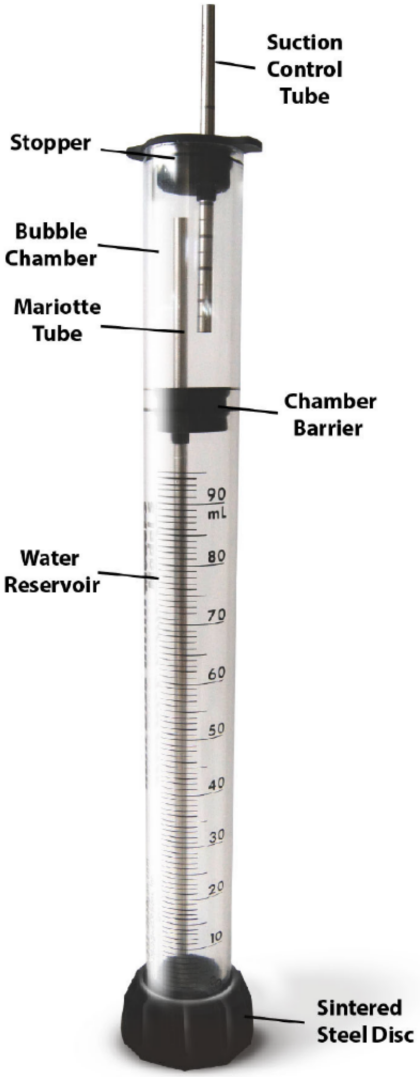
- derived for Richards equation for semi-infinite space

$$i = 1/2 S t^{-1/2} K_{fs}$$

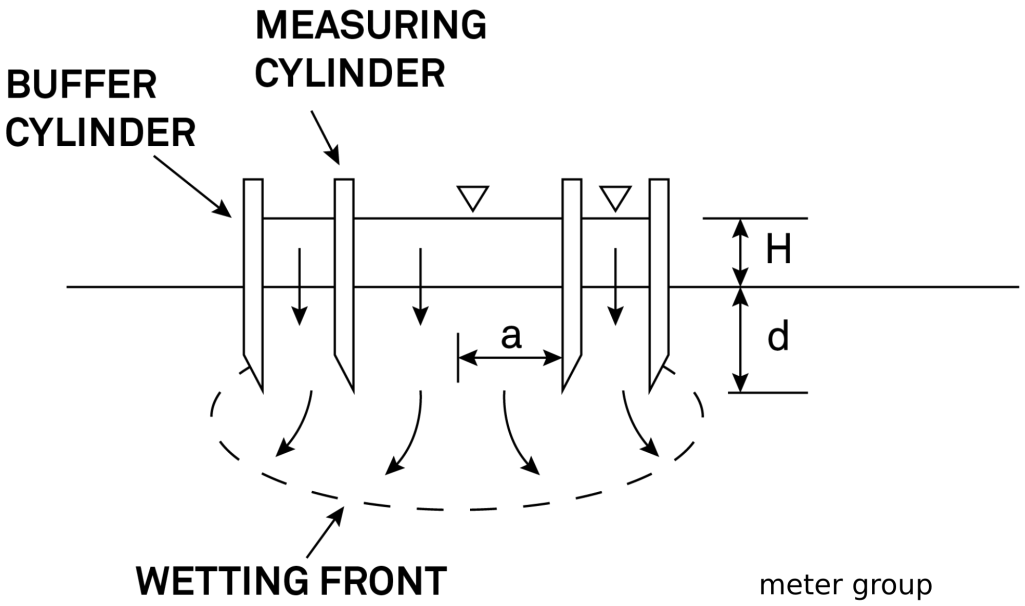


How to determine it?

In the field



Decagon



meter group

How to determine it?

from retention function

Mathematical model

- Introduction of relative capillary conductivity
- Predicted according to Mualem's or Burdine's capillary model
- **Relative K is inferred from retention curve and scaled by K_s**

Mualem's model

Relative unsaturated hydraulic conductivity (K_r)

$$K_r(h) = \begin{cases} \frac{(1-(-\alpha h)^{mn}(1+(-\alpha h)^n)^{-m})^2}{(1+(-\alpha h)^n)^{m/2}} & \text{if } h < 0 \\ 1 & \text{if } h \geq 0 \end{cases}$$

Inverse K_r

$$K_r(\theta_e) = \theta_e^{1/2}(1-(1-\theta_e^{1/m})^m)^2$$

Unsaturated hydraulic conductivity

$$K(h) = K_s K_r(h)$$

Assignment

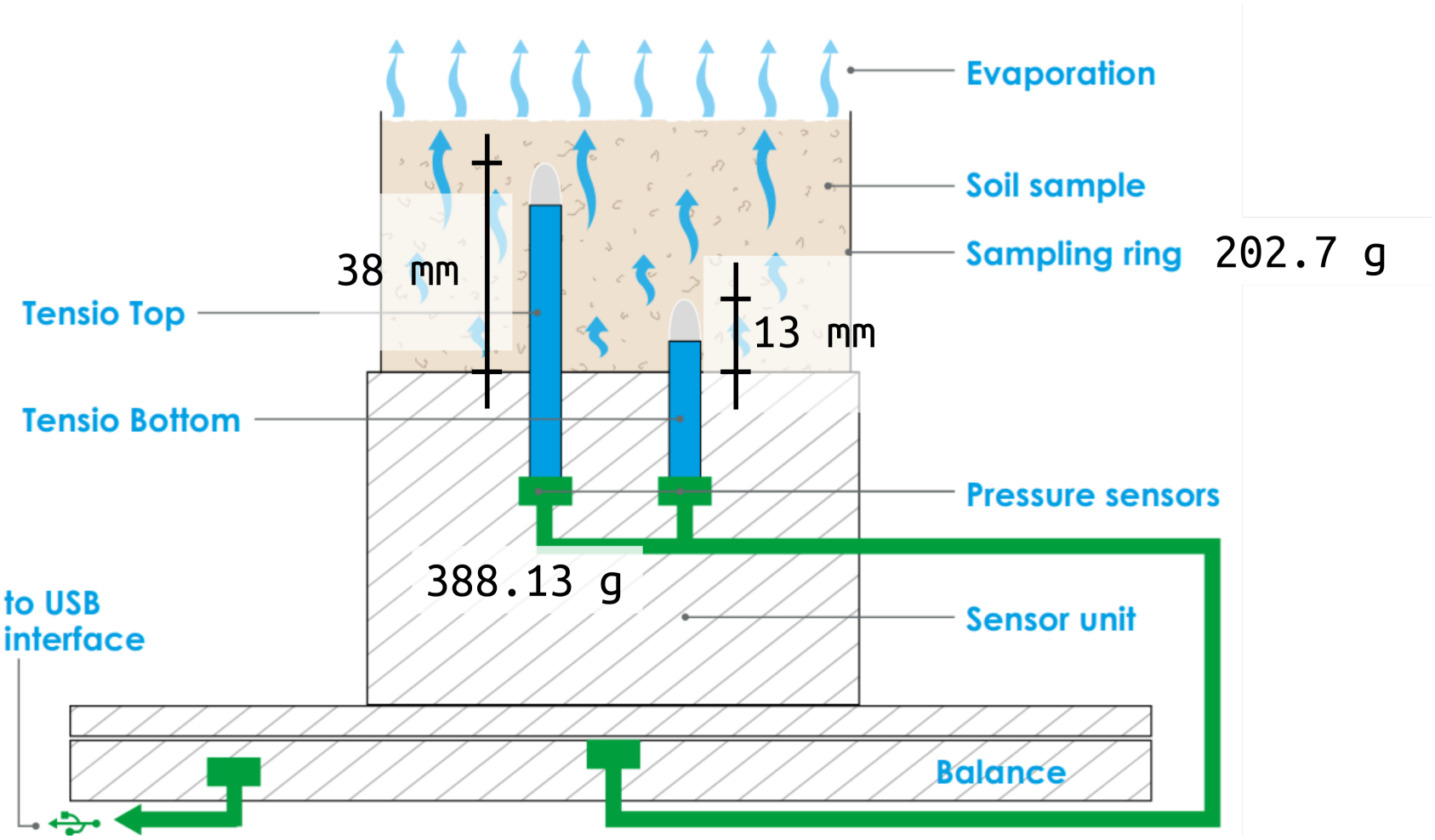
Infer curve of retention curve (van Genuchten's model) and hydraulic conductivity (Mualem's model) and form the evaporation experiment. Data provided in Excel sheet.

1. Have a look at the results of evaporation experiment.
2. Calculate tension mean and volumetric water content.
3. Calculate water flow and hydraulic gradient from the data.
4. Calculate curve of K(h) from the evaporation experiment with Darcy-Buckingham equation.
5. Get parameters of the van Genuchten-Mualem model using RETC. Use the option: `Both retention data and conductivity/diffusivity data`.
6. Calculate Mualem's relative hydraulic conductivity curve and van Genuchten's retention curve — use RECT parameters
7. Scale Mualem's K_r with K_s measured with the falling head method.
8. Make a graphs of both: retention curve and hydraulic conductivity curve including the measured data.

For comparison only: Ks of the soil sample is 2.05e-5 m/s based on the falling head experiment.

Hand in excel sheet with calculated retention and unsat. hyd. conductivity curve based on the evaporation experiment. Show the measured data in graphs. Include in the graphs also van Genuchten retention curve and Mualem unsaturated hydraulic conductivity curve with the RECT parameters. Please use subject: 143ESP.

Assignment


Evaporation
Soil sample
Sampling ring
202.7 g
38 mm
13 mm
Tensio Top
Tensio Bottom
Pressure sensors
388.13 g
Sensor unit
to USB interface
Balance

Effective water content

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

van Genuchten retention curve ($m = 1 - 1/n$) (VG)

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + (\alpha|h|)^n)^m}$$

Relative unsaturated hydraulic conductivity (K_r) :

$$K_r(h) = \begin{cases} \frac{(1-(\alpha|h|)^{mn}(1+(\alpha|h|)^n)^{-m})^2}{(1+(\alpha|h|)^n)^{m/2}} & \text{if } h < 0 \\ 1 & \text{if } h \geq 0 \end{cases}$$

Inverse K_r

$$K_r(\theta_e) = \theta_e^{1/2}(1 - (1 - \theta_e^{1/m})^m)^2$$

Unsaturated hydraulic conductivity

$$K(h) = K_s K_r(h)$$

h capillary pressure $[L, Pa]$, θ - volumetric water content $[L^3.L^{-3}]$, θ_r - residual water content$[L^3.L^{-3}]$, θ_s - saturated water content$[L^3.L^{-3}]$, α - VG parameter $[L^{-1}]$, n - VG parameter $[-]$, m - VG parameter $[-]$, K unsaturated hydraulic conductivity $[L.t^{-1}]$, K_r relative unsaturated hydraulic conductivity $[-]$, K_s saturated hydraulic conductivity $[L.t^{-1}]$