143ESP Soil physics for engineers

Hydraulic conductivity

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(Un)saturated hydraulic conductivity (K)

Measure how a given liquid flows through a given material.

- saturated hydraulic conductivity K_s
  - Scalar or tensor variable
- Unsaturated hydraulic conductivity K
  - function of suction pressure or vol. water content

Saturated hydraulic conductivity K_s

- permeability k is a property of the material alone
- $K = k\frac{\rho g}{\sigma}$
- k $\sim$ K/5

How to determine it?

(Un)saturated hydraulic conductivity can by obtained

1. by measurement in the field,
2. by measurement in the lab, or
3. from RC with a given mathematical model

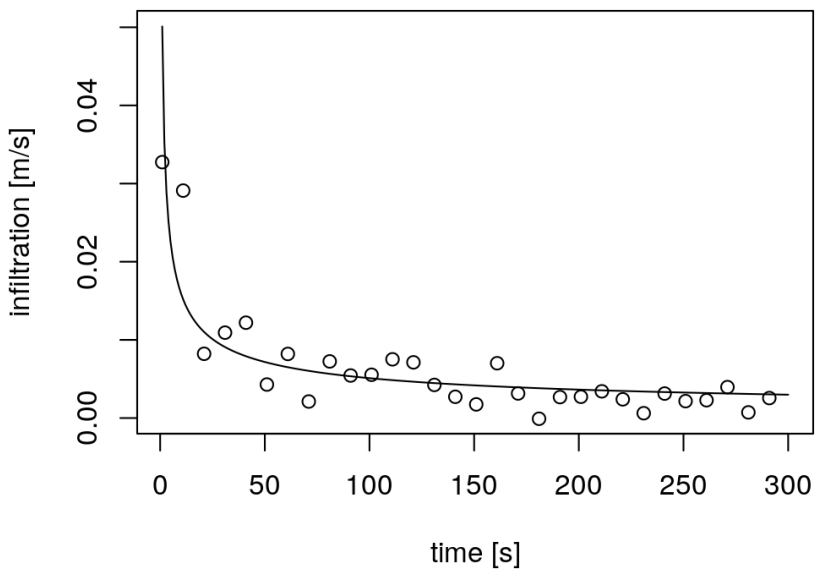
In the field

- Ponding experiment
- Mini-dist infiltration experiment

Philips infiltration

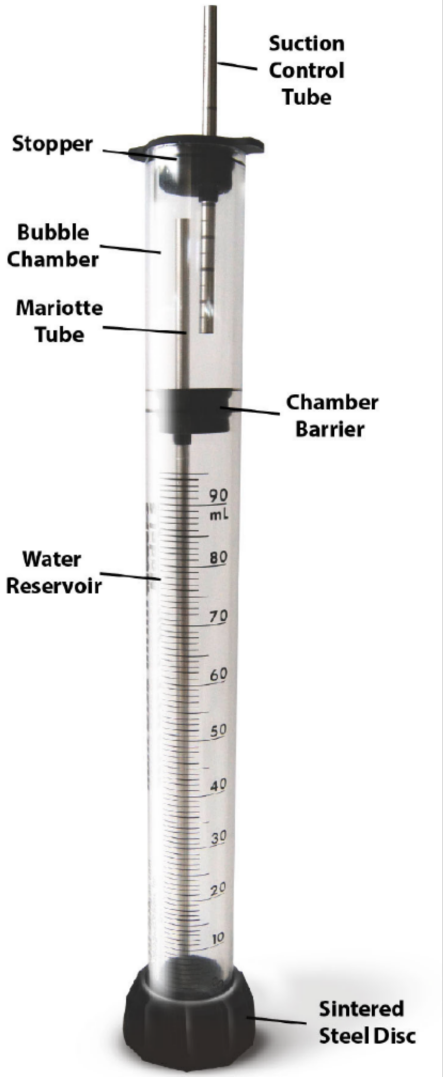
- derived for Richards equation for semi-infinite space

$$i = 1/2St^{-1/2}K_{fs}$$

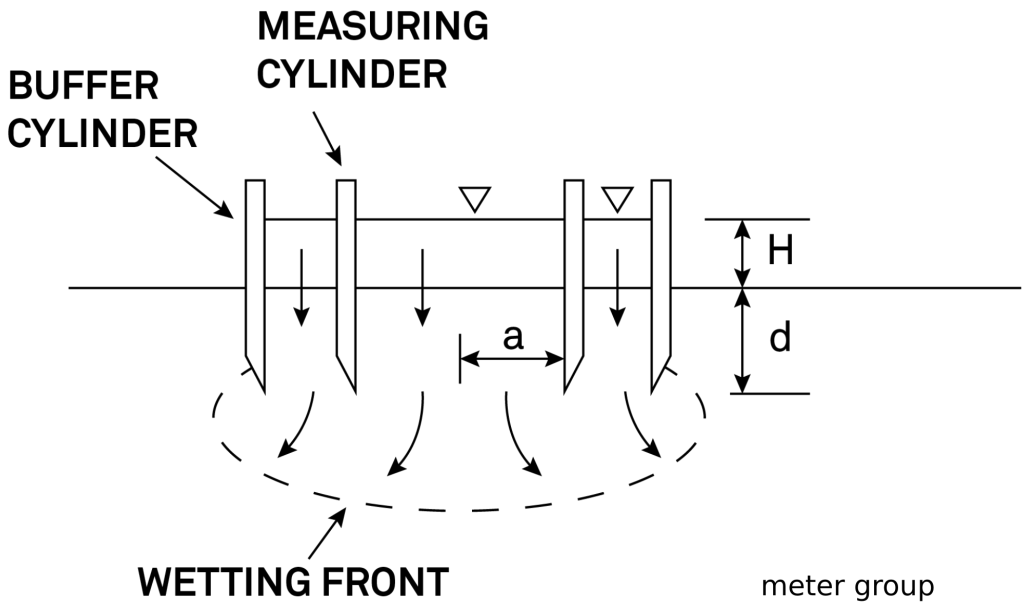


How to determine it?

In the field



Decagon



meter group

In the lab

- K_s is calculated with the Darcy formula
- K is calculated with the Darcy-Buckingham formula
- water flow measurement +
  - stepwise changes of hydraulic gradient
  - gradual changes of hydraulic head gradient (falling head experiment)
- evaporation method

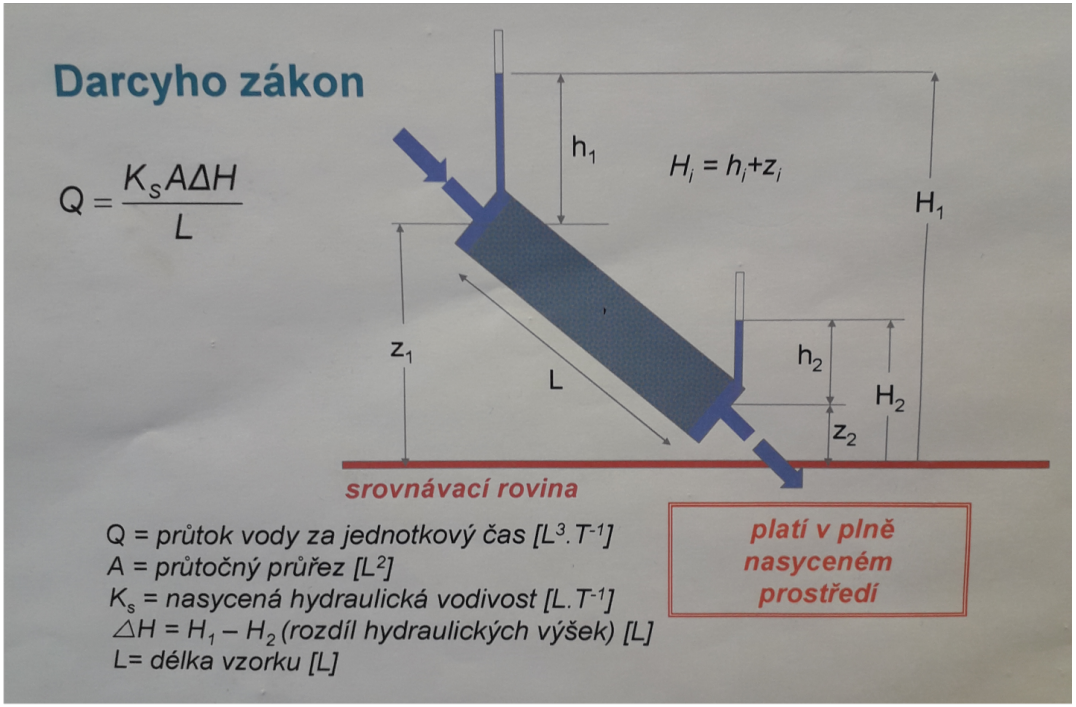
Darcy-Buckingham formula

$$q = -K(h)\nabla H$$

- H = h + z
- for x direction: $q = -K(h)\frac{\mathrm{d}H}{\mathrm{d}x}$

How to determine it?

In the lab



Constant and falling head exepriment to determine K_s

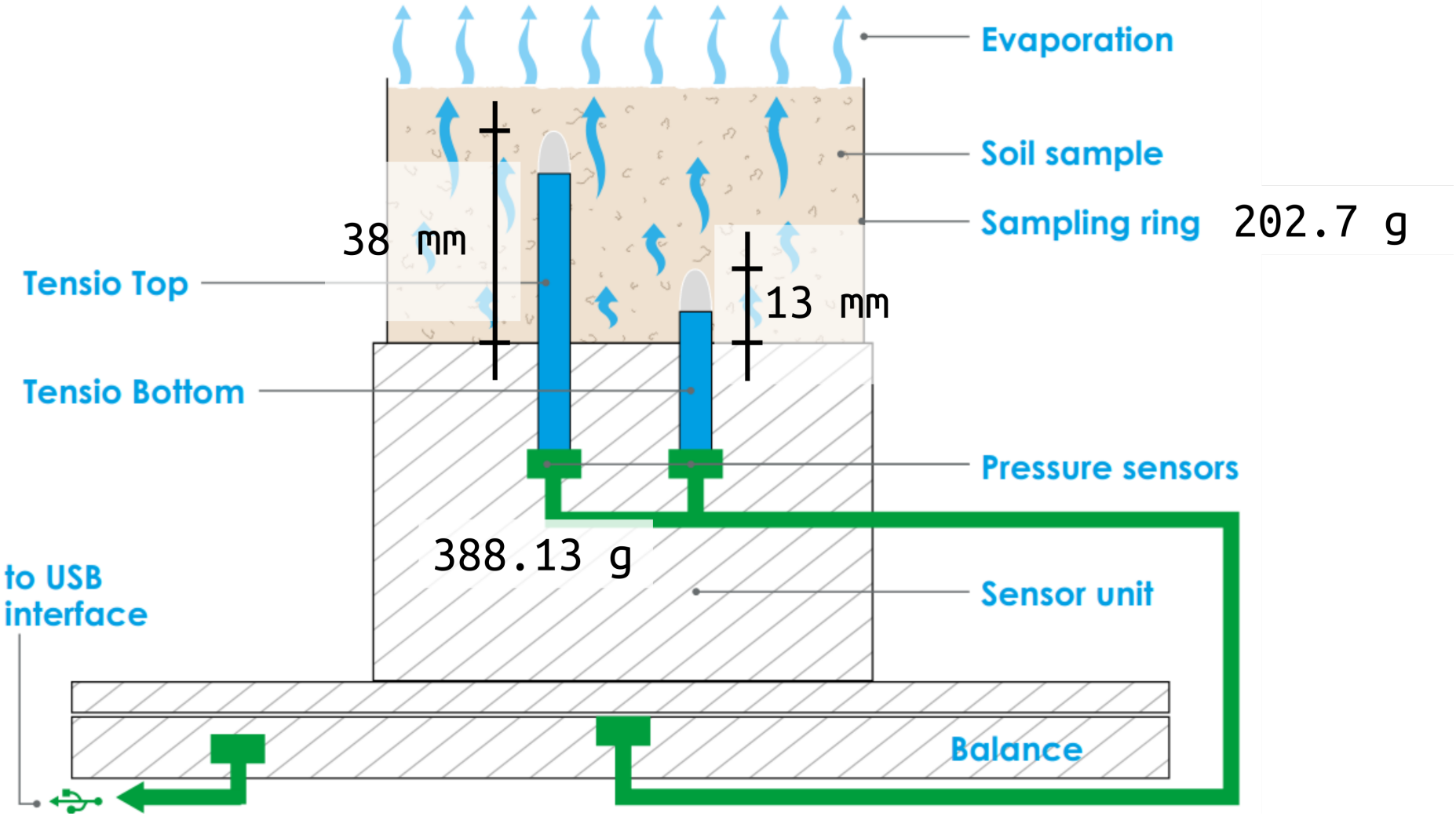
How to determine it?

In the lab

Evaporation experiment to determine K

Jak získat hydraulickou vodivost?

v laboratoři



Evaporation experiment to determine K

Mathematical model

- Introduction of relative capillary conductivity
- Predicted according to Mualem's or Burdine's capillary model
- **Relative K is inferred from retention curve and scaled by K_s**

Mualem's model

Relative unsaturated hydraulic conductivity (K_r)

$$K_r(h) = \begin{cases} \frac{(1-(-\alpha h)^{mn}(1+(-\alpha h)^n)^{-m})^2}{(1+(-\alpha h)^n)^{m/2}} & \text{if } h < 0 \\ 1 & \text{if } h \geq 0 \end{cases}$$

Inverse K_r

$$K_r(\theta_e) = \theta_e^{1/2}(1-(1-\theta_e^{1/m})^m)^2$$

Unsaturated hydraulic conductivity

$$K(h) = K_s K_r(h)$$

Assignment

1. Plot the Mualem relative hydraulic conductivity curve for 3 soils. Use the van Genuchten retention curve parameters from the previous exercise.
2. Scale the Mualem curve K_r with K_s.
3. Add the value of hydraulic conductivity to the soil profile in the previous exercise.
4. Optional: Calculate Darcian flow for depth $z_{i+1/2}$

Questions?

1. What is the order of the soils if you rank them by hydraulic conductivity values for a pressure of -20 hPa?
2. In which depth in the soil profile is K high? Why?
3. What processes can cause pressure head distribution as shown in the soil profile from previous exercise?

Submit the Excel with assignment with answers. In person or by email to jakub.jerabek@fsv.cvut.cz. Please use subject: **143ESP**.

Effective water content

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

van Genuchten retention curve ($m = 1 - 1/n$) (VG)

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + (\alpha|h|)^n)^m}$$

Relative unsaturated hydraulic conductivity (K_r) :

$$K_r(h) = \begin{cases} \frac{(1-(\alpha|h|)^{mn}(1+(\alpha|h|)^n)^{-m})^2}{(1+(\alpha|h|)^n)^{m/2}} & \text{if } h < 0 \\ 1 & \text{if } h \geq 0 \end{cases}$$

Inverse K_r

$$K_r(\theta_e) = \theta_e^{1/2}(1 - (1 - \theta_e^{1/m})^m)^2$$

Unsaturated hydraulic conductivity

$$K(h) = K_s K_r(h)$$

h capillary pressure $[L, Pa]$, θ - volumetric water content $[L^3.L^{-3}]$, θ_r - residual water content$[L^3.L^{-3}]$, θ_s - saturated water content$[L^3.L^{-3}]$, α - VG parameter $[L^{-1}]$, n - VG parameter $[-]$, m - VG parameter $[-]$, K unsaturated hydraulic conductivity $[L.t^{-1}]$, K_r relative unsaturated hydraulic conductivity $[-]$, K_s saturated hydraulic conductivity $[L.t^{-1}]$