Groundwater hydraulics and modelling - GWHM

Excercise 1 – Darcy´s law

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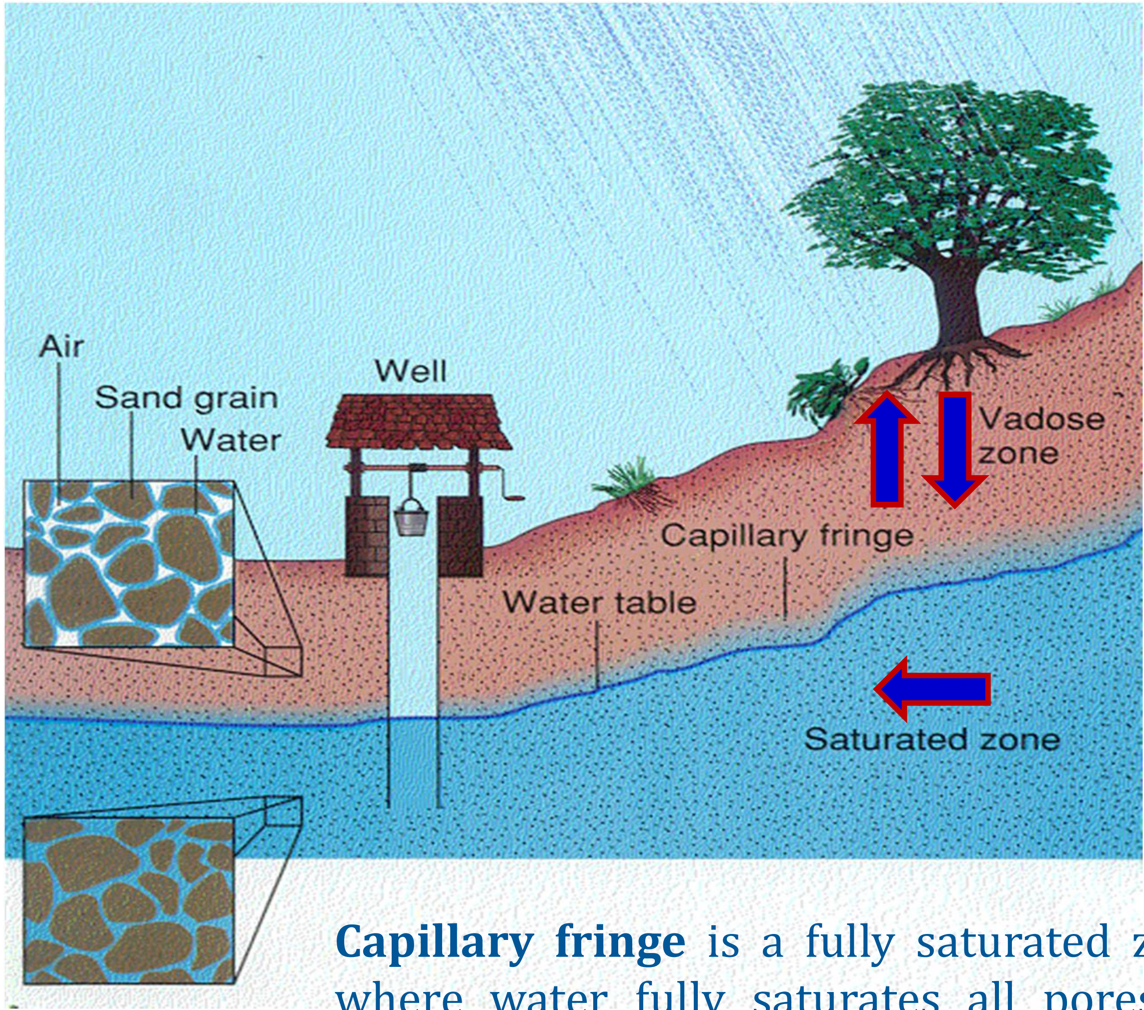
Themes of excercises

1. Terminology – saturated and unsaturated zone, porosity, hydraulic conductivity
2. Darcy’s law - examples

Groundwater in hydrological cycle



Soil above the water table:
solids – water - air
UNSATURATED ZONE

Soil below water table:
solids - water
SATURATED ZONE

Flow directions:
Vadose zone
Saturated zone

Capillary fringe is a fully saturated zone above the groundwater where water fully saturates all pores thanks to capillary forces greater in absolute than capillary pressure relevant for the biggest pore (see definition of *air entry value* in soil physics)

Porosity, saturated hydraulic conductivity

Pore volume: $V = V_s + V_p$

volume of solids volume of voids

Porosity:

$$n = \frac{V_p}{V}$$

volume of voids

total volume

Void ratio: defined as the ratio between the volume of voids and the volume of solids:

$$e = \frac{V_p}{V_s}$$

Hydraulic conductivity: indicates the ability of the aquifer to conduct water through it

K ... [m/s] $v = K * I$

Saturated hydraulic conductivity: **K_s ... [m/s]**

$K_s = 1*10^{-4}$ [m/s] ... coarse sand *$K_s = 1*10^{-11}$ [m/s] ... clay*

Groundwater movement

Flow potential for incompressible fluid (ρ=const.):

$$\Phi = gz + \frac{p - p_0}{\rho}$$

Water pressure is typically assigned at zero level equal to atmospheric pressure p_0, then

flow potential is defined as

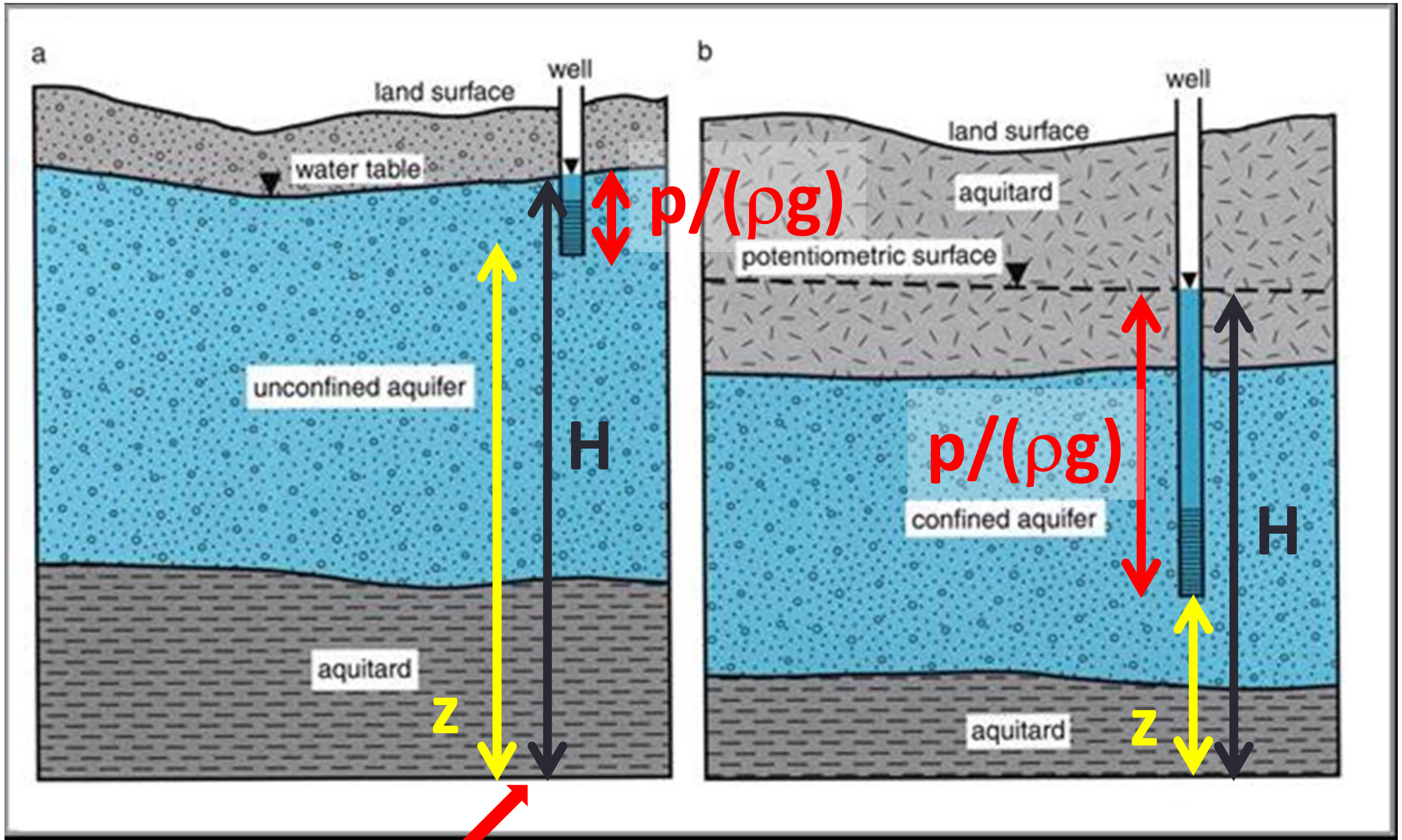
$$\boxed{\Phi = gz + \frac{p}{\rho}}$$

Water flow in between two points in porous space occurs in case of different potentials of water and simultaneously, there is continuous passage permeable for water. Water moves from level of higher to lower potential.

Groundwater hydraulics typically describes potential as **hydraulic head H**

$$\boxed{H = \frac{\Phi}{g} \qquad H = z + \frac{p}{\rho g}}$$

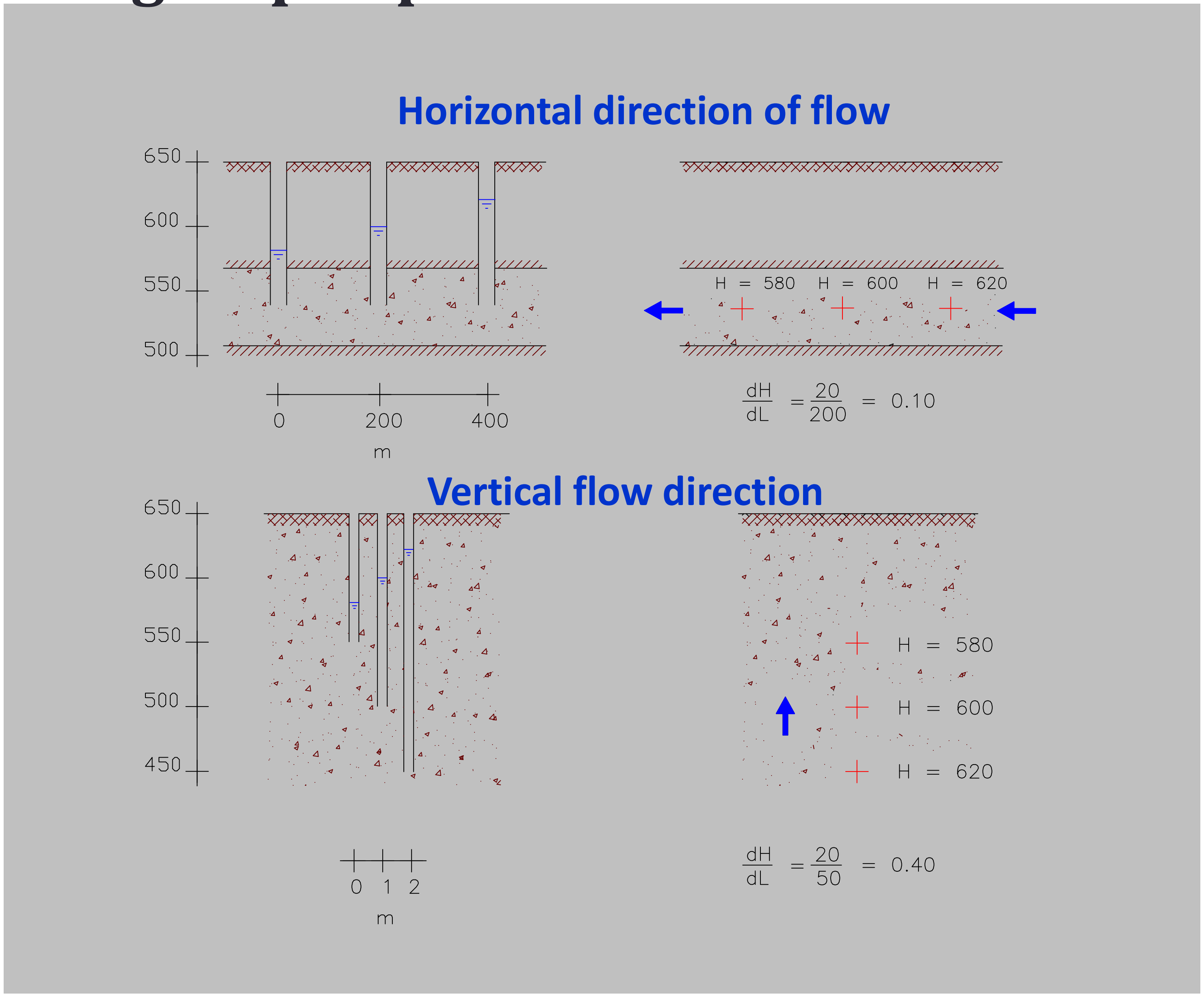
Measurement of hydraulic head



z=0

$$H = \frac{\Phi}{g} \qquad H = z + \frac{p}{\rho g}$$

Determination of groundwater flow direction based on group of piezometers



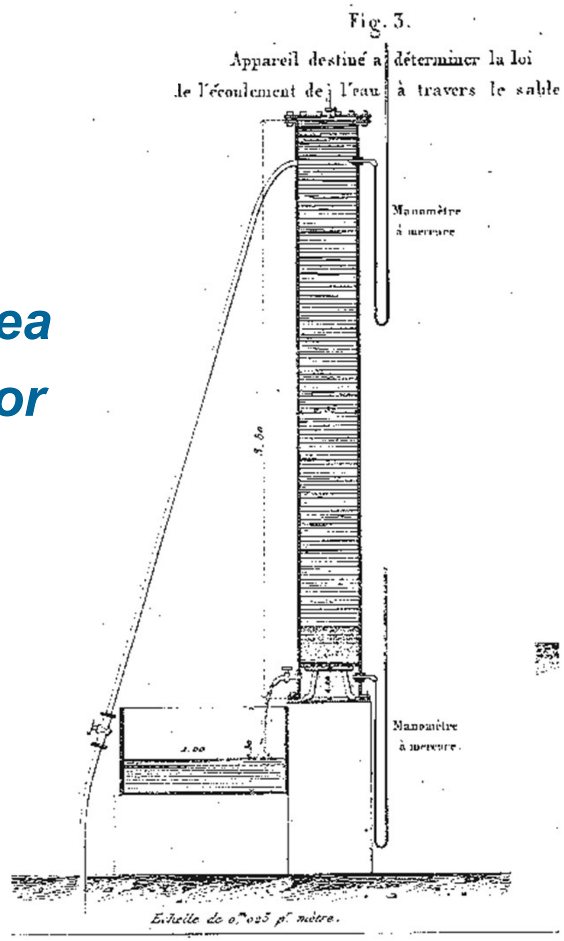
Saturated flow

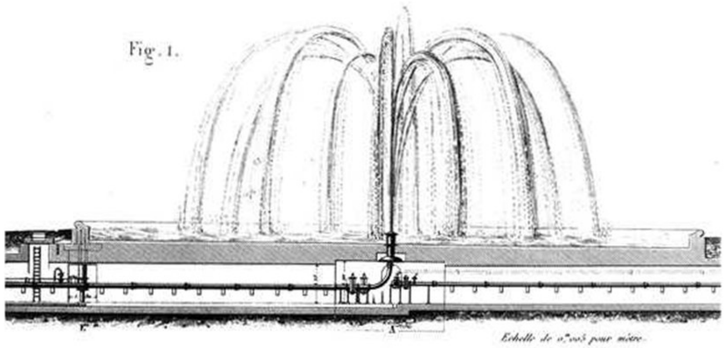
Henri Darcy (1856) *filtration of water for fountains in Dijon*

After many experiments he found that ***water flow*** *through the soil column depends on:*

- ***directly proportional to pressure drop***
- ***inversely proportional to the length***
- ***directly proportional to the crossectional area***
- ***dependent on coefficient which is specific for each media***

Henry Darcy

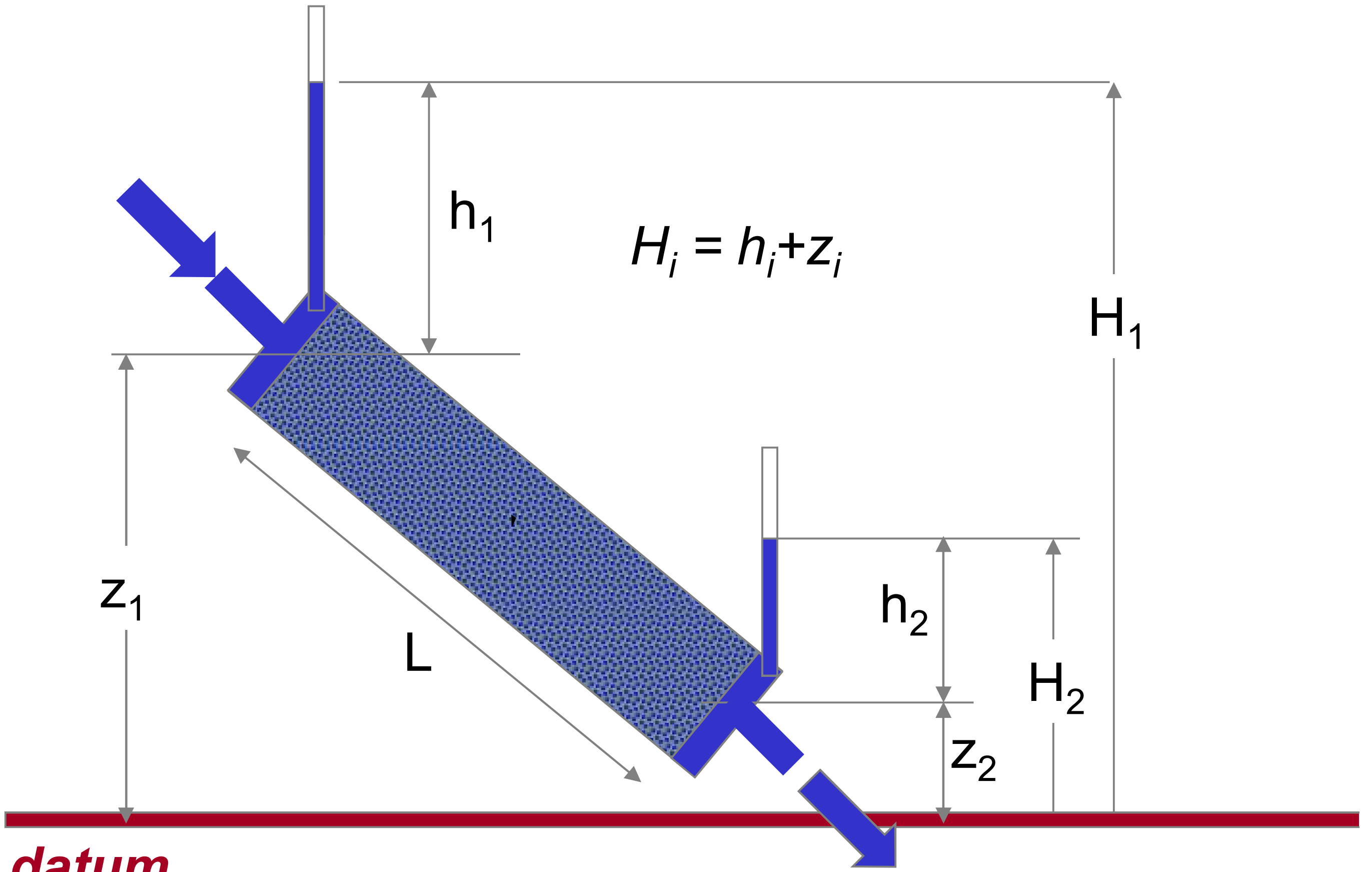




Darcy, H., 1856. *Les Fountaines de la Ville de Dijon*

Darcy law

$$Q = \frac{K_S A \Delta H}{L}$$



Q = flow [$L^3.T^{-1}$]
A = crossectional area [L^2]
K_s = saturated hydraulic conductivity [$L.T^{-1}$]
$\Delta H = H_1 - H_2$ (hydraulic head drop) [L]
L= sample lenght [L]

valid in fully saturated porous media
For example: under the ground water level

for:

$$q = \frac{Q}{A}$$

kde:
q ... Volume flux [L.T^{-1}]
Q ... Flow rate [L^{3}.T^{-1}]
A ... Crossectional area [L^{2}]

Transforms to the:

$$q = K_s \frac{\Delta H}{L}$$

note**: **negative sign** due to the fact **grad H** aims **against flow direction

More gereneral form:

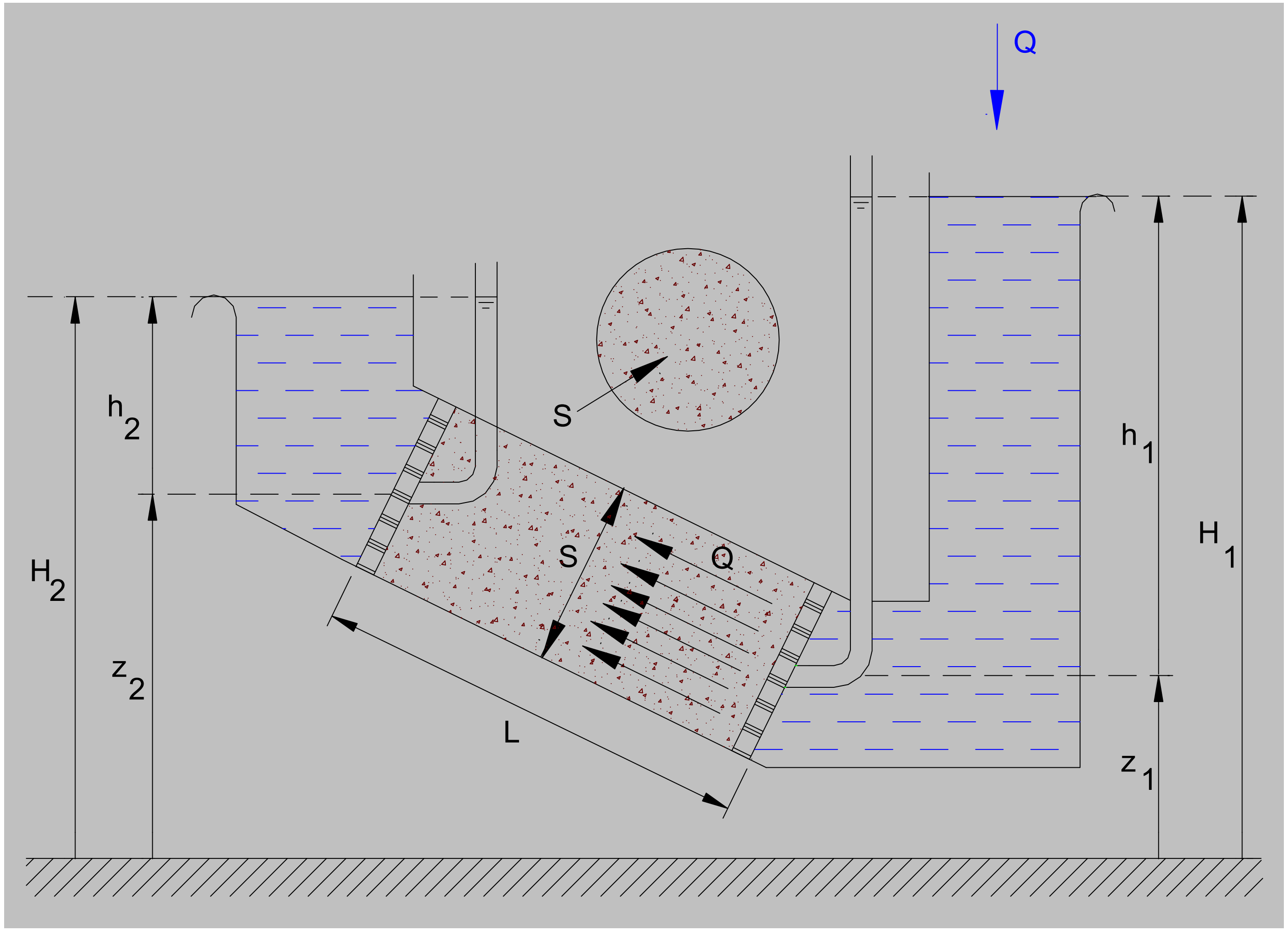
$$q = K_s \frac{dH}{dl}$$

Pro 1D vertical flow

$$q = -K_s \frac{dH}{dl} = -K_S \nabla H$$

Example 1 – Darcy law

Henri Darcy - 1856



$$\underset{\textbf{FLOW}}{Q} = \overset{\textbf{FLOW AREA}}{S} * \underset{\textbf{HYDRAULIC CONDUCTIVITY}}{K} * \overset{\textbf{HYDRAULIC GRADIENT}}{I}$$

$$v = \frac{Q}{S}$$

$$v_p = \frac{Q}{S_n}$$

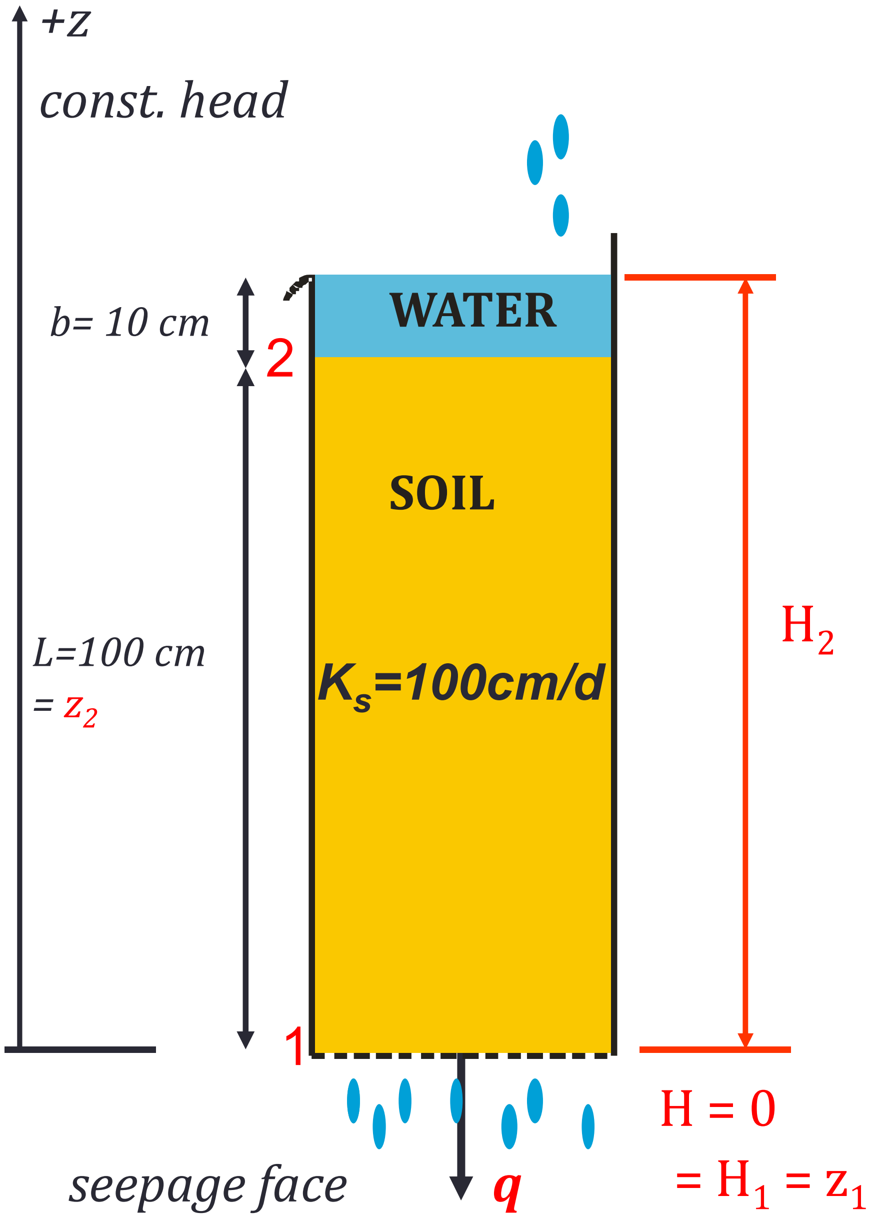
$$H = z + \frac{p}{\rho * g}$$

$$I = \frac{H_1 - H_2}{L}$$

Bernoulli´s equation $$z_1 + \frac{p_1}{\rho g} + \frac{v_1}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{v_2}{2g}$$

Example 1

Find q=?



Example 1

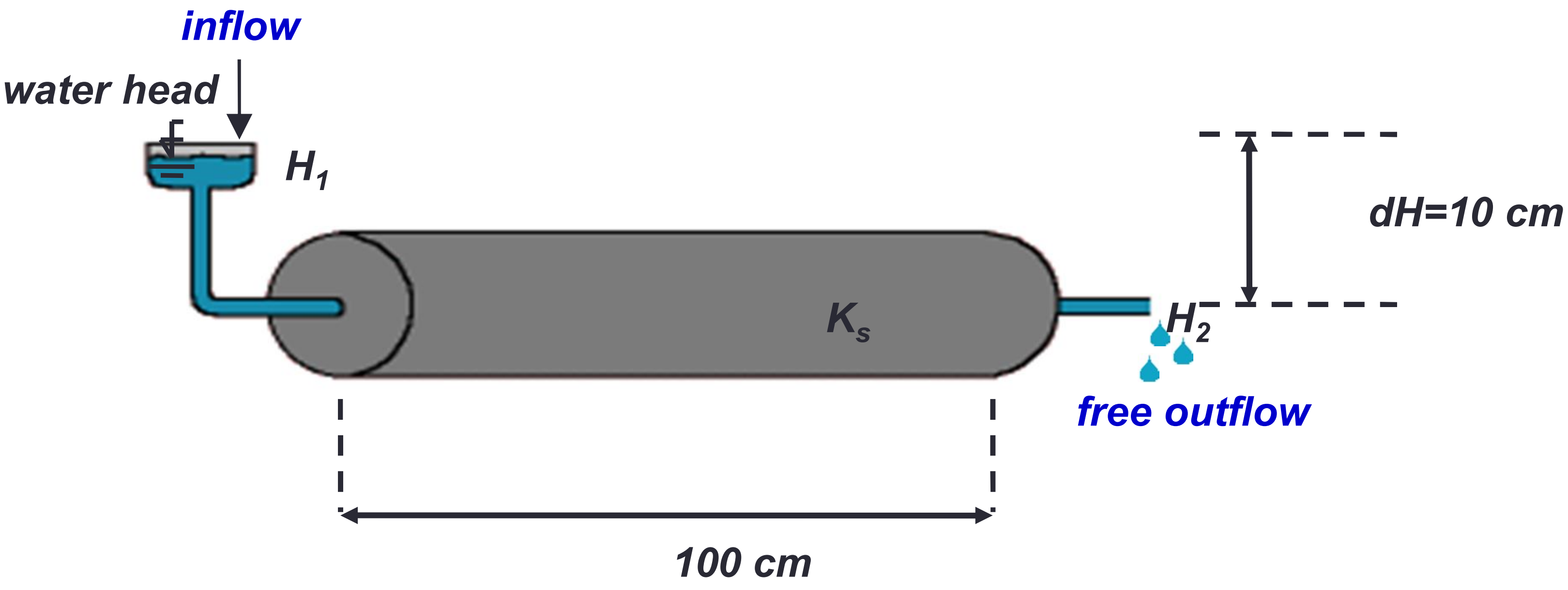
1) datum definition

2) points 1 and 2 with known hydraulic heads

3) Darcy's law

$$q = -K_s \frac{\Delta H}{L} = -K_s \frac{(H_2 - H_1)}{L} = -100 \frac{(110 - 0)}{100} = \mathbf{-110}\ \boldsymbol{cm.d^{-1}}$$

Example 2



Example 2

1) *Step 1 - definition of* ***coordination system***

2) *Definition of points 1 and 2).*

Then $x_1 = 0$ *and* $h_1 = 10$ *cm,* $x_2 = 100$*cm,* $h_2 = 0$*,* $z_1 = z_2 = 0$*,* $L = x_2 - x_1 = 100$ *cm*

3) *Hydraulics heads are then:* $H_1 = h_1 + z_1 = 10$ *cm,* $H_2 = h_2 + z_2 = 0$ *cm*

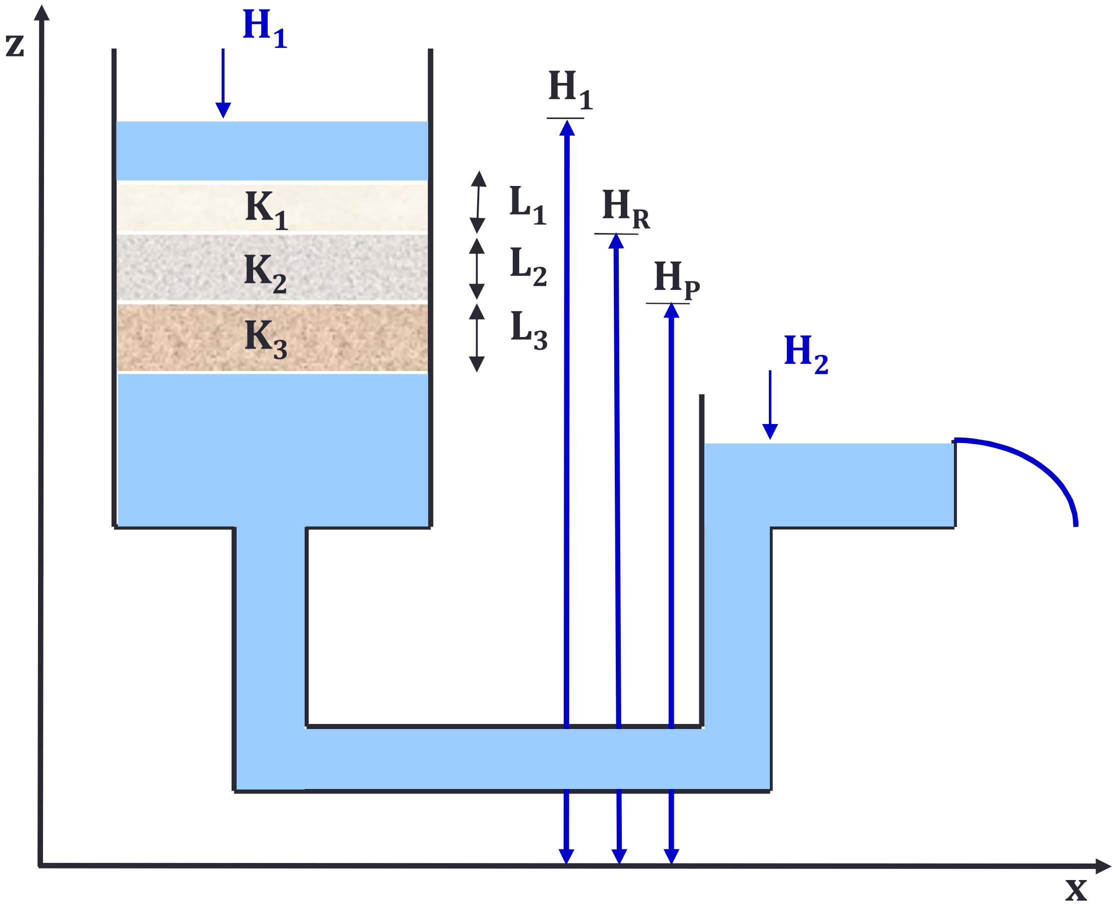
4) *Darcy's law*

$$q = -K_s \frac{\Delta H}{L} = -K_s \frac{(H_2 - H_1)}{L} = -100 \frac{(0-10)}{100} = 10\ cm.d^{-1}$$

Example 3

Calculate Q = ? (m^3/s) with known saturated hydraulic conductivities of three different materials $K_1 = 0.001$ m/s, $K_2 = 0.01$ m/s, $K_3 = 0.01$ m/s. Hydraulic heads are defined as: $H_1 = 1.2$ m, $H_R = 0.7$ m, $H_P = 0.6$ m, and $H_2 = 0.4$ m. Column area is $S = 0.0003$ m^2 and heights of each porous media are $L_1 = 0.1$ m, $L_2 = 0.2$ m, and $L_3 = 0.4$ m.

Example 3



Example 3

$$\mathbf{Q = Q_1 = Q_2 = Q_3}$$

$$Q_1 = K_1 S \frac{H_1 - H_R}{L_1} = 0.001 * 0.0003 * \frac{1.2 - 0.7}{0.1} = 1.5*10^{-6}\ m/s$$

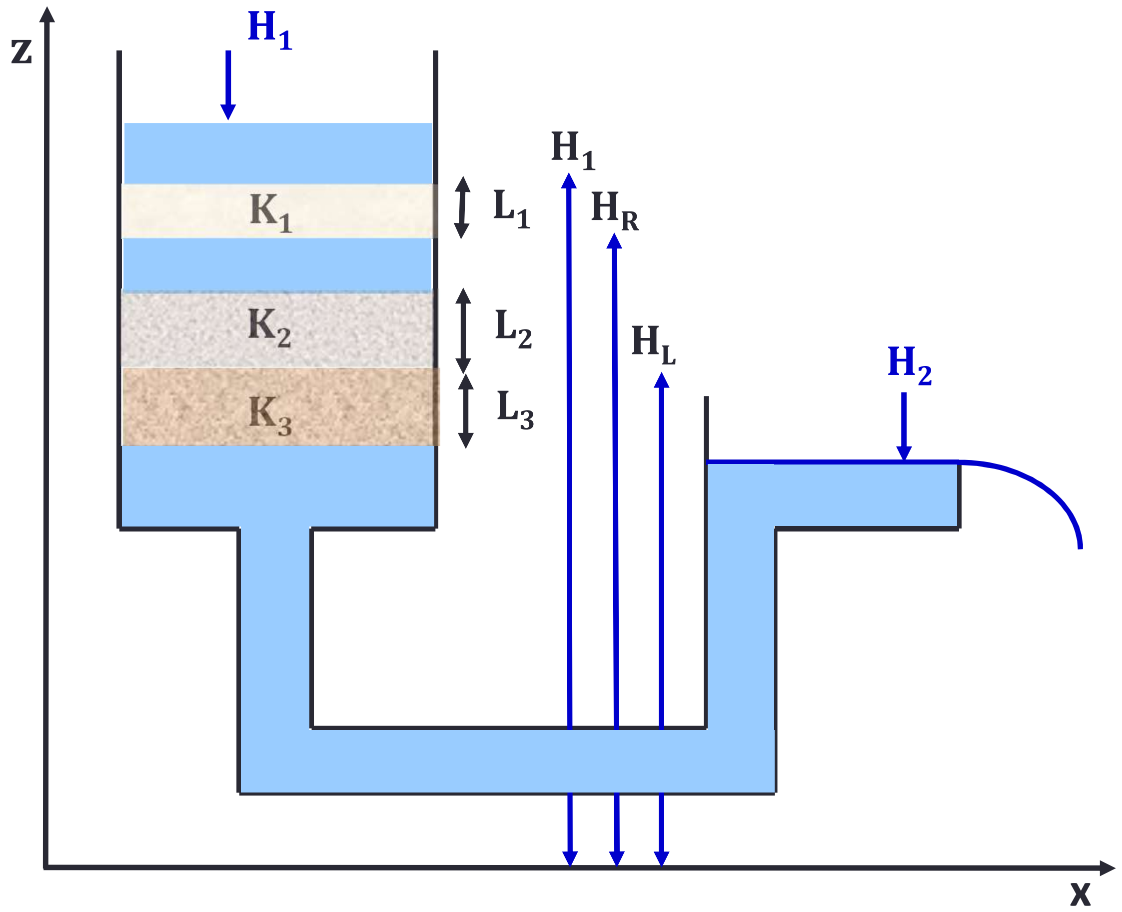
$$Q_2 = K_2 S \frac{H_R - H_P}{L_2} = 0.01 * 0.0003 * \frac{0.7 - 0.6}{0.2} = 1.5*10^{-6}\ m/s$$

$$Q_3 = K_3 S \frac{H_P - H_2}{L_3} = 0.01 * 0.0003 * \frac{0.6 - 0.4}{0.4} = 1.5*10^{-6}\ m/s$$

Example 3

Calculate $Q = ?$ (m^3/s) with known saturated hydraulic conductivities of three different materials $K_1 = 0.001$ m/s, $K_2 = 0.01$ m/s, $K_3 = 0.01$ m/s. Hydraulic heads are defined as: $H_1 = 1.2$ m, $H_R = 0.7$ m, $H_L = 0.6$ m, and $H_2 = 0.4$ m. Column area is $S = 0.0003$ m^2 and heights of each porous media are $L_1 = 0.1$ m, $L_2 = 0.2$ m, and $L_3 = 0.4$ m.

Example 4



Example 3

$$\mathbf{Q = Q_1 = Q_2 = Q_3}$$

$$Q_1 = K_1 S \frac{H_1 - H_R}{L_1} = 0.001 * 0.0003 * \frac{1.2 - 0.7}{0.1} = 1.5*10^{-6}\ m/s$$

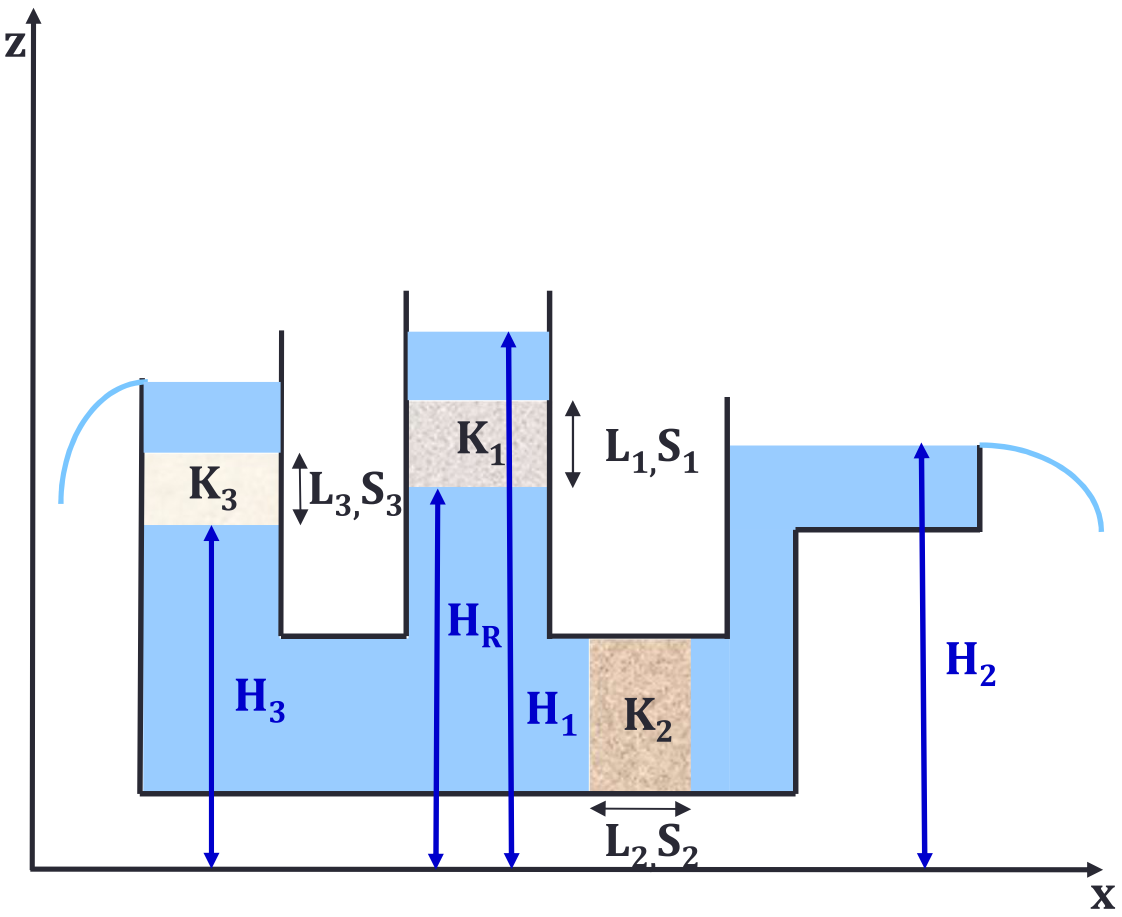
$$Q_2 = K_2 S \frac{H_R - H_L}{L_2} = 0.01 * 0.0003 * \frac{0.7 - 0.6}{0.2} = 1.5*10^{-6}\ m/s$$

$$Q_3 = K_3 S \frac{H_L - H_2}{L_3} = 0.01 * 0.0003 * \frac{0.6 - 0.4}{0.4} = 1.5*10^{-6}\ m/s$$

Example 5

Calculate Q_1 = ?, Q_2 = ?, Q_3 = ? m^3/s with known saturated hydraulic conductivities of three different materials K_1 = 0.05 m/s, K_2 = 0.1 m/s, K_3 = 0.04 m/s. Hydraulic heads are defined as: H_1 = 2.0 m, H_R = 1.5 m, H_2 = 1.3 m, and H_3 = 1.3 m. Column area is $S_1 = S_2 = S_3$ = 0.79 m^2 and heights of each porous media are L_1 = 0.020 m, L_2 = 0.022 m, and L_3 = 0.022 m.

Example 5



Example 5

$$\mathbf{Q_1 = Q_2 + Q_3}$$

$$Q_1 = K_1 S_1 \frac{H_1 - H_R}{L_1} = 0.05 * 0.79 * \frac{2.0-1.5}{0.02} = 1\ m/s$$

$$Q_2 = K_2 S_2 \frac{H_R - H_2}{L_2} = 0.1 * 0.79 * \frac{1.5-1.3}{0.022} = 0.7\ m/s$$

$$Q_3 = K_3 S_3 \frac{H_R - H_3}{L_3} = 0.04 * 0.79 * \frac{1.5-1.3}{0.022} = 0.3\ m/s$$

Example 5

Calculate Q_1 = ?, Q_2 = ?, Q_3 = ? m^3/s with known saturated hydraulic conductivities of three different materials K_1 = 4.35 m/s, K_2 = 0.80 m/s, K_3 = 0.90 m/s. Hydraulic heads are defined as: H_1 = 3.0 m, H_R = 2.1 m, H_2 = 1.0 m. Column area is $S_1 = S_2 = S_3$ = 2.54 m^2 and heights of each porous media are L_1 = 1.0 m, L_2 = 0.45 m, and L_3 = 0.50 m.

Example 6