Groundwater hydraulics – exercises 2

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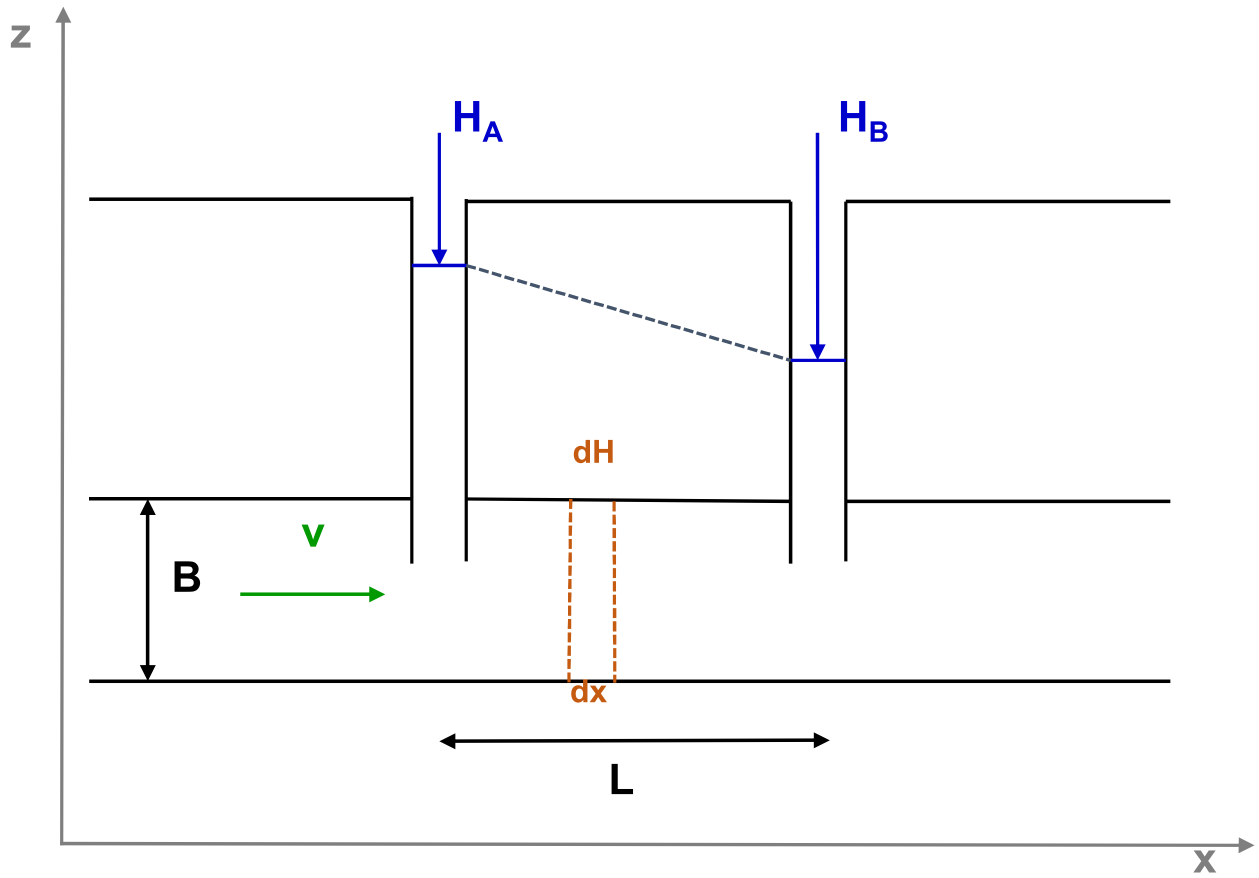
Themes of excercises

1. Water flow in confined aquifers

Calculate $q = ?$ (m^2/s) with known hydraulic conductivity $K = 0.1$ m/s. Hydraulic heads are defined as: $H_A = 100$ m, $H_B = 50$ m. Aquifer base is $B = 10$ m and distance of piezometers is $L = 20$ m.

Example 1



Example 1

B ... aquifer base … [L]
q ... specific flow rate for 1 m of aquifer … [T^2/L]

General solution:

$$v = -K\frac{dH}{dx}$$

$$q = \int_0^B v\,dz = \int_0^B (-K)\frac{dH}{dx}dz = -KB\frac{dH}{dx}$$

$$q\,dx = -KB\,dH$$

$$q\int dx = -KB\int dH$$

$$-\frac{q}{KB}x + C = H$$

Boundary conditions: 1) $x = 0 \quad H = HA$
2) $x = L \quad H = HB$

1) $-\frac{q}{KB}0 + C = HA \Rightarrow C = HA$ 2) $-\frac{q}{KB}L + H_A = HB$

$$-\frac{q}{KB}L + H_A = HB$$

$$-\frac{q}{KB} = \frac{H_B - HA}{L} \quad /*\text{-}1$$

$$\frac{q}{KB} = -\frac{H_B + H_A}{L}$$

$$\boxed{q = KB\frac{H_A - H_B}{L}}$$

Hydraulic head:

$$qx + C = -KBH$$

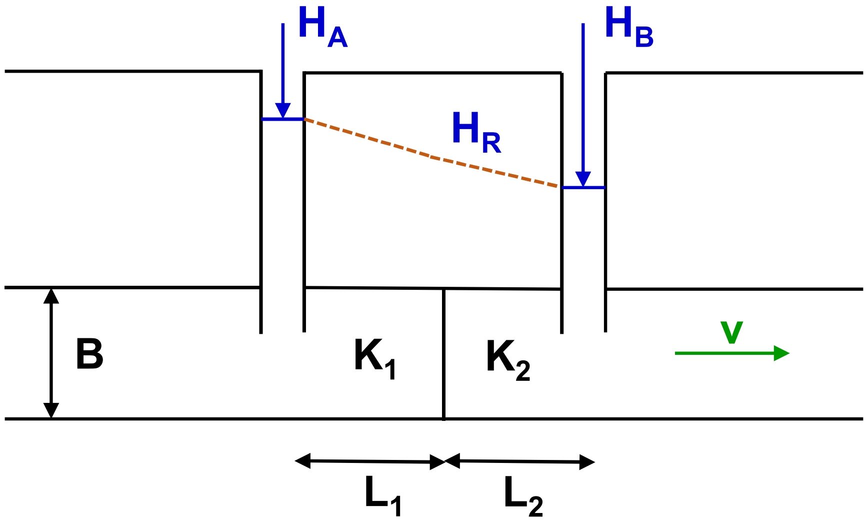
$$qx - KBHA = -KBH$$

$$\boxed{H = H_A - \frac{q}{KB}x}$$

Calculate $q = ?$ (m^2/s) with known hydraulic conductivity $K_1 = 0.1$ m/s and $K_2 = 0.2$ m/s . Hydraulic heads are defined as: $H_A = 100$ m, $H_B = 50$ m. Aquifer base is $B = 10$ m and distance of piezometers is $L_1 = 10$ m and $L_2 = 20$ m.

Example 2

- two different materials in aquifers



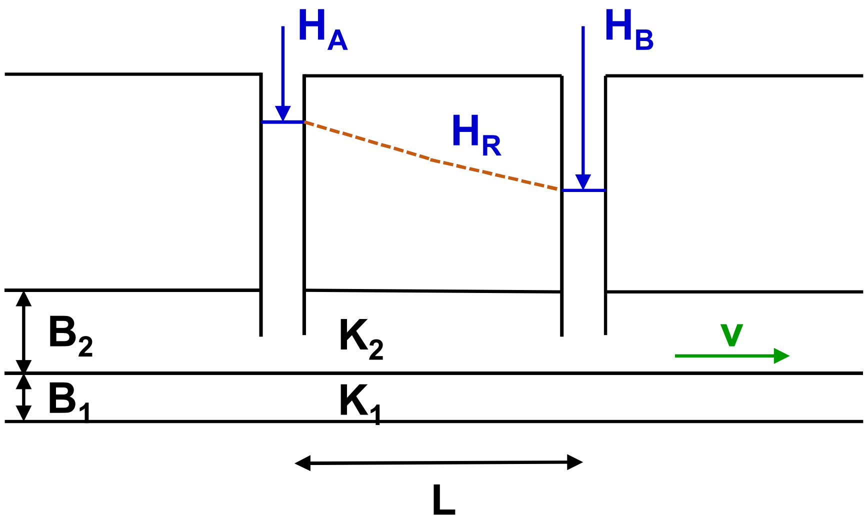
$$q = q_1 = q_2$$

$$q_1 = K_1 B \frac{H_A - H_B}{L_1}$$

$$q_2 = K_2 B \frac{H_A - H_B}{L_2}$$

Calculate $q = ?$ (m^2/s) with known hydraulic conductivity $K_1 = 0.1$ m/s and $K_2 = 0.2$ m/s . Hydraulic heads are defined as: $H_A = 100$ m, $H_B = 50$ m. Aquifer base is $B_1 = 5$ m, $B_2 = 10$ m and distance of piezometers is $L = 10$ m.

Example 3



$$q = q_1 + q_2$$

$$q_1 = K_1B_1 \frac{H_A - H_B}{L}$$

$$q_2 = K_2B_2 \frac{H_A - H_B}{L_2}$$

Hydraulic head

$$q = -K_1B_1 \frac{dH}{dx} - K_2B_2 \frac{dH}{dx}$$

$$-q = (K_1B_1 + K_2B_2) \frac{dH}{dx}$$

$$-q\ dx = (K_1B_1 + K_2B_2)\ dH$$

Boundary conditions: $1)\ x = 0 \quad H = HA$

$2)\ x = L \quad H = HB$

$$-q\,dx = (K_1B_1 + K_2B_2)\,dH$$

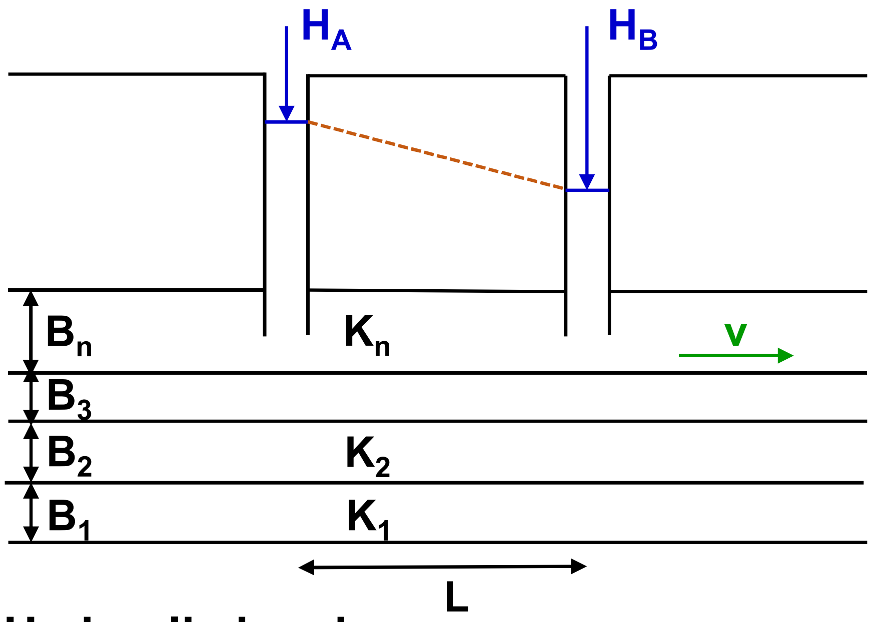
$$-qx + C = (K_1B_1 + K_2B_2)\,dH$$ ➡ **Boundary conditions:** 1) $x = 0 \quad H = HA$
2) $x = L \quad H = HB$

$$C = (K_1B_1 + K_2B_2)\,H_A$$

$$-qx + (K_1B_1 + K_2B_2)\,HA = (K_1B_1 + K_2B_2)\,H$$

$$H = H_A - \frac{q}{K_1B_1 + K_2B_2}\,x$$

Example 4



Hydraulic head

$$q = -K_1B_1\frac{dH}{dx} - K_2B_2\frac{dH}{dx}$$

$$-q = (K_1B_1 + K_2B_2)\frac{dH}{dx}$$

$$-q\,dx = (K_1B_1 + K_2B_2)\,dH$$

Boundary conditions: 1) $x = 0 \quad H = HA$
2) $x = L \quad H = HB$

$$q = \sum_{i=1}^{n} q_i \qquad B = \sum_{i=1}^{n} B_i$$

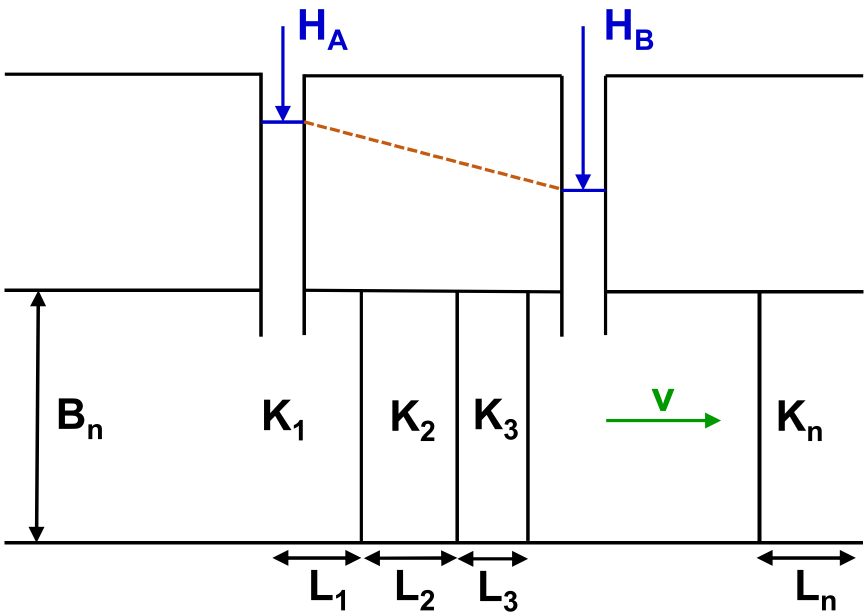
$$q_i = K_iB_i\frac{H_A - H_B}{L}$$

$$q = K_{eq}B\frac{H_A - H_B}{L}$$

$$K_{eq}B\frac{H_A - H_B}{L} = \sum_{i=1}^{n} K_iB_i\frac{H_A - HB}{L}$$

$$\boxed{K_{eq} = \frac{\sum_{i=1}^{n} K_iB_i}{B}}$$

Example 5



Hydraulic head

$$q = q_i = K_i B \frac{\Delta H_i}{\Delta L_i} \qquad L = \sum_{i=1}^{n} L_i$$

$$q = K_{eq} B \frac{\Delta H}{L} \qquad H = \sum_{i=1}^{n} H_i$$

$$q_i = K_i B_i \frac{H_A - H_B}{L}$$

$$K_{eq} B \frac{H_A - H_B}{L} = \sum_{i=1}^{n} K_i B_i \frac{H_A - H}{L}$$

$$\boxed{K_{eq} = \frac{\sum_{i=1}^{n} K_i B_i}{B}}$$

$$q = -K_1 B_1 \frac{dH}{dx} - K_2 B_2 \frac{dH}{dx}$$

$$-q = (K_1 B_1 + K_2 B_2) \frac{dH}{dx}$$

$$-q\, dx = (K_1 B_1 + K_2 B_2)\, dH$$

Boundary conditions: 1) $x = 0 \quad H = HA$

2) $x = L \quad H = HB$