CTU
CZECH TECHNICAL UNIVERSITY IN PRAGUE

Groundwater hydraulics – exercises 3

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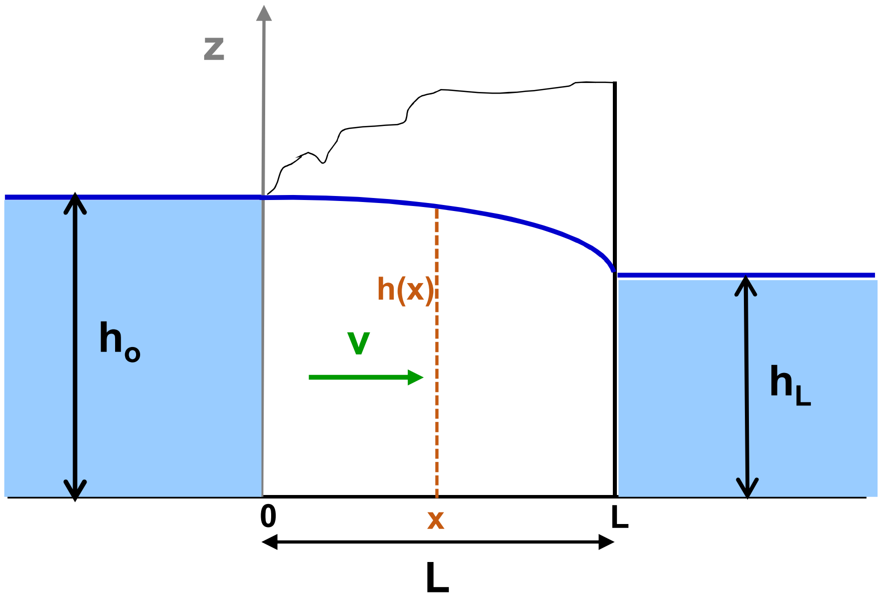
Themes of excercises

1. **Water flow in unconfined aquifers**

Example 1



General solution:

1. $\mathrm{v} = -\mathrm{K}\dfrac{dh(x)}{dx}$

2. $\mathrm{q} = \int_0^{\mathrm{h(x)}} \mathrm{v_x}\,\mathrm{dz} =$

$$= \int_0^{\mathrm{h(x)}} (-\mathrm{K})\frac{dh(x)}{dx}\,dz = -\mathrm{K}\frac{dh(x)}{dx}\,[\mathrm{z}]_0^{h(x)} =$$

$$= -\mathrm{K}\frac{dh(x)}{dx}h(x) - \mathrm{K}\frac{dh(x)}{dx}0 =$$

$$-\mathrm{K}\frac{dh(x)}{dx}h(x)$$

$$\mathrm{q} = -\mathrm{K}\frac{dh(x)}{dx}h(x)$$

$$\mathrm{q}\,dx = -\mathrm{K}\,h(x)\,dh$$

$$\mathrm{q}\int dx = -\mathrm{K}\int h(x)dh$$

$$\boxed{\mathrm{q\,x} + \mathrm{C} = -\mathrm{K}\frac{h(x)^2}{2}}$$

Boundary conditions:

1. $\mathrm{x} = 0 \quad \mathrm{h(x)} = h_0$
2. $\mathrm{x} = \mathrm{L} \quad \mathrm{h(x)} = h_L$

Specific flow *q*:

1. $x = 0 \quad h(x) = h_0$

$$q\,x + C = -K\frac{h(x)^2}{2}$$

$$q\,\cdot 0 + C = -K\frac{{h_0}^2}{2} \longrightarrow C = -K\frac{{h_0}^2}{2}$$

2. $x = L \quad h(x) = hL$

$$q\,L - K\frac{{h_0}^2}{2} = -K\frac{{h_L}^2}{2}$$

$$q\,L = -K\frac{{h_L}^2}{2} + K\frac{{h_0}^2}{2} \longrightarrow q\; = K\;\frac{{h_0}^2}{2L} - \frac{{h_L}^2}{2L} \longrightarrow \boxed{\mathbf{q\; = K\;\frac{h_0^2 - h_L^2}{2L}}}$$

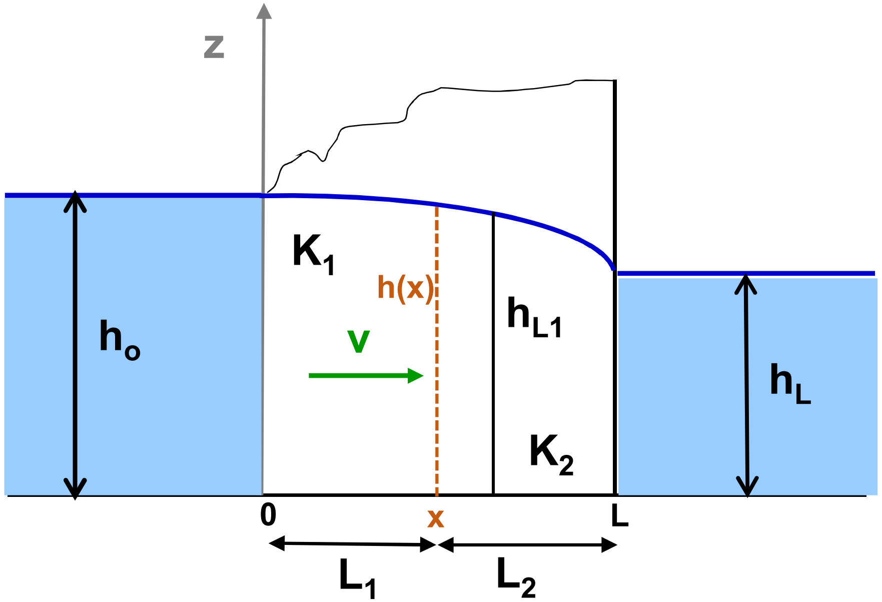
Hydraulic head *h(x)*:

$$q\,\cdot x - K\frac{{h_0}^2}{2} = -K\frac{h(x)^2}{2}$$

$$2 \cdot q\,\cdot x - \cancel{2} \cdot K\frac{{h_0}^2}{\cancel{2}} = -K \cdot h(x)^2$$

$$-\frac{2 \cdot q\,\cdot x}{K} - \frac{K \cdot h_0^2}{-K} = h(x)^2 \longrightarrow -\frac{2 \cdot q\,\cdot x}{K} + h_0^2 = h(x)^2 \longrightarrow \boxed{\mathbf{h(x) = \sqrt{h_0^2 - \frac{2 \cdot q\,\cdot \mathit{x}}{\mathit{K}}}}}$$

Example 2



General solution:

1. $v = -K\dfrac{dh}{dx}$

2. $q = \int_0^{h(x)} v_x \, dz =$

$= \int_0^{h(x)} (-K)\dfrac{dh}{dx} dz = -K\dfrac{dh}{dx} \, [z]_0^{h(x)} =$

$= -K\dfrac{dh}{dx}h(x) - K\dfrac{dh}{dx}0 = -K\dfrac{dh}{dx}h(x)$

3. $q = q_1 = q_2$

$$q = -K\frac{dh}{dx}h(x)$$

$$q_1 = -K_1\frac{dh}{dx}h(x) \qquad q_2 = -K_2\frac{dh}{dx}h(x)$$

$$q_1\int_0^{L_1} dx = -K_1\int_{h_0}^{h_{L1}} h(x)dh \qquad q_2\int_{L_1}^{L_2} dx = -K_2\int_{h_{L1}}^{h_L} h(x)dh$$

$$\boxed{q_1 = K_1 \cdot \frac{h_0^2 - h_{L1}^2}{2L_1}} \qquad \boxed{q_2 = K_2 \cdot \frac{h_{L1}^2 - h_L^2}{2L_2}}$$

Boundary conditions:

1. $x = 0$ $\quad h(x) = h_0$
2. $x = L_1$ $\quad h(x) = hL_1$

1. $x = L_1$ $\quad h(x) = hL_1$
2. $x = L_2$ $\quad h(x) = hL$

Hydraulic head *h(x)*:

$x \dots \langle 0, L_1 \rangle$

$$\mathbf{h(x)} = \sqrt{\mathbf{h_0^2} - \frac{\mathbf{h_0^2 - h_L^2}}{\mathbf{K_1 \cdot \left(\frac{L_2}{K_2} + \frac{L_1}{K_1}\right)}}} \cdot \mathbf{x}$$

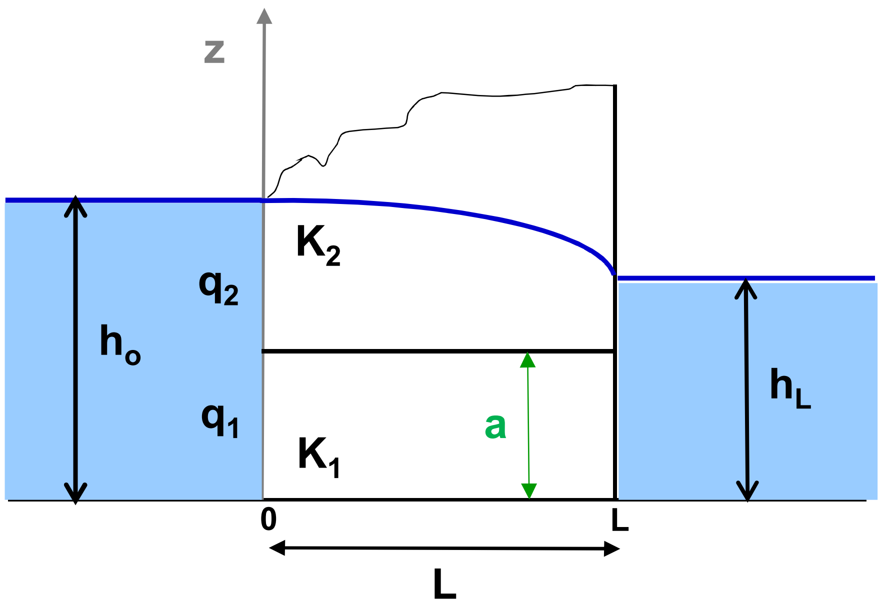
$x \dots \langle L_1, L \rangle$

$$\mathbf{h(x)} = \sqrt{\mathbf{h_{L1}^2} - \frac{\mathbf{h_0^2 - h_L^2}}{\mathbf{K_2 \cdot \left(\frac{L_2}{K_2} + \frac{L_1}{K_1}\right)}}} \cdot \mathbf{(L_1 - x)}$$

N materials:

$$\mathbf{q} = \frac{\mathbf{h_0^2 - h_L^2}}{\mathbf{2 \sum_{i=1}^{i=N} \left(\frac{L_i}{Ki}\right)}}$$

Example 3



General solution:

1. $v = -K\dfrac{dh}{dx}$

2. $q = \int_0^{h(x)} v_x \, dz =$

$= \int_0^{h(x)} (-K)\dfrac{dh}{dx} dz = -K\dfrac{dh}{dx}\,[z]_0^{h(x)} =$

$= -K\dfrac{dh}{dx}h(x) - K\dfrac{dh}{dx}0 = -K\dfrac{dh}{dx}h(x)$

3. $q = q_1 + q_2 = \text{konst.}$

$$q = -K_1 \cdot a \cdot \frac{dh}{dx} - K_2 \cdot h \cdot \frac{dh}{dx} = -(K_1 \cdot a \cdot + K_2 \cdot h) \cdot \frac{dh}{dx}$$

Boundary conditions:
1. $x = 0 \quad h(x) = h_0$
2. $x = L \quad h(x) = hL$

$$\boxed{q = \frac{K_2}{L}(h_0 - h_L)(h_0 + h_L - 2a + 2a\frac{K_1}{K_2})}$$