CTU
CZECH TECHNICAL UNIVERSITY IN PRAGUE

Vadose Zone Hydrology

Seminar 4
Hydraulic Conductivity

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Hydraulic Conductivity

- (Un)saturated hydraulic conductivity (K): Measure how a given liquid flows through a given material.
- Saturated hydraulic conductivity $\mathbf{K_s}$
  Scalar or tensor variable
- Unsaturated hydraulic conductivity **K**
  Function of suction pressure or vol. water content

Hydraulic Conductivity

- Saturated hydraulic conductivity K_s
  Permeability k is a property of the material alone
- $K = k\frac{\rho g}{\sigma}$
- $k \sim K/5$

How to determine Hydraulic Conductivity?

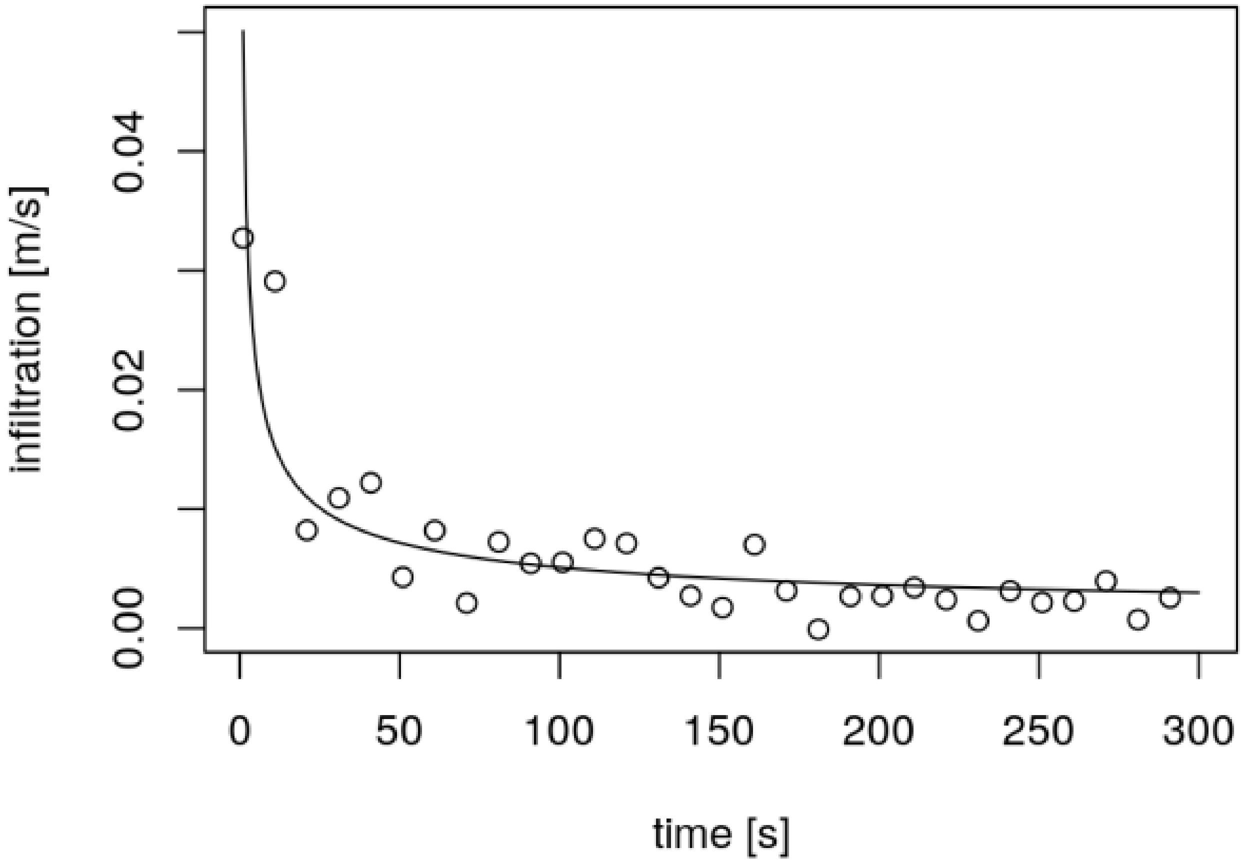
(Un)saturated hydraulic conductivity can by obtained

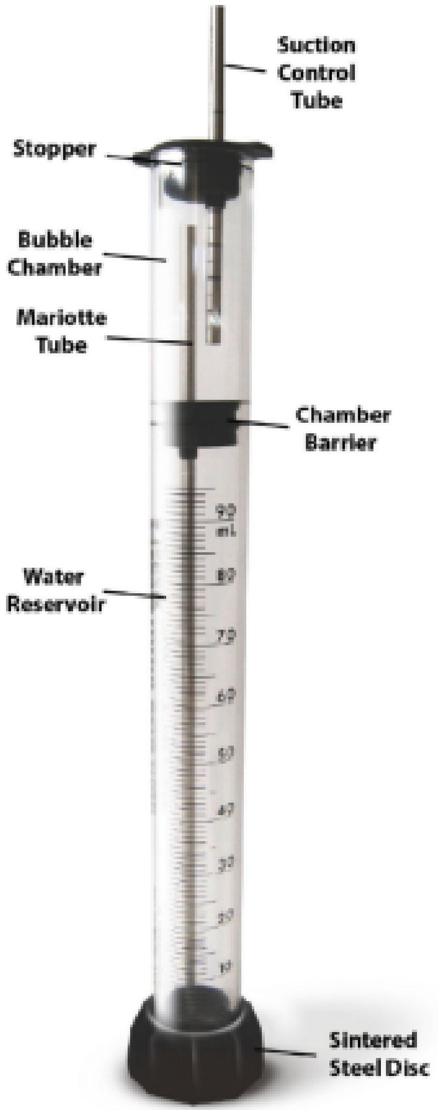
- by measurement in the field
- by measurement in the lab
- from RC with a given mathematical model

K_s Determination in the field

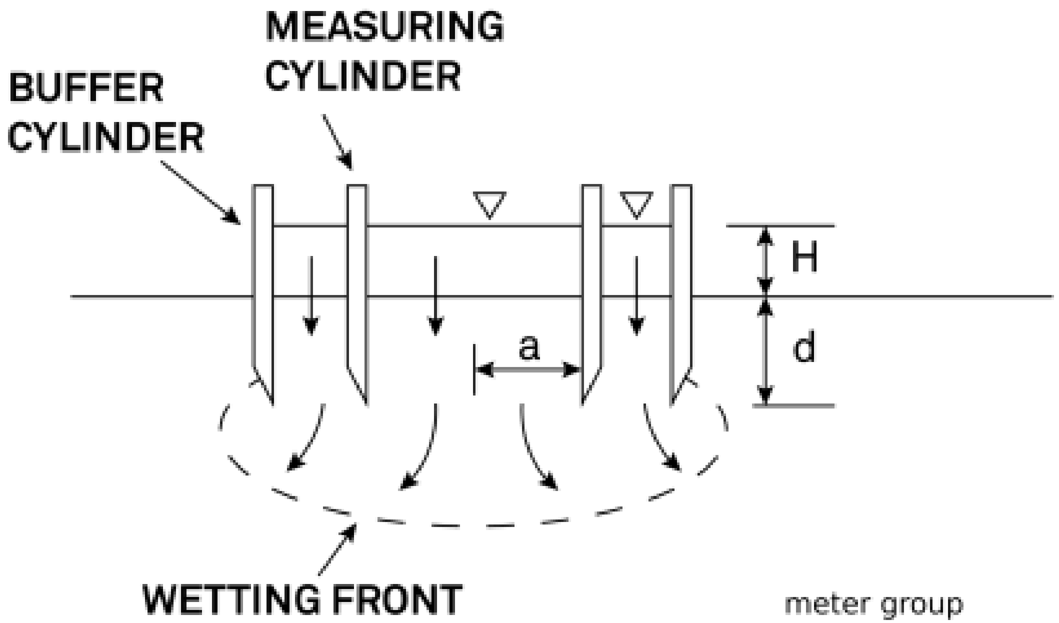
- In the field
  -Ponding experiment
  -Mini-disk infiltration experiment

- Philips infiltration
  derived for Richards equation for semi-infinite space
- $i = \frac{1}{2}\, S\, t^{-1/2}\, K_s$





Decagon



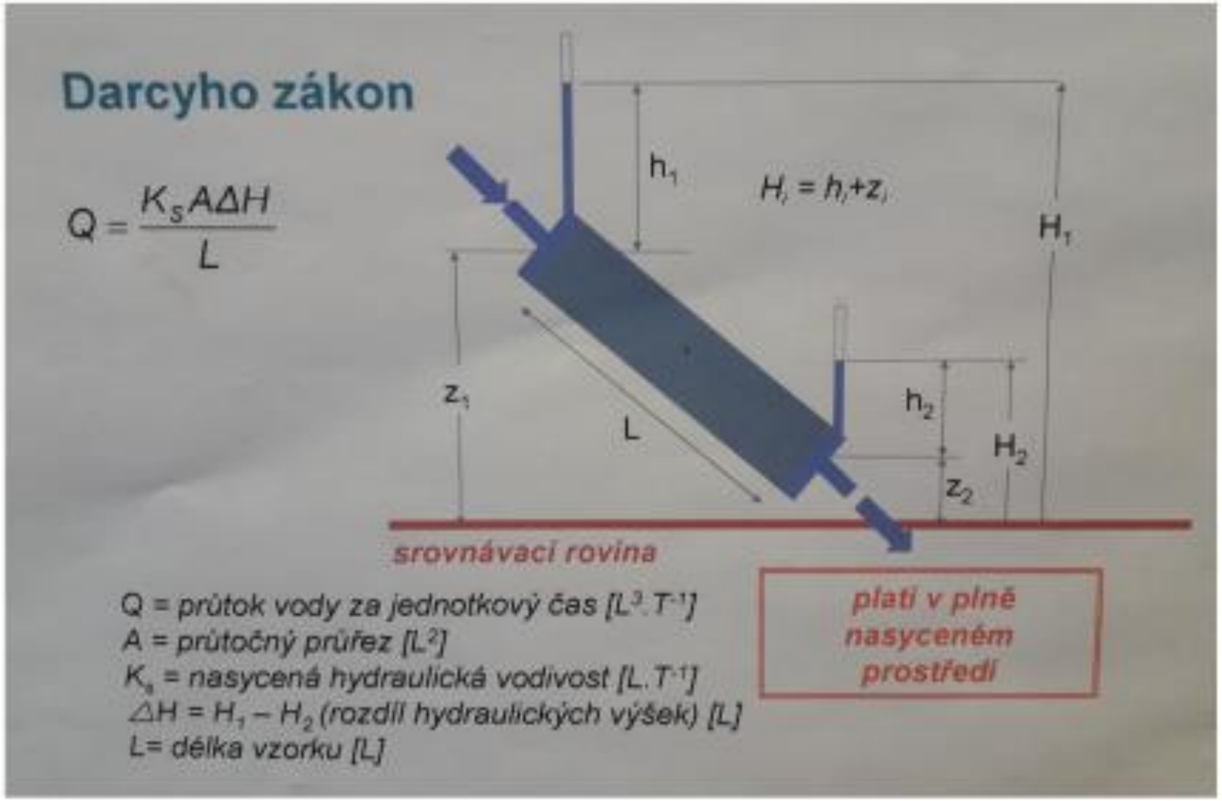
meter group

K_s determination in the lab

- Ks is calculated with the Darcy formula
- K is calculated with the Darcy-Buckingham formula
- water flow measurement +
  stepwise changes of hydraulic gradient
  gradual changes of hydraulic head gradient (falling head experiment)
- Evaporation method
- Darcy-Buckingham formula:

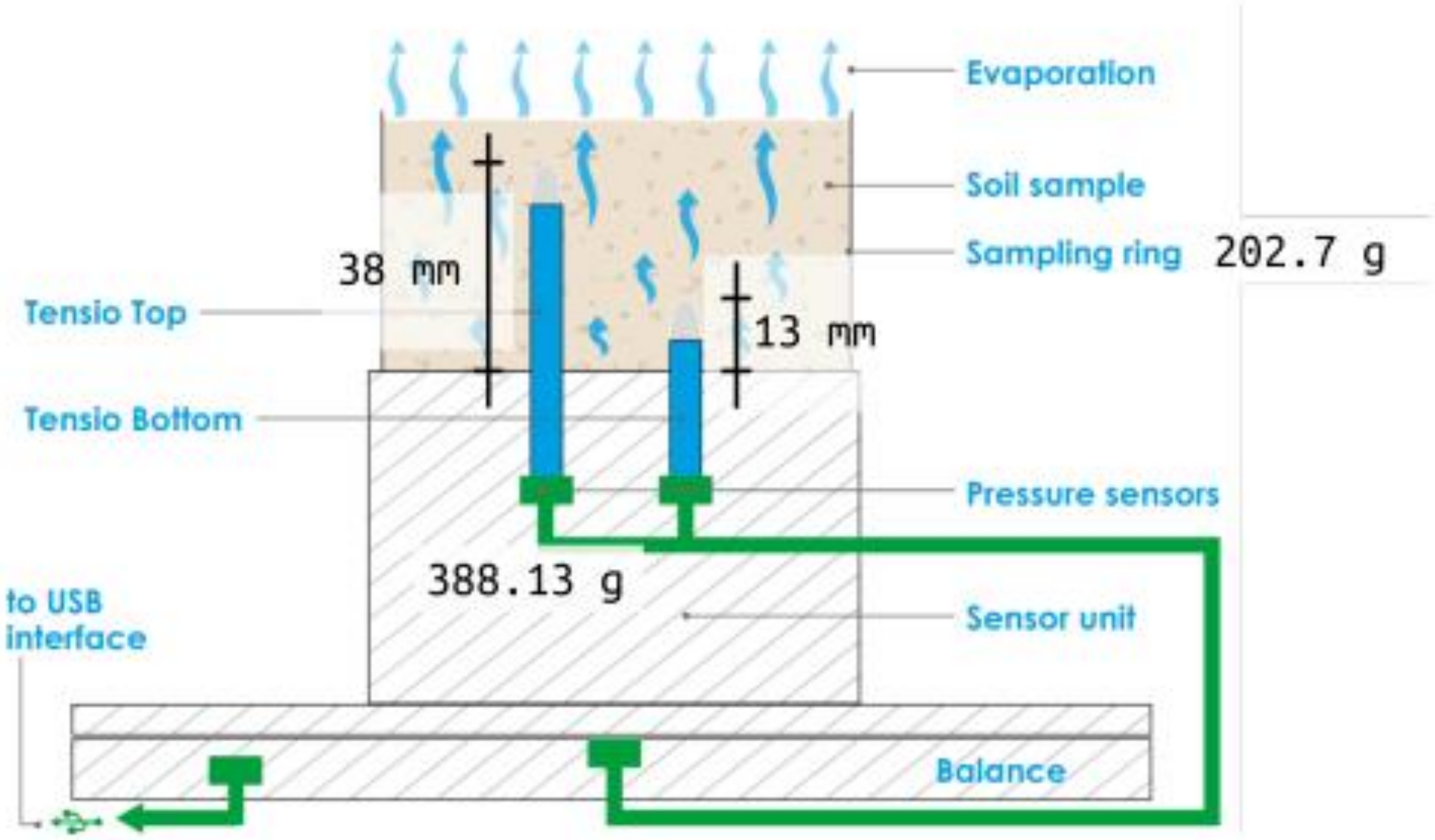
$$q = -K(h)\nabla H$$
$$H = h + z$$

For x direction: $q = -K(h)\frac{dH}{dx}$



Constant and falling head exepriment to determine K_s

Evaporation experiment to determine K



Limitations:

- averaging of pressure head; linearization assumption
- total potential gradient at the beginning

K_s determination from retention function

Mathematical model

- Introduction of relative capillary conductivity
- Predicted according to Mualem's or Burdine's capillary model
- Relative K is inferred from retention curve and scaled by Ks

- Mualem's model
  Relative unsaturated hydraulic conductivity (Kr)

$$K_r(h) = \begin{cases} \frac{(1-(-\alpha h)^{mn}(1+(-\alpha h)^n)^{-m})^2}{(1+(-\alpha h)^n)^{m/2}} & \text{if } h < 0 \\ 1 & \text{if } h \geq 0 \end{cases}$$

Inverse K_r

$$K_r(\theta_e) = \theta_e^{1/2}(1-(1-\theta_e^{1/m})^m)^2$$

Unsaturated hydraulic conductivity

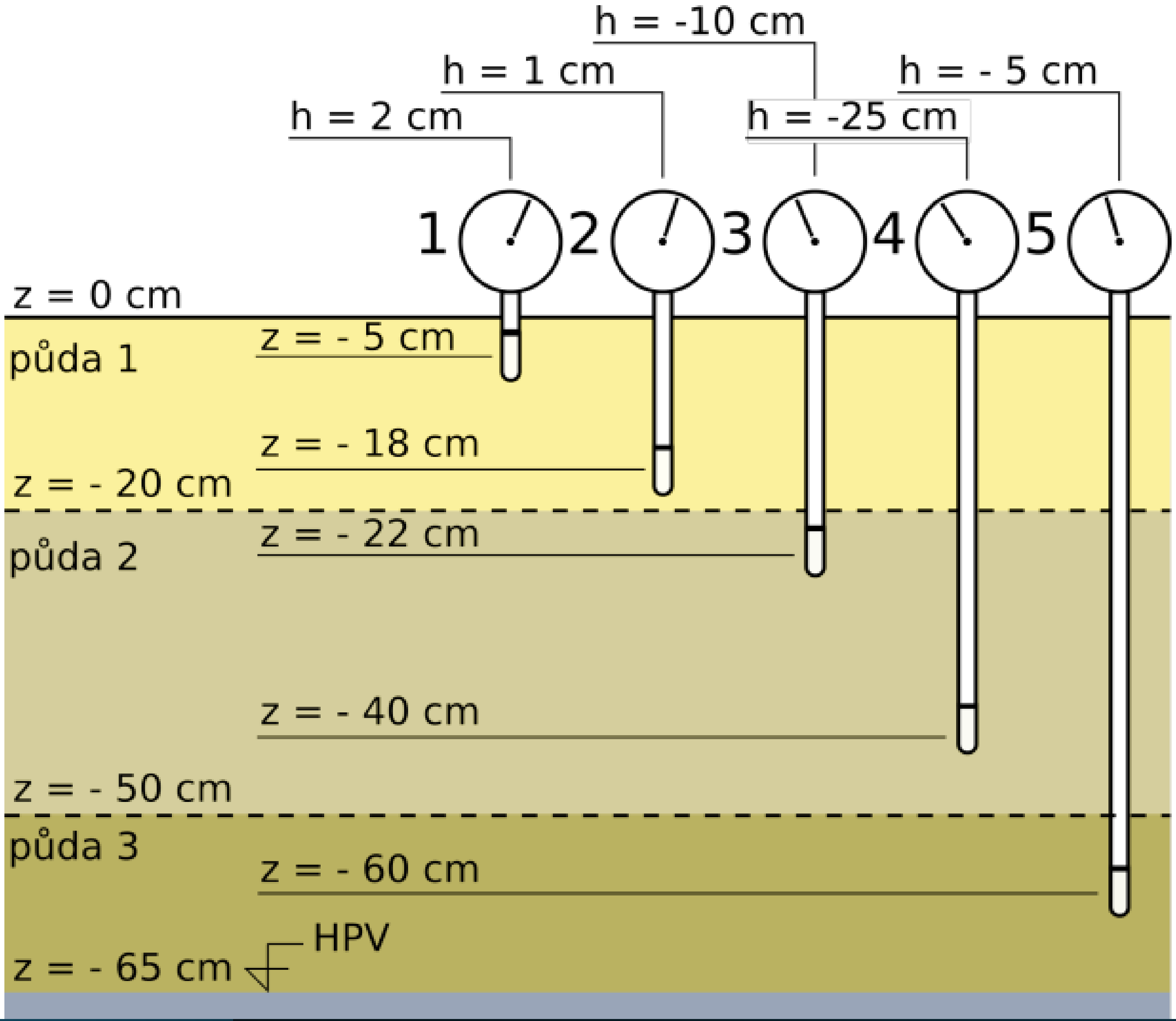
$$K(h) = K_s K_r(h)$$

Assignment

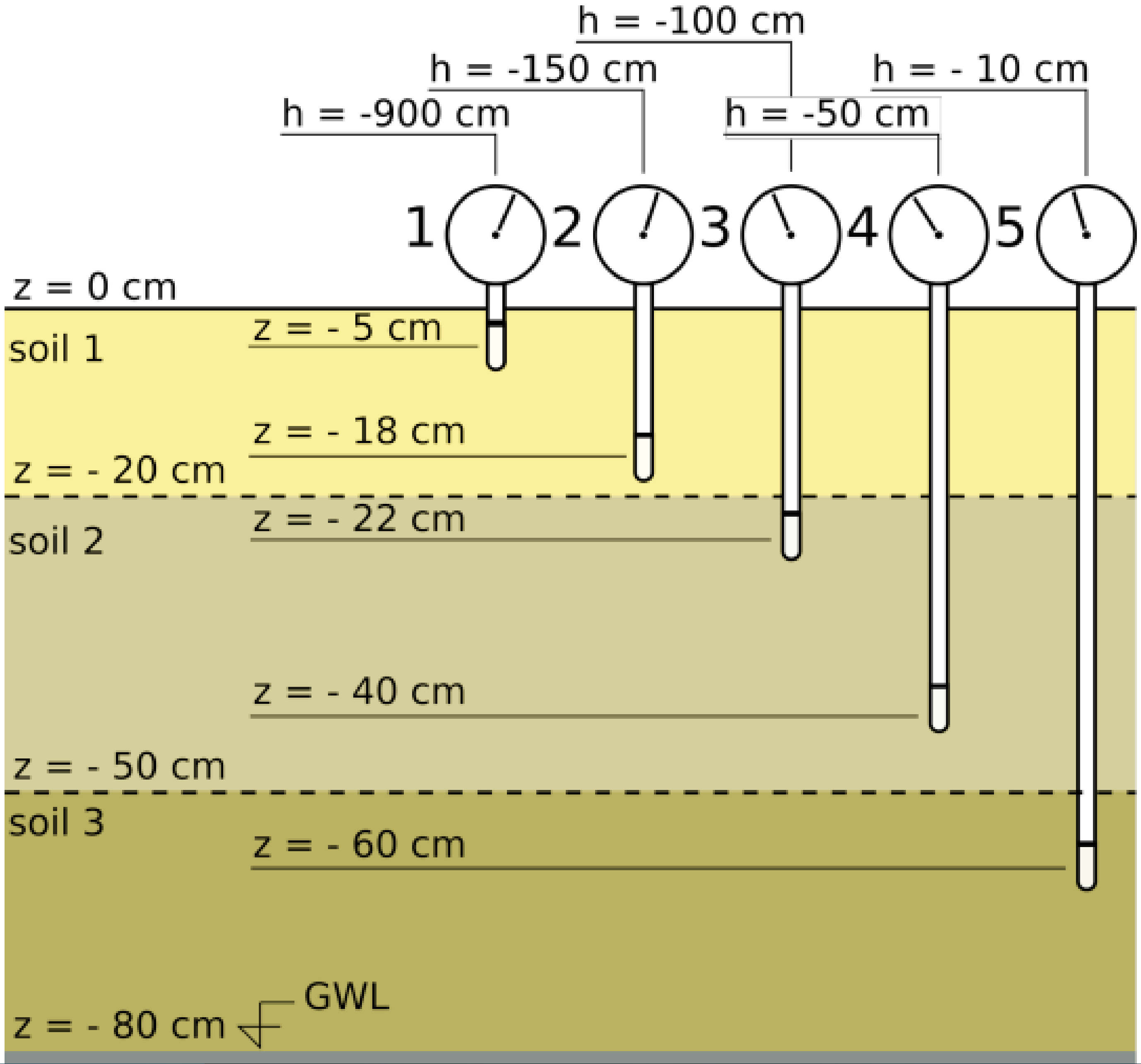
- Several tensiometers gauges were installed in the soil profile (tensiometer nest) at several depths. The soil profile is composed of three soils and the position of the water table is known. Using the van Genuchten RCs from previous problems, analyze the water regime in the soil at a given time.
- Use transiometer measurements and knowledge of the position of the water table.
- Plot pressure head, total potential, and volumetric water content as a function of depth.
- Determine the amount of water in the soil profile above the water table.

The values of the tensiometers and and their installation depths at 2 different times can be read from the figures below.

Time 1


h = -10 cm
h = 1 cm
h = - 5 cm
h = 2 cm
h = -25 cm
1
2
3
4
5
z = 0 cm
půda 1
z = - 5 cm
z = - 18 cm
z = - 20 cm
půda 2
z = - 22 cm
z = - 40 cm
z = - 50 cm
půda 3
z = - 60 cm
HPV
z = - 65 cm

Time 2



h = -100 cm
h = -150 cm
h = - 10 cm
h = -900 cm
h = -50 cm
1
2
3
4
5
z = 0 cm
soil 1
z = - 5 cm
z = - 18 cm
z = - 20 cm
soil 2
z = - 22 cm
z = - 40 cm
z = - 50 cm
soil 3
z = - 60 cm
GWL
z = - 80 cm

Formulas

Total potential

$$H = h + z \quad (1)$$

Effective water content

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (2)$$

van Genuchten retention curve (VG)

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + (\alpha|h|)^n)^m} \quad (3)$$

Evaluation of m

$$m = 1 - 1/n \quad (4)$$

Relative unsaturated hydraulic conductivity (K_r)

$$K_r(h) = \begin{cases} \frac{(1-(\alpha|h|)^{mn}(1+(\alpha|h|)^n)^{-m})^2}{(1+(\alpha|h|)^n)^{m/2}} & \text{if } h < 0 \\ 1 & \text{if } h \geq 0 \end{cases} \quad (5)$$

Inverse K_r

$$K_r(\theta_e) = \theta_e^{1/2}(1 - (1 - \theta_e^{1/m})^m)^2$$

Unsaturated hydraulic conductivity

$$K(h) = K_s K_r(h) \quad (6)$$

h capillary pressure $[L, Pa]$, θ - volumetric water content $[L^3.L^{-3}]$, θ_r - residual water content$[L^3.L^{-3}]$, θ_s - saturated water content$[L^3.L^{-3}]$, α - VG parameter $[L^{-1}]$, n - VG parameter $[-]$, m - VG parameter $[-]$, K unsaturated hydraulic conductivity $[L.t^{-1}]$, K_r relative unsaturated hydraulic conductivity $[-]$, K_s saturated hydraulic conductivity $[L.t^{-1}]$

Formulas

Darcy-Buckingham law

$$q = -K(h)\nabla H$$

Mean porous velocity

$$v = q/\theta \qquad (7)$$

Evaluate of Darcian flow between measuring depths i and $i+1$

$$q_{i+1/2} = K_{i+1/2}\nabla H_{i+1/2} \qquad (8)$$

Evaluate the derivative with difference for $i+1/2$ depth

$$\nabla H_{i+1/2} \approx \frac{H_i - H_{i-1}}{z_i - z_{i-1}}$$

Averaging of K between depths i and $i+1$

$$K_{i+1/2} = (K_i(h_i) + K_{i+1}(h_{i+1}))/2$$

geometrical, harmonic or weighted mean is more precise in complex problems

q Darcian flow $[L.t^{-1}]$, v mean porous velocity$[L.t^{-1}]$, H total potential $[L, Pa]$, z depth $[L]$